June 2005
6666 Core C4
Mark Scheme

Question Number	Scheme	Marks
1.	$(4-9x)^{\frac{1}{2}} = 2\left(1-\frac{9x}{4}\right)^{\frac{1}{2}}$	B1
	$= 2\left(1+\frac{\frac{1}{2}}{1}\left(-\frac{9x}{4}\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}\left(-\frac{9x}{4}\right)^2+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}\left(-\frac{9x}{4}\right)^3+\ ...\right)$	M1
	$= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ ...\right)$	
	$= 2-\frac{9}{4}x, -\frac{81}{64}x^2, -\frac{729}{512}x^3+\ ...$	A1, A1, A1
		[5]
	Note The M1 is gained for $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1.2}(\ ...\ )^2$ or $\frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1.2.3}(\ ...\ )^3$	
	Special Case	
	If the candidate reaches $= 2\left(1-\frac{9}{8}x-\frac{81}{128}x^2-\frac{729}{1024}x^3+\ ...\right)$ and goes no further allow A1 A0 A0	

Question Number	Scheme	Marks
2.	$2x+\left(2x\frac{dy}{dx}+2y\right)-6y\frac{dy}{dx}=0$	M1 (A1) A1
	$\frac{dy}{dx}=0 \Rightarrow x+y=0$ or equivalent	M1
	Eliminating either variable and solving for at least one value of x or y.	M1
	$y^2-2y^2-3y^2+16=0$ or the same equation in x	
	$y=\pm 2$ or $x=\pm 2$	A1
	$(2,-2),(-2,2)$	A1
		[7]
	Note: $\frac{dy}{dx}=\frac{x+y}{3y-x}$	
	Alternative	
	$3y^2-2xy-\left(x^2+16\right)=0$	
	$y=\frac{2x\pm\sqrt{\left(16x^2+192\right)}}{6}$	
	$\frac{dy}{dx}=\frac{1}{3}\pm\frac{1}{3}.\frac{8x}{\sqrt{\left(16x^2+192\right)}}$	M1 A1± A1
	$\frac{dy}{dx}=0 \Rightarrow \frac{8x}{\sqrt{\left(16x^2+192\right)}}=\pm 1$	M1
	$64x^2=16x^2+192$	
	$x=\pm 2$	M1 A1
	$(2,-2),(-2,2)$	A1
		[7]

Question Number	Scheme		Marks	
3.	(a) $\frac{5x+3}{(2x-3)(x+2)} = \frac{A}{2x-3} + \frac{B}{x+2}$			
	$5x+3 = A(x+2) + B(2x-3)$			
	Substituting $x = -2$ or $x = \frac{3}{2}$ and obtaining A or B; or equating coefficients and solving a pair of simultaneous equations to obtain A or B.		M1	
	$A = 3,\ B = 1$		A1, A1	**(3)**
	If the cover-up rule is used, give M1 A1 for the first of A or B found, A1 for the second.			
	(b) $\int \frac{5x+3}{(2x-3)(x+2)}\,dx = \frac{3}{2}\ln(2x-3) + \ln(x+2)$		M1 A1ft	
	$\left[\ \ldots\ \right]_2^6 = \frac{3}{2}\ln 9 + \ln 2$		M1 A1	
	$= \ln 54$	cao	A1	**(5)**
				[8]

Question Number	Scheme	Marks
4.	$\int \frac{1}{(1-x^2)^{\frac{1}{2}}}\,dx = \int \frac{1}{(1-\sin^2\theta)^{\frac{3}{2}}}\cos\theta\,d\theta$ Use of $x=\sin\theta$ and $\frac{dx}{d\theta}=\cos\theta$	M1
	$= \int \frac{1}{\cos^2\theta}\,d\theta$	M1 A1
	$= \int \sec^2\theta\,d\theta = \tan\theta$	M1 A1
	Using the limits 0 and $\frac{\pi}{6}$ to evaluate integral	M1
	$[\tan\theta]_0^{\frac{\pi}{6}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$ cao	A1
		[7]
	Alternative for final M1 A1	
	Returning to the variable x and using the limits 0 and $\frac{1}{2}$ to evaluate integral	M1
	$\left[\frac{x}{\sqrt{(1-x^2)}}\right]_0^{\frac{1}{2}} = \frac{1}{\sqrt{3}} \quad \left(=\frac{\sqrt{3}}{3}\right)$ cao	A1

Question Number	Scheme	Marks
5.	(a) $\int x\mathrm{e}^{2x}\mathrm{d}x = \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{2}\int \mathrm{e}^{2x}\,\mathrm{d}x$ Attempting parts in the right direction	M1 A1
	$= \frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}$	A1
	$\left[\frac{1}{2}x\mathrm{e}^{2x} - \frac{1}{4}\mathrm{e}^{2x}\right]_0^1 = \frac{1}{4} + \frac{1}{4}\mathrm{e}^2$	M1 A1
		(5)
	(b) $x = 0.4 \Rightarrow y \approx 0.89022$	
	$x = 0.8 \Rightarrow y \approx 3.96243$ Both are required to 5 d.p.	B1
		(1)
	(c) $I \approx \frac{1}{2} \times 0.2 \times [\ \dots\ ]$	B1
	$\approx \dots \times \left[0 + 7.38906 + 2\left(0.29836 + .89022 + 1.99207 + 3.96243\right)\right]$	M1 A1ft
	ft their answers to (b)	
	$\approx 0.1 \times 21.67522$	
	≈ 2.168 cao	A1 **(4)**
		[10]
	Note $\frac{1}{4} + \frac{1}{4}\mathrm{e}^2 \approx 2.097\ \dots$	

Question Number	Scheme		Marks
6.	(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = -2\operatorname{cosec}^2 t,\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = 4\sin t\cos t$ both	M1 A1	
	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2\sin t\cos t}{\operatorname{cosec}^2 t}$ $\left(= -2\sin^3 t\cos t\right)$	M1 A1	
			(4)
	(b) At $t = \frac{\pi}{4}$, $x = 2, y = 1$ both x and y	B1	
	Substitutes $t = \frac{\pi}{4}$ into an attempt at $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to obtain gradient $\left(-\frac{1}{2}\right)$	M1	
	Equation of tangent is $y - 1 = -\frac{1}{2}(x - 2)$	M1 A1	
	Accept $x + 2y = 4$ or any correct equivalent		**(4)**
	(c) Uses $1 + \cot^2 t = \operatorname{cosec}^2 t$, or equivalent, to eliminate t	M1	
	$1 + \left(\dfrac{x}{2}\right)^2 = \dfrac{2}{y}$ correctly eliminates t	A1	
	$y = \dfrac{8}{4 + x^2}$ cao	A1	
	The domain is $x \ldots 0$	B1	**(4)**
			[12]
	An alternative in (c)		
	$\sin t = \left(\dfrac{y}{2}\right)^{\frac{1}{2}}$; $\cos t = \dfrac{x}{2}\sin t = \dfrac{x}{2}\left(\dfrac{y}{2}\right)^{\frac{1}{2}}$		
	$\sin^2 t + \cos^2 t = 1 \Rightarrow \dfrac{y}{2} + \dfrac{x^2}{4} \times \dfrac{y}{2} = 1$	M1 A1	
	Leading to $y = \dfrac{8}{4 + x^2}$	A1	

Question Number	Scheme	Marks
7.	(a) **k** component $\quad 2+4\lambda=-2 \quad \Rightarrow \quad \lambda=-1$	M1 A1
	Note $\mu=2$	
	Substituting their λ (or μ) into equation of line and obtaining B	M1
	B: $(2, 2, -2)$ $\quad$ Accept vector forms	A1
		(4)
	(b) $\left\lvert\begin{pmatrix}1\\-1\\4\end{pmatrix}\right\rvert=\sqrt{18}; \quad \left\lvert\begin{pmatrix}1\\-1\\0\end{pmatrix}\right\rvert=\sqrt{2}$ $\quad$ both	B1
	$\begin{pmatrix}1\\-1\\4\end{pmatrix}\cdot\begin{pmatrix}1\\-1\\0\end{pmatrix}=1+1+0\,(=2)$	B1
	$\cos\theta=\dfrac{2}{\sqrt{18}\sqrt{2}}=\dfrac{1}{3}$ $\quad$ cao	M1 A1
		(4)
	(c) $\overrightarrow{AB}=-\mathbf{i}+\mathbf{j}-4\mathbf{k}\Rightarrow\left\lvert\overrightarrow{AB}\right\rvert^2=18$ or $\left\lvert\overrightarrow{AB}\right\rvert=\sqrt{18}$ $\quad$ ignore direction of vector	M1
	$\overrightarrow{BC}=3\mathbf{i}-3\mathbf{j}\Rightarrow\left\lvert\overrightarrow{BC}\right\rvert^2=18$ or $\left\lvert\overrightarrow{BC}\right\rvert=\sqrt{18}$ $\quad$ ignore direction of vector	M1
	Hence $\left\lvert\overrightarrow{AB}\right\rvert=\left\lvert\overrightarrow{BC}\right\rvert$ $\ast$	A1 **(3)**
	(d) $\overrightarrow{OD}=6\mathbf{i}-2\mathbf{j}+2\mathbf{k}$ $\quad$ Allow first B1 for any two correct	B1 B1
	Accept column form or coordinates	**(2)**
		[13]

Question Number	Scheme	Marks
8.	(a) $\frac{dV}{dt}$ is the rate of increase of volume (with respect to time)	B1
	$-kV$: k is constant of proportionality and the negative shows decrease (or loss)	
	giving $\frac{dV}{dt} = 20 - kV$ ∗ These Bs are to be awarded independently	B1
		(2)
	(b) $\int \frac{1}{20-kV}\,dV = \int 1\,dt$ separating variables	M1
	$-\frac{1}{k}\ln(20-kV) = t \quad (+C)$	M1 A1
	Using $V=0, t=0$ to evaluate the constant of integration	M1
	$c = -\frac{1}{k}\ln 20$	
	$t = \frac{1}{k}\ln\left(\frac{20}{20-kV}\right)$	
	Obtaining answer in the form $V = A + Be^{-kt}$	M1
	$V = \frac{20}{k} - \frac{20}{k}e^{-kt}$ Accept $\frac{20}{k}\left(1-e^{-kt}\right)$	A1 **(6)**
	(c) $\frac{dV}{dt} = 20e^{-kt}$ Can be implied	M1
	$\frac{dV}{dt} = 10, t = 5 \Rightarrow 10 = 20e^{-kt} \Rightarrow k = \frac{1}{5}\ln 2 \approx 0.139$	M1 A1
	At $t = 10$, $V = \frac{75}{\ln 2}$ awrt 108	M1 A1 **(5)**
		[13]
	Alternative to (b)	
	Using printed answer and differentiating $\frac{dV}{dt} = -kBe^{-kt}$	M1
	Substituting into differential equation	
	$-kBe^{-kt} = 20 - kA - kBe^{-kt}$	M1
	$A = \frac{20}{k}$	M1 A1
	Using $V=0, t=0$ in printed answer to obtain $A + B = 0$	M1
	$B = -\frac{20}{k}$	A1 **(6)**