GCE

Further Mathematics A

Y541/01: Pure Core 2

A Level

Mark Scheme for June 2022

Oxford Cambridge and RSA Examinations

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. Annotations and abbreviations

Annotation in RM assessor	Meaning
✓and✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. Subject-specific Marking Instructions for A Level Mathematics A

a Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and applied in a manner which shows thatthe method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.

- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.**unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.
Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g Rules for replaced work and multiple attempts:

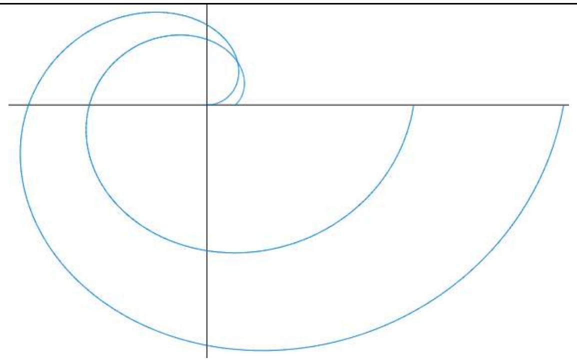
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.
If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, providedthat there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AO	Guidance	
1	(a)		$(3\mathbf{i} - 5\mathbf{j} - \mathbf{k})\times(\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}) = 23\mathbf{i} + 11\mathbf{j} + 14\mathbf{k}$	**B1** **[1]**	1.1	or any non-zero multiple . ISW	
	(b)		x: $2 + \lambda = 1 - \mu$ y: $3 - 2\lambda = 11 + 3\mu$ or z: $3 + \lambda = -4 - 2\mu$ eg $4 + 2\lambda = 2 - 2\mu \Rightarrow 7 = 13 + \mu$	**M1** **M1**	1.1 1.1	Any correct numeric equation Using any 2 equations to eliminate either λ or μ.	 Must indicate which equations are used to find λ and μ.
			$\mu = -6$, $\lambda = 5$	**A1**	1.1		
			eg z: LHS = 3 + 5 = 8 RHS = –4 – 2×(–6) = –4 – (–12) = 8	**B1FT**	1.1	Check in unused (or all) equation(s) FT their values of λ and μ provided that LHS = RHS.	Calculations must be correct and answer must be evaluated. LHS = 8, RHS = 8 is insufficient; some substitution must be evident.
			PoI is (7, –7, 8)	**B1** **[5]**	1.1	Condone as (position) vector	

Question			Answer	Marks	AO	Guidance	
2	(a)		At A $\theta = 2\pi$ so A[..., 2π] r = $2\theta = 2\times 2\pi = 4\pi$ so A[4π, ...]	**B1** **B1** **[2]**	2.2a 1.1	or just $\theta = 2\pi$. ISW or just r = 4π. ISW	
	(b)		At PoI $2\theta = \theta + 1$ $=> \theta = 1$ so [2, 1]	**M1** **A1** **[2]**	1.1 1.1	Correct condition for PoI or r = 2 and $\theta = 1$. ISW	
	(c)			**B1** **[1]**	1.1	C_2 drawn as a smooth curve spiralling out from [1, 0] outside C_1 until a single point in the 1st quadrant and then inside C_1. Must stop on initial line.	Start at [1, 0]. Intersection in 1st quadrant (by eye). r increasing (by eye). Reaches initial line and stops. Ignore labels.

Question			Answer	Marks	AO	Guidance	
3			DR $\sum\alpha=-\frac{6}{4}=-\frac{3}{2},\ \sum\alpha\beta=\frac{-3}{4},\ \alpha\beta\gamma=-\frac{9}{4}$	**B1**	1.1		
			$\sum\alpha'=\alpha+\beta+\beta+\gamma+\gamma+\alpha=2(\alpha+\beta+\gamma)=-3$	**B1**	1.1	(ie sum of new roots = –3)	or $\frac{b}{a}=3$
			$\sum\alpha'\beta'=$ $(\alpha+\beta)(\beta+\gamma)+(\beta+\gamma)(\gamma+\alpha)+(\gamma+\alpha)(\alpha+\beta)$ $=\alpha\beta+\alpha\gamma+\beta\gamma+\beta^2+\beta\gamma+\beta\alpha$ $+\gamma\alpha+\gamma^2+\gamma\alpha+\gamma\beta+\alpha\beta+\alpha^2$ $=\alpha^2+\beta^2+\gamma^2+3(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=\alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha)+(\alpha\beta+\beta\gamma+\gamma\alpha)$ $=(\alpha+\beta+\gamma)^2+\alpha\beta+\beta\gamma+\gamma\alpha$ or $\alpha'\beta'\gamma'=(\alpha+\beta)(\beta+\gamma)(\gamma+\alpha)$ $=(\alpha\beta+\alpha\gamma+\beta^2+\beta\gamma)(\gamma+\alpha)$ $=\left(\sum\alpha\beta+\beta^2\right)(\gamma+\alpha)$ $=(\gamma+\alpha)\sum\alpha\beta+\beta^2(\gamma+\alpha)$ $=(\alpha+\beta+\gamma)\sum\alpha\beta-\beta\sum\alpha\beta+\beta(\alpha\beta+\beta\gamma)$ $=\sum\alpha\sum\alpha\beta-\beta\sum\alpha\beta+\beta(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma$ $=\sum\alpha\sum\alpha\beta-\alpha\beta\gamma$	**M1**	1.1	Correctly writing either $\alpha'\beta'\gamma'$ or $\Sigma\alpha'\beta'$ in a form involving the basic symmetrical forms	
			$\therefore\sum\alpha'\beta'=\left(-\frac{3}{2}\right)^2+\frac{-3}{4}=\frac{6}{4}=\frac{3}{2}$	**A1**	1.1		
			& $\alpha'\beta'\gamma'=-\frac{3}{2}\times\frac{-3}{4}-\frac{-9}{4}=\frac{9}{8}+\frac{18}{8}=\frac{27}{8}$	**A1**	1.1		

			$8u^3 + 24u^2 + 12u - 27 = 0$	A1	2.5	Must be an equation with integer coefficients. Allow use of any unknown.

Question			Answer	Marks	AO	Guidance	
			Alternative method:				
			$u = \sum\alpha - x$	B1		Can be implied by 2nd **B1**.	
			$u = -\frac{3}{2} - x$	B1			
			$x = -\frac{3}{2} - u$				
			$4\left(-\frac{3}{2} - u\right)^3 + 6\left(-\frac{3}{2} - u\right)^2 - 3\left(-\frac{3}{2} - u\right) + 9 = 0$	M1		Rearrange and substitute into original equation	
			$-\frac{27}{2} - 27u - 18u^2 - 4u^3 + \frac{27}{2} + 18u + 6u^2 + \frac{27}{2} + 3u = 0$				
			$8u^3 + 24u^2 + 12u - 27 = 0$	A3		**A3** for fully correct equation in the correct form.	Award **A1** only for coefficients of u^3 and one other correct in any form Award **A2** only for coefficients of u^3 and two others correct in any form
				[6]			
4			DR $\frac{1^2 + 2^2 + \ldots + n^2}{1 + 2 + \ldots + n} = \frac{\frac{1}{6}n(n+1)(2n+1)}{\frac{1}{2}n(n+1)}$	M1	3.1a	Identifying the two series and quoting the standard series results	
			$= \frac{2n+1}{3} \Rightarrow \frac{2n+1}{3} > 341$ oe	M1	2.2a	Correctly cancelling to set up inequality with two layer fraction(s) (NB **M0** for ≥ 342).	If cubic inequality formed then it must be factorised before **M1**...

			$\therefore n > 511$ $\therefore n_{min} = 512$	**A1** **A1** **[4]**	1.1 3.2a	 512 from equation rather than inequality then **M1M1A0A1**	...and must be fully solved for **A1**.

Question			Answer	Marks	AO	Guidance	
5	**(a)**		$\text{RHS} = \cosh^2 x + \sinh^2 x$ $= \left(\dfrac{e^x + e^{-x}}{2}\right)^2 + \left(\dfrac{e^x - e^{-x}}{2}\right)^2$ $= \dfrac{1}{4}\left(e^{2x} + e^{-2x} + 2 + e^{2x} + e^{-2x} - 2\right)$ $= \dfrac{1}{4}\left(2e^{2x} + 2e^{-2x}\right) = \dfrac{1}{2}\left(e^{2x} + e^{-2x}\right)$ = cosh 2x = LHS	**M1** **A1** **[2]**	2.1 1.1	Using definitions of cosh x **and** sinh x. AG. Intermediate working must be seen.	
	(b)		$\cosh^2 x + \sinh^2 x = \cosh 2x$ & $\cosh^2 x - \sinh^2 x = 1$ => $2\cosh^2 x = \cosh 2x + 1$ => $\cosh 2x = 2\cosh^2 x - 1$	**B1** **[1]**	2.2a		
	(c)		$10\cosh^2 x - 5 = 16\cosh x + 21$ => $10c^2 - 16c - 26 = 0$ => $5c^2 - 8c - 13 = 0$ $c = -1$ rejected since $\cosh x \geq 1$ (or ≥ 0 oe) or $c = 13/5$ $\cosh^{-1}\dfrac{13}{5} = \ln\left(\dfrac{13}{5} + \sqrt{\left(\dfrac{13}{5}\right)^2 - 1}\right) = \ln 5$ $\therefore x = \pm\ln 5$	**M1** **A1** **M1** **A1**	1.1 2.3 1.1 2.2a	Using the identity from **(b)** to reduce equation to 3 term quadratic Both solutions found… … and –1 rejected explicitly with valid reason. E.g "–1 is outside the range of cosh x" Use of formula for $\cosh^{-1}$ (or by solving quadratic in e^x). x = ln5 or ln(1/5). Must have both solutions.	 Could be BC

Question			Answer	Marks	AO	Guidance	
			Alternative method: $\frac{5}{2}\left(e^{2x}+e^{-2x}\right)=8\left(e^{x}+e^{-x}\right)+21$ $5e^{4x}-16e^{3x}-42e^{2x}-16e^{x}+5=0$	**M1**		Using the exponential definition of cosh to reduce the given equation to a quartic equation in e^x.	
			$e^x = e \Rightarrow$ $5e^3(e-5)+9e^2(e-5)+3e(e-5)-(e-5)=0$ $(e-5)(5e^3+9e^2+3e-1)=0$ $(e-5)(e^2(5e-1)+2e(5e-1)+(5e-1))=0$ $(e-5)(5e-1)(e^2+2e+1)=0$ $(e-5)(5e-1)(e+1)^2=0$ $\therefore e=-1,\ \frac{1}{5}$ or 5	**M1**		Factorising or using quartic solver.	
			e = –1 => e^x = –1 which is not possible since e^x > 0 for all (real) x.	**A1**		Negative solution must be rejected and a valid reason given.	
			So e^x = 5 or 1/5 => x = ln5 or ln(1/5)	**A1**		x = ±ln5. Must have both solutions.	
				[4]			

Question			Answer	Marks	AO	Guidance	
6	(a)		$x = A\sin\omega t + B\cos\omega t$ or $R\cos(\omega t + \phi)$ or $R\sin(\omega t + \phi)$	B1 [1]	1.2	Correct form with 2 arbitrary constants. Must be "x =". Do not ISW; consider final answer as GS unless explicitly labelled otherwise.	Candidates may derive GS from, eg, auxiliary equation but GS must be in real form.
	(b)		$t = 0, x = 0 \Rightarrow B = 0$ (so $x = A\sin\omega t$) Stops when $x = d \Rightarrow A = d$ so $x = d\sin\omega t$ $v = \omega d\cos\omega t$	M1 A1 A1 [3]	3.3 3.4 2.2a	Using one boundary condition (may be seen in **(a)**). Using other boundary condition	
	(c)		$\text{RHS} = \omega^2(d^2 - x^2) = \omega^2(d^2 - d^2\sin^2\omega t)$ $= \omega^2 d^2(1-\sin^2\omega t) = \omega^2 d^2\cos^2\omega t$ $= (\omega d\cos\omega t)^2 = v^2 = \text{LHS}$	B1 [1]	3.4	AG. Sufficient working must be shown.	
	(d)		$z_m = \frac{1}{d-0}\int_0^d z\,dx = \frac{1}{d}\int_0^d \frac{1}{v}dx$ $= \frac{1}{d}\int_0^d \frac{1}{\omega\sqrt{d^2-x^2}}dx$ $= \frac{1}{\omega d}\left[\sin^{-1}\frac{x}{d}\right]_0^d = \frac{1}{\omega d}\left[\sin^{-1}1 - \sin^{-1}0\right]$ $= \frac{\pi}{2\omega d}$	M1 A1 [2]	3.3 3.4	Use of mean formula, over x, with correct limits and z substituted	
	(e)		Using $d = 0.25$ and $\omega = 0.75$ leads to $z_m = 8.38...$, which suggests that the model is valid, but d could be as high as 0.255 and ω as high as 0.77 which would lead to $z_m = 7.99998$. So it is highly likely that the model is valid (although just possible that it is not).	B1 [1]	2.2b	Indication that for most, but not all, of the possible combinations of values of d and ω the value of z_m exceeds 8.	$7.99998129... < z_m < 8.782758327...$
	(f)		Initial velocity = $\omega d\cos 0$ (or ωd) so $u_{min} = 0.73 \times 0.245 = 0.17885$ so $0.18\ \text{ms}^{-1}$ cao	M1 A1	3.4 3.4	Putting t = 0 in expression for v Must include units.	Or x = 0 in expression for v^2.

				Marks	AO	Guidance	
				[2]			

Question			Answer	Marks	AO	Guidance	
7	(a)		det **A** $=2((10-4a)\times4-9\times4)-4(-3\times4-9\times7)$ $+(-6)(-3\times4-(10-4a)\times7)$ $=2(40-16a-36)-4(-12-63)$ $-6(-12-70+28a)$ $=8-32a+300+492-168a$ $=800-200a$ or $200(4-a)$	**M1** **A1** **[2]**	1.1 1.1	Expanding the determinant ISW once all a terms and all number terms collected	Condone one **calculation** error
	(b)	(i)	$\begin{pmatrix}4-16a & 75 & 28a-82\\ -40 & 50 & 20\\ 96-24a & 0 & 32-8a\end{pmatrix}$ $\begin{pmatrix}4-16a & -40 & 96-24a\\ 75 & 50 & 0\\ 28a-82 & 20 & 32-8a\end{pmatrix}$ $\mathbf{A}^{-1}=\frac{1}{800-200a}\begin{pmatrix}4-16a & -40 & 96-24a\\ 75 & 50 & 0\\ 28a-82 & 20 & 32-8a\end{pmatrix}$ $\mathbf{A}^{-1}\begin{pmatrix}6\\-9\\11\end{pmatrix}$ $=\frac{1}{\Delta}\begin{pmatrix}4-16a & -40 & 96-24a\\ 75 & 50 & 0\\ 28a-82 & 20 & 32-8a\end{pmatrix}\begin{pmatrix}6\\-9\\11\end{pmatrix}$ $=\frac{1}{800-200a}\begin{pmatrix}1440-360a\\0\\-320+80a\end{pmatrix}$	***M1** **dep*M1** **A1FT** **M1**	1.1 1.1 1.1 1.1	Correctly finding at least 5 cofactors or minors (need not be in matrix) Transposing and changing signs in correct way FT their determinant Forming correct product and attempting multiplication, resulting in a vector	

Question			Answer	Marks	AO	Guidance	
			$=\frac{1}{200(4-a)}\begin{pmatrix}360(4-a)\\0\\-80(4-a)\end{pmatrix}$ $=\frac{40(4-a)}{200(4-a)}\begin{pmatrix}9\\0\\-2\end{pmatrix}=\frac{1}{5}\begin{pmatrix}9\\0\\-2\end{pmatrix}=\begin{pmatrix}\frac{9}{5}\\0\\-\frac{2}{5}\end{pmatrix}$ So $x=\frac{9}{5},\ y=0,\ z=-\frac{2}{5}$	**A1** **[5]**	1.1	Simplification to correct numerical solution (can be left as an unmultiplied vector)	**i.e. A1** can be awarded for $\begin{pmatrix}\frac{9}{5}\\0\\-\frac{2}{5}\end{pmatrix}$
	(b)	**(ii)**	Singular when det**A** = 0 => a = 4 so the second equation is $-3x-6y+9z=-9$... ...which has the same normal (direction) as the first equation and is consistent with it... oe ...so the first two planes are identical and the third intersects them in a line.	**M1** **M1** **A1** **[3]**	2.1 1.1 2.4	Finding a from det**A** = 0 and subbing in to 2nd equation. Could just find the appropriate normal eg The second equation is a multiple of the first.	
	(c)	**(i)**	Orientation reversed if det**A** < 0 so $200(4-a)<0$ so $a>4$	**B1FT** **[1]**	3.1a	Must be strict inequality	FT their expression for determinant if linear function of a.
	(c)	**(ii)**	Image volume smaller than object volume => $-1<\det\mathbf{A}<1$... ... but $a\neq 4$ so $\frac{799}{200}<a<4$ or $4<a<\frac{801}{200}$ oe	**M1** **A1** **[2]**	3.1a 3.2a	Understanding that $\lvert\det\mathbf{A}\rvert$ represents the volume scale factor and so $\lvert\det\mathbf{A}\rvert<1$... ...and that $a\neq 4$. Must be strict inequalities	$\frac{799}{200}<a<\frac{801}{200}$ and $a\neq 4$

Question			Answer	Marks	AO	Guidance	
8			DR				
			$\frac{5r+2}{r(r+1)(r+2)}=\frac{A}{r}+\frac{B}{r+1}+\frac{C}{r+2}$	**M1**	3.1a	Correct form for partial fractions	
			$=\frac{1}{r}+\frac{3}{r+1}-\frac{4}{r+2}$	**A1**	1.1	or A = 1, B = 3, C = –4	
			$\sum_{r=k}^{98}\frac{5r+2}{r(r+1)(r+2)}=\sum_{r=k}^{98}\left(\frac{1}{r}+\frac{3}{r+1}-\frac{4}{r+2}\right)$ $=\frac{1}{k}\quad+\frac{3}{k+1}-\frac{4}{k+2}$ $+\frac{1}{k+1}+\frac{3}{k+2}-\frac{4}{k+3}$ $+\frac{1}{k+2}+\frac{3}{k+3}-\frac{4}{k+4}$ $+\ldots$ $+\frac{1}{96}+\frac{3}{97}-\frac{4}{98}$ $+\frac{1}{97}+\frac{3}{98}-\frac{4}{99}$ $+\frac{1}{98}+\frac{3}{99}-\frac{4}{100}$	**M1**	3.1a	Writing out sufficient terms so that cancellation pattern becomes evident. Could see N rather than 98.	If use r = a to N then **M1** awarded when N = 98 and N = k – 1 both considered.
			$=\frac{1}{k}+\frac{4}{k+1}-\frac{124}{2475}$ or $\frac{1}{k}+\frac{4}{k+1}-\frac{1}{99}-\frac{4}{100}$ oe	**A1**	2.2b	Could see $\frac{5k+1}{k(k+1)}-\frac{124}{2475}$	
			$\frac{1}{k}+\frac{4}{k+1}-\frac{124}{2475}=\frac{20539}{34650}\Rightarrow\frac{1}{k}+\frac{4}{k+1}=\frac{9}{14}$ $\Rightarrow 14(k+1)+56k=9k(k+1)$ $\Rightarrow 9k^2-61k-14=0$	**M1**	1.1	Equating their expression for LHS to 20539/34650 and rearranging to 3 term quadratic	

Question			Answer	Marks	AO	Guidance	
			$(9k+2)(k-7)=0 \Rightarrow k=-\frac{2}{9}$ or 7	**A1**	1.1	$\frac{--61\pm\sqrt{(-61)^2-4\times 9\times(-14)}}{2\times 9}$ $=\frac{61\pm\sqrt{4225}}{18}=\frac{61\pm 65}{18}$	or $9\left(\left(k-\frac{61}{18}\right)^2-\left(\frac{61}{18}\right)^2\right)-14=0$ $9\left(k-\frac{61}{18}\right)^2-\frac{4225}{36}=0$
			But k is an integer so $k \neq -\frac{2}{9}$ so $k=7$	**A1** **[7]**	3.2a	Explicitly rejecting non-integer value, with reason, and deducing correct value... ...or stating that the original function is undefined if r is allowed to take the value 0.	NB $k \geq 0$ (or $k > 0$) alone is **not** a valid reason to reject –2/9
9	**(a)**		DR $e^{i\theta}+e^{-i\theta}=2\cos\theta$ oe $e^{4i\theta}=\cos 4\theta+i\sin 4\theta$ $\mathrm{Re}\left(e^{4i\theta}\left(e^{i\theta}+e^{-i\theta}\right)^4\right)$ $=\mathrm{Re}\left((\cos 4\theta+i\sin 4\theta)(2\cos\theta)^4\right)$ $=\cos 4\theta\times(2\cos\theta)^4$ $=16\cos 4\theta\cos^4\theta$ (so a = 16)	**B1** **M1** **A1** **[3]**	2.1 1.1 2.2a	 Use of DMT Must equate terms to complete demonstration	

Question		Answer	Marks	AO	Guidance	
	(b)	DR $\left(e^{i\theta}+e^{-i\theta}\right)^4=\left(e^{i\theta}\right)^4+4\left(e^{i\theta}\right)^3e^{-i\theta}$ $+6\left(e^{i\theta}\right)^2\left(e^{-i\theta}\right)^2+4e^{i\theta}\left(e^{-i\theta}\right)^3+\left(e^{-i\theta}\right)^4$	*M1	3.1a	Expands correct brackets using binomial theorem. Terms can be unsimplified but must have correct numerical coefficients.	eg $\left(e^{2i\theta}+1\right)^4$ or $\left(z+z^{-1}\right)^4$ or $\left(\frac{\sqrt{3}}{2}+1+\frac{1}{2}i\right)^4$ etc if expansion seen in **9(a)**, must be used in **9(b)** to gain mark here
		$\therefore e^{4i\theta}\left(e^{i\theta}+e^{-i\theta}\right)^4$ $=e^{8i\theta}+4e^{6i\theta}+6e^{4i\theta}+4e^{2i\theta}+1$ $e^{8i\theta}=\cos 8\theta+i\sin 8\theta$ or $\mathrm{Re}\left(e^{8i\theta}\right)=\cos 8\theta$ etc	dep*M1	1.1	Use of Euler's formula to convert exponential form to trigonometric form	
		$\therefore 16\cos 4\theta\cos^4\theta=$ $\cos 8\theta+4\cos 6\theta+6\cos 4\theta+4\cos 2\theta+1$	dep*M1	1.1	Taking real parts	
		$\theta=\frac{\pi}{12}\Rightarrow 16\cos\frac{\pi}{3}\cos^4\frac{\pi}{12}=$ $\cos\frac{2\pi}{3}+4\cos\frac{\pi}{2}+6\cos\frac{\pi}{3}+4\cos\frac{\pi}{6}+1$	dep*M1	3.1a	Choice of θ soi and substituted into identity with their 16.	
		$\Rightarrow\cos\frac{\pi}{12}=$ $\sqrt[4]{\frac{1}{8}\left(-\frac{1}{2}+4\times 0+6\times\frac{1}{2}+4\times\frac{\sqrt{3}}{2}+1\right)}$	dep*M1	1.1	Gives correct numerical values to all $\cos\frac{n\pi}{6}$ terms. Also dependent on use of Euler's formula and choice of θ.	
		$=\sqrt[4]{\frac{1}{8}\left(\frac{7}{2}+2\sqrt{3}\right)}=\sqrt[4]{\frac{1}{16}\left(7+4\sqrt{3}\right)}$ $=\frac{1}{\sqrt[4]{16}}\sqrt[4]{7+4\sqrt{3}}=\frac{1}{2}\sqrt[4]{7+4\sqrt{3}}$	A1 **[6]**	2.2a	So b = 7 and c = 4 (can be embedded). cao.	

10			$\mathbf{AB}=\begin{pmatrix}10\\12\\10\end{pmatrix}$	*M1		Finding **AB** or an expression for $p_B - p_A$ or $d_B - d_A$	from $\mathbf{r.n} = p_A = 3a - 2a - c$ and $\mathbf{r.n} = p_B = 13a + 10a + 9c$ or $\mathbf{r.\hat{n}} = d_A$ and $\mathbf{r.\hat{n}} = d_B$ ie $22a + 10c$
			$\pm 2 = \text{AB}\cos\theta = \frac{\lvert\mathbf{AB.n}\rvert}{\lvert\mathbf{n}\rvert}$ with **AB** from above used	dep*M1		Use of dot product to express correct shortest distance in terms of **AB** and the normal to the planes or seeing an appropriate difference between plane constants	or $d_B - d_A = \pm 2$ or $\frac{p_B}{\sqrt{a^2+b^2+c^2}} - \frac{p_A}{\sqrt{a^2+b^2+c^2}} = \pm 2$ allow omission of $\pm$
			$\mathbf{n}=\begin{pmatrix}a\\a\\c\end{pmatrix}$ or $\mathbf{n}=\begin{pmatrix}1\\1\\s\end{pmatrix}$ oe	*M1		$\begin{pmatrix}a\\a\\c\end{pmatrix}$ used consistently	
			$119a^2 + 110ac + 24c^2 = 0$	A1		Quadratic formed	Or $171a^2 \mp 22a - 24 = 0$ $171c^2 \mp 20c - 119 = 0$ $24s^2 + 110s + 119 = 0$
			$(7a+4c)(17a+6c)=0$ e.g. $a = 4, c = -7 \Rightarrow \mathbf{n}_1 = \begin{pmatrix}4\\4\\-7\end{pmatrix}$	dep*M1		Using one solution of quadratic to obtain a normal of one of the solution planes.	Dependent on all previous M marks
			e.g. $a = 6, c = -17 \Rightarrow \mathbf{n}_2 = \begin{pmatrix}6\\6\\-17\end{pmatrix}$	A1		Both correct.	
			$\begin{pmatrix}4\\4\\-7\end{pmatrix} . \begin{pmatrix}6\\6\\-17\end{pmatrix} = 167$	Dep*M1		Finding the dot product of their solution normals.	Dependent on all previous M marks
			$\cos\theta = \frac{167}{9\times 19} = \frac{167}{171}$ $\Rightarrow$ acute angle is awrt 12.4°	A1 **(8)**		or awrt 0.217 rads	

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