Oxford Cambridge and RSA

GCE

Further Mathematics B (MEI)

Y431/01: Mechanics minor

Advanced GCE

Mark Scheme for Autumn 2021

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Question			Answer	Marks	AOs	Guidance	
1	(a)		MLT^{-2}	B1	1.2		
				[1]			
	(b)		$[RHS] = \frac{M(LT^{-1})^2}{L}$	M1	3.4	Using given formula with $[m] = M$ and $[r] = L$ and $[v] = LT^{-1}$	
			$= ML^2T^{-2}L^{-1} = MLT^{-2} = [LHS]$	A1	2.2a	must see $(LT^{-1})^2$ expanded	
				[2]			
	(c)		$1.1^2 \div 0.9$	M1	1.1	Using correct formula with 1.1 and 0.9	
			$= 1.34444\ldots$ so 34.4%	A1	2.2b		34.44444…
				[2]			
	(d)		$1016 \times 1609 \div 60^2$	M1	1.1	Condone denominator which is not squared e.g 60 or 60^3 for the M mark	
			$= 454$ N	A1	1.1		454.0955…
				[2]			

Question			Answer	Marks	AOs	Guidance	
2			Let the components of the force at A be F_A (parallel to floor) and R_A (parallel to wall).				
			$F_A = 20$	B1	1.1		
			$R_A = \sqrt{70^2 - 20^2} = \sqrt{4500} = 30\sqrt{5}$	M1	3.1b	Using Pythagoras to find the normal contact force at A	
			$\Rightarrow W = R_A = 30\sqrt{5}$ or 67	A1	1.1	Accept exact or to at least 2 sf	67.082039…
			$\mu = \frac{F_A}{R_a} = \frac{20}{30\sqrt{5}} = \frac{2}{3\sqrt{5}} = \frac{2\sqrt{5}}{15}$ or 0.30	B1	3.4	Using $F = \mu R$ - accept any equivalent exact form or 0.30 (2 sf or better)	0.29814… $\frac{2\sqrt{5}}{15}$
			e.g. Taking moments about A: $Wa\cos\theta = F_B 2a\sin\theta$	M1*	3.3	Taking moments about A (or B etc.) – correct number of terms. Allow cos/sin errors but must reflect ratio of distances.	
			$\Rightarrow \tan\theta = \frac{3}{4}\sqrt{5}$	M1dep*	1.1	Substituting $F_A = 20$ and their value for W and then obtain a value for tan	
			$\Rightarrow \theta = 59°$	A1	1.1	2 sf or better	59.19301…
				[7]			

Question			Answer	Marks	AOs	Guidance	
3							
			$P = 65gv$	**B1**	**3.1b**	Use of $P = Fv$ (either one)	
			$P = 40g(v+3)$	**B1**	**1.1**		
			$65gv = 40g(v+3)$	**M1**	**3.4**	Equating their two expressions for P – with at least one correct equation	
			$v = 4.8\ ms^{-1}$	**A1**	**1.1**		
			$P = 3060\,W$	**A1**	**1.1**	or 3.06 kW (but must state kW in this case)	3057.6
				[5]			

Question			Answer	Marks	AOs	Guidance	
4	(a)		$\frac{1}{2}m \times 7.2^2 = mgh$	M1	3.4	Must use energy method as directed in question.	
			$h = 2.64489\ldots$ so maximum height is 4.24 m	A1	1.1		$4.2448979\ldots$
				[2]			
	(b)		Let the amount of work done per metre against air resistance be W				
			$\frac{1}{2}m \times 7.2^2 - 2.5W = 2.5mg$	M1	3.1b	Work-energy principle. All three terms present. Condone sign errors.	
			$W = 0.568m$	A1	1.1	**AG** – sufficient working must be shown	
				[2]			
	(c)		Let v be the speed of the ball just before impact with ground				
			$4.1mg - 4.1W = \frac{1}{2}mv^2$	M1 B1	3.3 1.1	Work-energy principle: all three terms present (or all four if starting from when ball leaves the hand). $4.1W$ with $W = 0.568m$	
			$\frac{1}{2}mv^2 = 37.8512m \Rightarrow v = 8.70\,\text{ms}^{-1}$	A1	1.1		$8.7007126\ldots$
				[3]			
	(d)		Let V be the speed of the ball just after impact with ground.				
			$\frac{1}{2}mV^2 - 2.8W = 2.8mg$	M1	3.3	Work-energy principle - all three terms present	
			$V = 7.6197\ldots$	A1	1.1		
			Coefficient of restitution = $\frac{7.6197\ldots}{8.7007\ldots} = 0.876$	A1ft	3.4	FT their answer to **(c)**	$0.87576318\ldots$
				[3]			
	(e)		$-mv + 12 = mV$	M1	3.3	Using impulse = change in momentum. Correct number of terms but allow sign errors. FT their values for v and V	
			$m = \frac{12}{v+V} = 0.735$	A1	1.1		
				[2]			

Question			Answer	Marks	AOs	Guidance	
5	(a)		Let the coordinates of centre of mass be $(\bar{x}, \bar{y}, \bar{z})$				
			$\bar{z} = 1.5$	B1	1.1		
			$210\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 120\begin{pmatrix}2\\5\end{pmatrix} + 90\begin{pmatrix}9\\8.5\end{pmatrix}$	M1	1.1	Any correct equation, using correct ratio of masses of the constituent parts.	e.g. could also have $28\begin{pmatrix}2\\3.5\end{pmatrix}+42\begin{pmatrix}7\\8.5\end{pmatrix}$
			$\bar{x} = 5$	A1	1.1		
			$\bar{y} = 6.5$	A1	1.1		
				[4]			
	(b)		$\theta_{\min} = \arctan\frac{1}{6.5}$ or $\theta_{\max} = \arctan\frac{5}{6.5}$	M1	3.1b	$\tan\theta = \frac{1}{\bar{y}}$ or $\tan\theta = \frac{\bar{x}}{\bar{y}}$ - condone reciprocal fractions for this mark	
			$\theta_{\min} = 8.75$	A1	1.1		8.746162…
			$\theta_{max} = 37.6$	A1	1.1		37.568592…
				[3]			
	(c)		Let the thresholds for breaking equilibrium be sliding and toppling be P_s and P_t				
			$R + P_s\sin 30° = mg \Rightarrow R = mg - P_s\sin 30°$	M1*	3.3	Resolve vertically – correct number of terms (allow sin/cos errors)	
			$P_s\cos 30° = F_{max} = \mu(mg - P_s\sin 30°)$	M1*	3.4	Resolve horizontally and use of $F = \mu R$	
			$P_s = \dfrac{\mu mg}{\cos 30° + \mu\sin 30°}$	A1	1.1	oe	
			$14P_t\sin 30° + 10P_t\cos 30° = 5mg$	M1*	3.1b	Moment – correct number of terms. Condone sin/cos errors (and sign errors)	
			$P_t = \dfrac{5mg}{14\sin 30° + 10\cos 30°}$	A1	1.1	oe	

Question			Answer	Marks	AOs	Guidance	
			$P_s > P_t$, so $\dfrac{\mu mg}{\cos 30° + \mu \sin 30°} > \dfrac{5mg}{14\sin 30° + 10\cos 30°}$ $\mu mg(14\sin 30° + 10\cos 30°) > 5mg(\cos 30° + \mu \sin 30°)$ $\mu(9\sin 30° + 10\cos 30°) > 5\cos 30°$	**M1dep***	**2.1**	Dependent on all previous M marks	
			$\mu > \dfrac{5}{9\tan 30° + 10}$ $\mu > \dfrac{5}{3\sqrt{3}+10}$ $So\ \mu_{min} = \dfrac{5}{3\sqrt{3}+10} = \dfrac{50-15\sqrt{3}}{73}$	**A1**	**2.2a**	Accept any equivalent exact form, or 0.329 (or better) Need not be explicitly stated if inequality is present.	0.3290306…
				[7]			
	(d)		Either				
			If the angle. θ, were smaller then $\tan\theta$ would be smaller …	**M1**	**2.1**		
			… so μ_{min} would be larger.	**A1**	**2.2a**		
			Or				
			If the angle were smaller then P would have a larger horizontal component **and** a smaller anticlockwise turning effect …	**M1**	**2.1**		
			… so μ_{min} would be larger.	**A1**	**2.2a**		
				[2]			

Question			Answer	Marks	AOs	Guidance	
6	(a)		Let the block move x metres before coming to rest.				
			$\frac{1}{2}mv^2 - Fx = 0$	M1	1.1	Work-energy principle – correct number of terms	
			Since block is sliding, $F = F_{max} = \mu mg$	M1	3.4	Use of $F = \mu R$	
			$\frac{1}{2}mv^2 - \mu mgx = 0 \Rightarrow x = \frac{v^2}{2\mu g}$	A1	1.1	N.B. answer given.	
			Alternative method:				
			Since block is sliding, $F = F_{max} = \mu mg$	M1	3.4		
			$a = -\frac{\mu mg}{m} = -\mu g$	M1	1.1		
			So $0^2 = v^2 + 2\ (-\mu g)\ x \Rightarrow x = \frac{v^2}{2\mu g}$	A1	1.1		
				[3]			
	(b)		$mu = mv_S + 8mv_B$	M1*	3.3	Conservation of linear momentum – correct number of terms (allow sign errors)	
			$v_B - v_S = 0.8u$	M1*	3.3	Newton's experimental law – must be consistent with CoLM (so signs of v_S in the two equations must be different)	
			$u = v_s + 8\ (0.8u + v_s)\ = 9v_s + 6.4u$	M1dep*	3.4	Attempt at eliminating either variable – dependent on both previous M marks	
			$\Rightarrow v_S = -0.6u$ and $v_B = 0.2u$	A1	1.1	Ignore incorrect signs.	
			S has speed $0.6u$ towards the wall B has speed $0.2u$ away from the wall	A1	2.4	Both correct. Accept other appropriate descriptions of direction (e.g. 'opposite to original direction of travel', etc.)	
				[5]			

Question			Answer	Marks	AOs	Guidance	
	(c)		Each time S returns for impact it has $\frac{3}{5}$ the speed it had previously; therefore after impact, the block will have also have $\frac{3}{5}$ the speed it had just after the previous impact …	M1	3.5a	Argument using their value of v_S from **(b)** – must relate this value to B	
			… so by part **(a)**, the block will move only $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ of the distance moved after the previous impact.	A1	2.2a	Must reference result in part **(a)** (or convincingly explain where the squaring comes from)	May consider that the ratio of successive distances travelled by B is v^2
				[2]			
	(d)		After first impact, speed of block is $11.2 \times 0.2 = 2.24$	B1ft	3.4	Follow through their value of v_B from **(b)**	
			So $x_1 = \dfrac{2.24^2}{2 \cdot \frac{1}{7} \cdot (9.8)} = 1.792$	M1	1.1	Using given result in **(a)** to find distance travelled after first collision	
			$\sum_{n=1}^{\infty} x_n = \dfrac{1.792}{1 - \frac{9}{25}} = 2.8\ (\text{m})$	A1	2.2a	**AG** Use of infinite sum of a GP to derive required result	
				[3]			

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