



Oxford Cambridge and RSA

**Monday 18 October 2021 – Afternoon**

**A Level Mathematics A**

**H240/03 Pure Mathematics and Mechanics**

**Time allowed: 2 hours**



**You must have:**

- the Printed Answer Booklet
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [ ].
- This document has **12** pages.

**ADVICE**

- Read each question carefully before you start your answer.

**Formulae**  
**A Level Mathematics A (H240)**

**Arithmetic series**

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small angle approximations**

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

**Trigonometric identities**

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

**Numerical methods**

Trapezium rule:  $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$ , where  $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving  $f(x) = 0$ :  $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

**Probability**

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , mean of  $X$  is  $np$ , variance of  $X$  is  $np(1-p)$

**Hypothesis test for the mean of a normal distribution**

If  $X \sim N(\mu, \sigma^2)$  then  $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$  and  $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

**Percentage points of the normal distribution**

If  $Z$  has a normal distribution with mean 0 and variance 1 then, for each value of  $p$ , the table gives the value of  $z$  such that  $P(Z \leq z) = p$ .

$p$	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
$z$	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

## Section A: Pure Mathematics

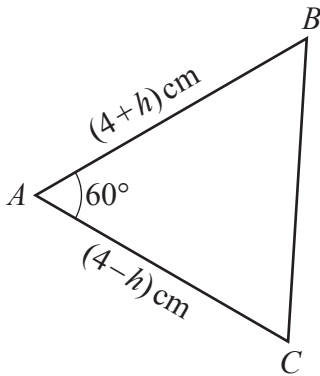
Answer **all** the questions.

- 1 Show in a sketch the region of the  $x$ - $y$  plane within which all three of the following inequalities hold.

$$y \geq x^2, \quad x + y \leq 2, \quad x \geq 0.$$

You should indicate the region for which the inequalities hold by labelling the region  $R$ . [3]

2



The diagram shows triangle  $ABC$  in which angle  $A$  is  $60^\circ$  and the lengths of  $AB$  and  $AC$  are  $(4+h)$  cm and  $(4-h)$  cm respectively.

- (a) Show that the length of  $BC$  is  $p$  cm where

$$p^2 = 16 + 3h^2. \quad [2]$$

- (b) Hence show that, when  $h$  is small,  $p \approx 4 + \lambda h^2 + \mu h^4$ , where  $\lambda$  and  $\mu$  are rational numbers whose values are to be determined. [4]

- 3 An arithmetic progression has first term 2 and common difference  $d$ , where  $d \neq 0$ . The first, third and thirteenth terms of this progression are also the first, second and third terms, respectively, of a geometric progression.

By determining  $d$ , show that the arithmetic progression is an increasing sequence. [5]

- 4 (a) Sketch, on a single diagram, the following graphs.
- $y = |x - 1|$
  - $y = \frac{k}{x}$ , where  $k$  is a negative constant [2]
- (b) Hence explain why the equation  $x|x - 1| = k$  has exactly one real root for any negative value of  $k$ . [1]
- (c) Determine the real root of the equation  $x|x - 1| = -6$ . [2]

- 5 A particle  $P$  moves along a straight line in such a way that at time  $t$  seconds  $P$  has velocity  $v$  m s<sup>-1</sup>, where

$$v = 12 \cos t + 5 \sin t.$$

- (a) Express  $v$  in the form  $R \cos(t - \alpha)$ , where  $R > 0$  and  $0 < \alpha < \frac{1}{2}\pi$ . Give the value of  $\alpha$  correct to 4 significant figures. [3]
- (b) Hence find the two smallest positive values of  $t$  for which  $P$  is moving, in either direction, with a speed of 3 m s<sup>-1</sup>. [3]

- 6 The equation  $6 \arcsin(2x - 1) - x^2 = 0$  has exactly one real root.

- (a) Show by calculation that the root lies between 0.5 and 0.6. [2]

In order to find the root, the iterative formula

$$x_{n+1} = p + q \sin(rx_n^2),$$

with initial value  $x_0 = 0.5$ , is to be used.

- (b) Determine the values of the constants  $p$ ,  $q$  and  $r$ . [2]
- (c) Hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [2]

- 7 A curve  $C$  in the  $x$ - $y$  plane has the property that the gradient of the tangent at the point  $P(x, y)$  is three times the gradient of the line joining the point  $(3, 2)$  to  $P$ .

(a) Express this property in the form of a differential equation. [2]

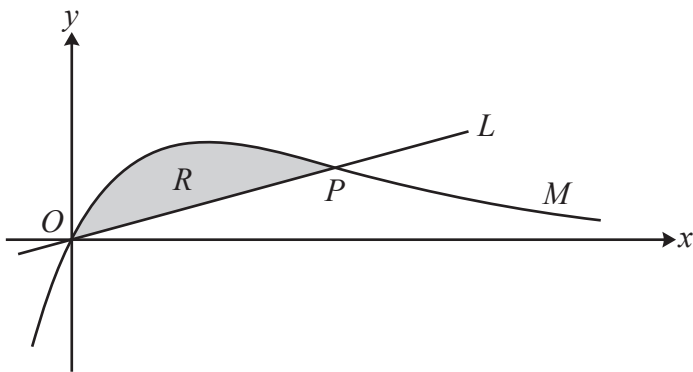
It is given that  $C$  passes through the point  $(4, 3)$  and that  $x > 3$  and  $y > 2$  at all points on  $C$ .

(b) Determine the equation of  $C$  giving your answer in the form  $y = f(x)$ . [4]

The curve  $C$  may be obtained by a transformation of part of the curve  $y = x^3$ .

(c) Describe fully this transformation. [2]

8



The diagram shows the curve  $M$  with equation  $y = xe^{-2x}$ .

(a) Show that  $M$  has a point of inflection at the point  $P$  where  $x = 1$ . [5]

The line  $L$  passes through the origin  $O$  and the point  $P$ . The shaded region  $R$  is enclosed by the curve  $M$  and the line  $L$ .

(b) Show that the area of  $R$  is given by

$$\frac{1}{4}(a + be^{-2}),$$

where  $a$  and  $b$  are integers to be determined. [6]

## Section B: Mechanics

Answer **all** the questions.

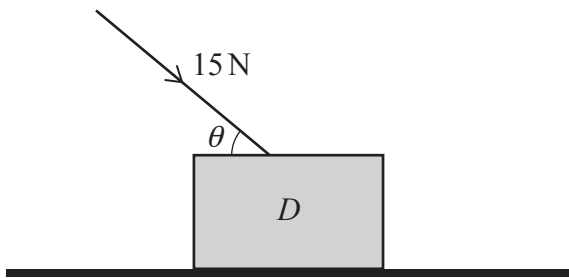
- 9 There are three checkpoints,  $A$ ,  $B$  and  $C$ , in that order, on a straight horizontal road. A car travels along the road, in the direction from  $A$  to  $C$ , with constant acceleration. The car takes 20 s to travel from  $B$  to  $C$ . The speed of the car at  $B$  is  $14 \text{ m s}^{-1}$  and the speed of the car at  $C$  is  $18 \text{ m s}^{-1}$ .

(a) Find the acceleration of the car. [1]

It is given that the distance between  $A$  and  $B$  is 330 m.

(b) Determine the speed of the car at  $A$ . [2]

10



A block  $D$  of weight 50 N lies at rest in equilibrium on a fixed rough horizontal surface. A force of magnitude 15 N is applied to  $D$  at an angle  $\theta$  to the horizontal (see diagram).

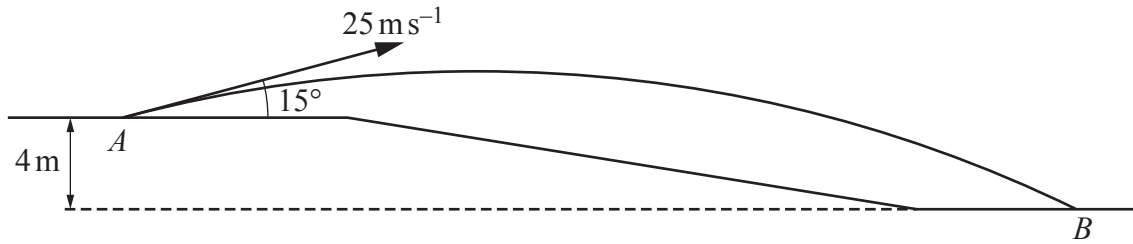
(a) Complete the diagram in the Printed Answer Booklet showing all the forces acting on  $D$ . [1]

It is given that  $D$  remains at rest and the coefficient of friction between  $D$  and the surface is 0.2.

(b) Show that

$$15 \cos \theta - 3 \sin \theta \leq 10. \quad [5]$$

11



A golfer hits a ball from a point  $A$  with a speed of  $25 \text{ m s}^{-1}$  at an angle of  $15^\circ$  above the horizontal. While the ball is in the air, it is modelled as a particle moving under the influence of gravity. Take the acceleration due to gravity to be  $10 \text{ m s}^{-2}$ .

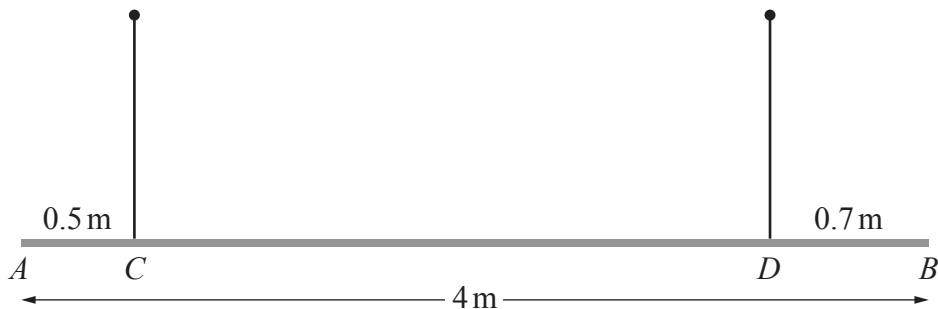
The ball first lands at a point  $B$  which is  $4 \text{ m}$  below the level of  $A$  (see diagram).

- (a) Determine the time taken for the ball to travel from  $A$  to  $B$ . [3]
- (b) Determine the horizontal distance of  $B$  from  $A$ . [2]
- (c) Determine the direction of motion of the ball  $1.5$  seconds after the golfer hits the ball. [4]

The horizontal distance from  $A$  to  $B$  is found to be greater than the answer to part (b).

- (d) State one factor that could account for this difference. [1]

12



A beam,  $AB$ , has length  $4 \text{ m}$  and mass  $20 \text{ kg}$ . The beam is suspended horizontally by two vertical ropes. One rope is attached to the beam at  $C$ , where  $AC = 0.5 \text{ m}$ . The other rope is attached to the beam at  $D$ , where  $DB = 0.7 \text{ m}$  (see diagram).

The beam is modelled as a non-uniform rod and the ropes as light inextensible strings.

It is given that the tension in the rope at  $C$  is three times the tension in the rope at  $D$ .

- (a) Determine the distance of the centre of mass of the beam from  $A$ . [5]

A particle of mass  $m \text{ kg}$  is now placed on the beam at a point where the magnitude of the moment of the particle's weight about  $C$  is  $3.5mg \text{ N m}$ . The beam remains horizontal and in equilibrium.

- (b) Determine the largest possible value of  $m$ . [2]



**13** In this question the unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in the directions east and north respectively.

At time  $t$  seconds, where  $t \geq 0$ , a particle  $P$  of mass 2 kg is moving on a smooth horizontal surface under the action of a constant horizontal force  $(-8\mathbf{i} - 54\mathbf{j})\text{N}$  and a variable horizontal force  $(4t\mathbf{i} + 6(2t - 1)^2\mathbf{j})\text{N}$ .

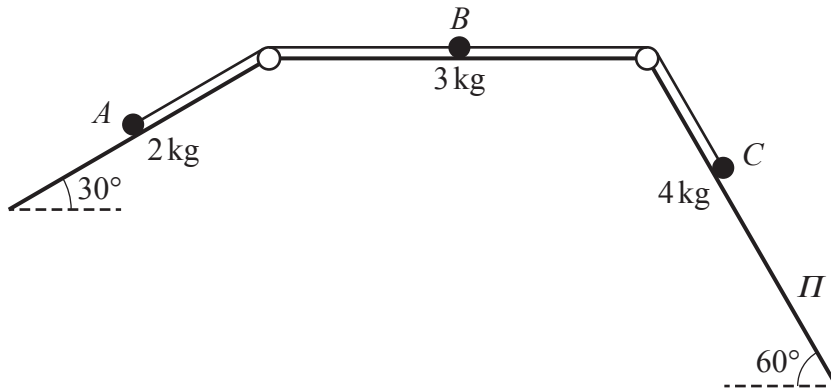
(a) Determine the value of  $t$  when the forces acting on  $P$  are in equilibrium. [2]

It is given that  $P$  is at rest when  $t = 0$ .

(b) Determine the speed of  $P$  at the instant when  $P$  is moving due north. [6]

(c) Determine the distance between the positions of  $P$  when  $t = 0$  and  $t = 3$ . [5]

**Turn over for question 14**



One end of a light inextensible string is attached to a particle  $A$  of mass 2 kg. The other end of the string is attached to a second particle  $B$  of mass 3 kg. Particle  $A$  is in contact with a smooth plane inclined at  $30^\circ$  to the horizontal and particle  $B$  is in contact with a rough horizontal plane.

A second light inextensible string is attached to  $B$ . The other end of this second string is attached to a third particle  $C$  of mass 4 kg. Particle  $C$  is in contact with a smooth plane  $\Pi$  inclined at an angle of  $60^\circ$  to the horizontal.

Both strings are taut and pass over small smooth pulleys that are at the tops of the inclined planes. The parts of the strings from  $A$  to the pulley, and from  $C$  to the pulley, are parallel to lines of greatest slope of the corresponding planes (see diagram).

The coefficient of friction between  $B$  and the horizontal plane is  $\mu$ . The system is released from rest and in the subsequent motion  $C$  moves down  $\Pi$  with acceleration  $a \text{ m s}^{-2}$ .

- (a) By considering an equation involving  $\mu$ ,  $a$  and  $g$  show that  $a < \frac{1}{9}g(2\sqrt{3} - 1)$ . [7]
- (b) Given that  $a = \frac{1}{9}g$ , determine the magnitude of the contact force between  $B$  and the horizontal plane. Give your answer correct to 3 significant figures. [4]

**END OF QUESTION PAPER**





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