AQA

A-LEVEL
Mathematics

MS2B Statistics 2B
Mark scheme

6360

June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
Adep	mark is dependent on A or B marks as detailed and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
ft or F	follow through from previous incorrect result
CAO	correct answer only
AWFW	anything which falls within
AWRT	anything which rounds to
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Marks	Total	Comments
(a)	$\frac{1}{6}(x-3) = 0.75$ $x = 7.5$ (or 7½ or $\frac{15}{2}$)	M1 A1	**2**	CAO B2 for unsupported answer
(b)	$f(x) = \frac{1}{6}$ ($3 \le x \le 9$) **Or** recognition as uniform + formula	M1		Correct differentiation seen Clear use of F(x) = 0.5 OE to find **median** rather than **mean** scores M0
	Mean = 6	A1		CAO
	Variance = $\frac{1}{12}(9-3)^2$ (= 3) **Or** integration leading to $39 - 6^2$ (= 3)	M1		Formula applied or by integration
	s.d. = $\sqrt{3}$ (= 1.73)	A1	**4**	Exact or AWRT 3 s.f.
			6	

Q2	Solution	Marks	Total	Comments
(a)	The **sample** must be **random(**ly selected)	E1		Not 'it' or 'they' or 'the data' 'is/are random'
	(the April) **rainfall** must be **normal**(ly distributed)	E1		Referring to the population not the sample
			2	
(b)	$\bar{x} = 35.6$	B1		CAO
	$s = 32.50$ or 30.40 (or $s^2 = 1056.5$ or 924.5)	B1		AWRT 32.5 or 30.4 (or AWFW 1056 to 1057 or 924 to 925)
	$t_7 = 3.499$	B1		AWRT 3.50
	$35.6 \pm 3.499 \times$ se where se = $32.5/\sqrt{8}$ or $30.4/\sqrt{7}$	M1		PI by correct answer
	-4.6, 75.8	A1		CAO
			5	
(c)	Since mean **rainfall cannot be negative**	M1		Recognition of the problem in context (as long as their c.i. includes negative)
	Assumption(s) (probably) not valid	A1		OE And its consequence
			2	
			9	

Note. Use of *z* or wrong *t* value in (b) scores maximum of B1 B1 B0 M0 A0

Q3	Solution	Marks	Total	Comments
(a)(i)	$7 \times 3 \times 0.1 = 2.1$. So Po(2.1)	M1		Stated or implied by formula or correct final answer
	$\left(e^{-2.1} \times \frac{2.1^4}{4!}\right) = 0.0992$	A1		AWRT. Formula or calculator.
			2	
(a)(ii)	$5 \times 4 \times 0.3 = 6$. **Use** of Po(6)	M1		Must see use of 0.4457, 0.6063, 0.7440, 0.9161, 0.9574 or 0.9799 to at least 3 s.f.
	We require $P(\leq 9) - P(\leq 5)$	M1		Stated or implied by either of following lines
	$0.9161 - 0.4457$	B1		Either, to at least 3 s.f.
	$= 0.470(4)$	A1		AWRT 0.470. Accept 0.47
			4	
(a)(iii)	$1 - e^{-2.1}$ (= 0.8775)	M1		Possibly implied
	$\times$ 0.002478 (0.0025)	B1		AWRT 0.0025 Must multiply
	$= 0.00217$ or 2.17×10^{-3}	A1		AWFW 0.00217 to 0.00220
			3	
(b)	$A = \pi r^2$ so $\lambda = 0.1\pi r^2$	M1		Must identify $0.1\pi r^2$ as λ or σ^2
	$\sigma = \sqrt{\text{variance}} = \sqrt{\lambda} = \sqrt{(0.1\pi r^2)} = \sqrt{(0.1\pi)}r$	A1		Accept $0.560r$ for this mark
	$\sigma = 0.560r$ **or** $k = 0.560$	A1		Accept 0.560 or 0.561 only.
			3	
			12	

Q4	Solution	Marks	Total	Comments
(a)	Evidence of totals being calculated: (43, 47, 28, 14, 48 and 90) – either table	M1		At least 4 seen or implied by 4 correct values in table
	<table><tr><td></td><td>French</td><td>German</td><td>Spanish</td></tr><tr><td>Boys</td><td>13.38</td><td>6.69</td><td>22.93</td></tr><tr><td>Girls</td><td>14.62</td><td>7.31</td><td>25.07</td></tr></table>	A2,1,0	**3**	CAO A2 for all correct, A1 for any 4 correct.
(b)(i)	$\frac{(18-14)^2}{14}+\frac{(9-7)^2}{7}+\frac{(15-21)^2}{21}+\frac{(12-16)^2}{16}+\frac{(6-8)^2}{8}+\frac{(30-24)^2}{24}$	B2		All six
	$\frac{4^2}{14}+\frac{2^2}{7}+\frac{6^2}{21}+\frac{4^2}{16}+\frac{2^2}{8}+\frac{6^2}{24}$ or $\frac{8}{7}+\frac{4}{7}+\frac{12}{7}+1+\frac{1}{2}+\frac{3}{2}$ or 1.143 + 0.571 + 1.714 + 1 + 0.5 + 1.5	(B1)		If B2 not earned. All six. Can do a mixture of fractions and decimals
			2	
(b)(ii)	Two degrees of freedom Critical value = 4.605 6.43 > 4.605 so reject Ho and conclude that there is (significant evidence of) an association between gender and language choice.	B1 B1 B1dep	**3**	CAO Implied by 4.60 or 4.61 AWFW 4.60 to 4.61 Comparison or diagram Dep on B1 B1.Conclusion in context.
(c)(i)	(Expected) girls doing German less than 5 4.675 less than 5	E1	**1**	Either of these two
(c)(ii)	(Yates correction must be used so calculate) $\sum\frac{(\lvert O-E\rvert-0.5)^2}{E}$ or $\frac{N}{mnrs}\left(\lvert ad-bc\rvert-\frac{N}{2}\right)^2$	E1	**1**	Condone omission of Σ
(iii)	2.706	B1	**1**	AWFW 2.70 to 2.71
			11	

Q5	Solution	Marks	Total	Comments
(a)	$c(7+6+5+4+3+2+1) = 1$ so $c = \frac{1}{28}$	B1	**1**	CAO
(b)	$P(X > 4) = \frac{1}{28}(3+2+1) = \frac{6}{28}$ $(= \frac{3}{14})$ Or $P(X \leq 4) = \frac{11}{14}$ We require $1 - (\frac{11}{14})^3$ or $3\left(\frac{3}{14}\right)\left(\frac{11}{14}\right)^2 + 3\left(\frac{3}{14}\right)^2\left(\frac{11}{14}\right) + \left(\frac{3}{14}\right)^3$ $= \frac{1413}{2744} = 0.515$	B1 M1 A1	**3**	CAO any exact form P.I. by correct answer Exact or AWRT 3sf
(c)	$E(X) = 'c'(1\times7+2\times6+3\times5+4\times4+5\times3+6\times2+7\times1)$ $= 3$ x^2 | 1 | 4 | 9 | 16 | 25 | 36 | 49 P | $7c$ | $6c$ | $5c$ | $4c$ | $3c$ | $2c$ | c $E(X^2) = 'c'(1^2\times7+2^2\times6+3^2\times5+4^2\times4+5^2\times3+6^2\times2+7^2\times1)$ Complete set of 7 correct numerical products added (= 12 or $\frac{336}{28}$ OE) $Var(X) = E(X^2) - (E(X))^2 = 12 - 3^2 = 3$	M1 A1 M1 A1 A1	**5**	PI by correct answer CAO Not necessary to see table With multiplications, or results of multiplications, shown CAO No longer c CAO
(d)	y | 42 | 21 | 14 | 10.5 | 8.4 | 7 | 6 $P(Y=y)$ | $7c$ | $6c$ | $5c$ | $4c$ | $3c$ | $2c$ | c $E(Y) = 42\times7c + 21\times6c$ etc. $= 20\frac{43}{70}$ or $\frac{1443}{70}$ or 20.6(143) **Alternative** Table to find $E\left(\frac{1}{X}\right)$ $= \frac{481}{980} = 0.49(08\ldots)$ $E(Y) = 42 \times E\left(\frac{1}{X}\right)$ $= 20\frac{43}{70}$ or $\frac{1443}{70}$ or 20.6(143)	M1 A1 m1 A1 (M1) (A1) (m1) (A1)	**4**	For Y values PI by correct answer For correct p values linked to correct y values. PI by correct answer Correct $y \times$ their p. PI by correct answer. CAO/AWRT Complete PI by correct answer Exact or at least 2sf. PI by correct answer PI by correct answer CAO/AWRT
			13	

Note. Where a candidate has an incorrect value for c or uses decimal equivalent to $\frac{1}{28}$ award marks as follows:

(b) B1F for P($X > 4$) = 6c or P($X \leq 4$) = 1 – 6c, exact, or decimal equivalent to 3sf, and then maybe M1 A1 if using the decimal equivalent of $\frac{1}{28}$.
(c) M1 A0 M1 A0 A0 for correct method from their table.
(d) M1 m1 M1 A0 for correct method from their table

Q6	Solution	Marks	Total	Comments
(a)	H_0: $\mu = 1.25$ H_1: $\mu \neq 1.25$	B1		Both
	Sample mean = 1.66	B1		CAO
	$s^2 = 0.53978$ ($s = 0.735$)	B1		AWRT 0.54 (AWFW 0.734 to 0.735)
	Test statistic = $\frac{(1.66-1.25)}{\frac{s}{\sqrt{10}}}$	M1 m1		For denominator (c's s) PI by ts For numerator (ignore sign) PI by ts
	= 1.765	A1		AWFW 1.76 to 1.77
	$t_9 = 1.833$	B1		AWRT 1.83
	(1.765 < 1.833 so accept H_0)			
	There is not sufficient evidence that the mean weight of adult trout has changed.	A1dep		Dep on preceding A1 and B1 In context (at least 'weight'). Must specify "mean".
			8	
(b)	Marks in (b) dependent on A1dep in (a) apart from the **S**pecial **C**ase			
	The mean weight **has** changed We accepted H_0 when H_0 false We accepted H_0 when H_1 true We rejected H_1 when H_1 true	M1		Any of these four lines or close equiv
	…. so a Type II error has been made.	E1		
	SC If A0(dep) but 'The mean weight **has** changed' scores M1 A0			
			2	
			10	

Q7	Solution	Marks	Total	Comments
(a)(i)	Graph zero for $x \leq -a$ and $x \geq a$	B1		Not given for a totally blank graph! Drawn graph must pass through (-a, 0) and (a, 0).
	 Correct shape from $-a$ to a.	B1		"Bell shaped" Must have a rounded top at $x = 0$, condone gradient not reducing at bottom
	Shape drawn passes through $(0, ka^4)$	B1	**3**	Labelled as ka^4
(ii)	$1 = \int_{-a}^{a} k(x-a)^2(x+a)^2 \, dx$ $= \int_{-a}^{a} k(x^4 - 2a^2x^2 + a^4) dx$	M1		Correct integral (either of these two) equal to 1 here or later. Condone missing limits until A1
	(or $0.5 = \int_{0}^{a} k(x-a)^2(x+a)^2 \, dx$)			Must have the limits for this
	$= [k(\frac{1}{5}x^5 - \frac{2}{3}a^2x^3 + a^4x]_{-a}^{a}$	m1		Correct integration
	$1 = \frac{16}{15}ka^5$ (or $0.5 = \frac{8}{15}ka^5$)	A1		Limits inserted and simplified to this single fraction
	So $k = \frac{15}{16a^5}$	A1	**4**	CAO
(iii)	$E(X) = 0$	B1	**1**	Irrespective of working
(iv)	$E(X^2) = \int_{-a}^{a} kx^2(x^4 - 2a^2x^2 + a^4) dx$	M1		Correct integral. k or their value for k provided constant. Condone missing limits until A1
	$= [k(\frac{1}{7}x^7 - \frac{2}{5}a^2x^5 + \frac{1}{3}a^4x^3]_{-a}^{a}$	m1		Correct integration
	$= k \times \frac{16a^7}{105}$	A1		Limits inserted and simplified to this single fraction
	$= \frac{a^2}{7}$ and $\text{Var}(X) = E(X^2) - 0^2 = \frac{a^2}{7}$	A1	**4**	CAO
(b)(i)	$E(Y) = 7a$	B1	**1**	CAO
(ii)	$\text{Var}(Y) = 7a^2$	B1	**1**	CAO
			14	