



Oxford Cambridge and RSA

**Tuesday 25 June 2019 – Morning**

**A Level Further Mathematics B (MEI)**

**Y436/01 Further Pure with Technology**

**Time allowed: 1 hour 45 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae Further Mathematics B (MEI)
- Computer with appropriate software

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**COMPUTING RESOURCES**

- Candidates will require access to a computer with a computer algebra system, a spreadsheet, a programming language and graph-plotting software throughout the examination.

**INFORMATION**

- The total number of marks for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

**1** A family of curves is given by the parametric equations

$$x(t) = \cos(t) - \frac{\cos((m+1)t)}{m+1} \quad \text{and} \quad y(t) = \sin(t) - \frac{\sin((m+1)t)}{m+1}$$

where  $0 \leq t < 2\pi$  and  $m$  is a positive integer.

**(a)** **(i)** Sketch the curves in the cases  $m = 3$ ,  $m = 4$  and  $m = 5$  on separate axes in the Printed Answer Booklet. **[3]**

**(ii)** State one common feature of these three curves. **[1]**

**(iii)** State a feature for the case  $m = 4$  which is absent in the cases  $m = 3$  and  $m = 5$ . **[1]**

**(b)** **(i)** Determine, in terms of  $m$ , the values of  $t$  for which  $\frac{dx}{dt} = 0$  but  $\frac{dy}{dt} \neq 0$ . **[4]**

**(ii)** Describe the tangent to the curve at the points corresponding to such values of  $t$ . **[1]**

**(c)** **(i)** Show that the curve lies between the circle centred at the origin with radius

$$1 - \frac{1}{m+1}$$

and the circle centred at the origin with radius

$$1 + \frac{1}{m+1}. \quad \text{[2]}$$

**(ii)** Hence, or otherwise, show that the area  $A$  bounded by the curve satisfies

$$\frac{m^2\pi}{(m+1)^2} < A < \frac{(m+2)^2\pi}{(m+1)^2}. \quad \text{[1]}$$

**(iii)** Find the limit of the area bounded by the curve as  $m$  tends to infinity. **[1]**

**(d)** The arc length of a curve defined by parametric equations  $x(t)$  and  $y(t)$  between points corresponding to  $t = c$  and  $t = d$ , where  $c < d$ , is

$$\int_c^d \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

Use this to show that the length of the curve is independent of  $m$ . **[6]**

- 2 (a) Prove that if  $x$  and  $y$  are integers which satisfy  $x^2 - 2y^2 = 1$ , then  $x$  is odd and  $y$  is even. [3]
- (b) Create a program to find, for a fixed positive integer  $s$ , all the positive integer solutions  $(x, y)$  to the equation  $x^2 - 2y^2 = 1$  where  $x \leq s$  and  $y \leq s$ . Write out your program in the Printed Answer Booklet. [3]
- (c) Use your program to find all the positive integer solutions  $(x, y)$  to the equation  $x^2 - 2y^2 = 1$  where  $x \leq 600$  and  $y \leq 600$ . Give the solutions in ascending order of the value of  $x$ . [1]
- (d) By writing the equation  $x^2 - 2y^2 = 1$  in the form  $(x + \sqrt{2}y)(x - \sqrt{2}y) = 1$  show how the first solution (the one with the lowest value of  $x$ ) in your answer to part (c) can be used to generate the other solutions you found in part (c). [4]
- (e) What can you deduce about the number of positive integer solutions  $(x, y)$  to the equation  $x^2 - 2y^2 = 1$ ? [1]

In the remainder of this question  $T_m$  is the  $m^{\text{th}}$  triangular number, the sum of the first  $m$  positive integers, so that  $T_m = \frac{m(m+1)}{2}$ .

- (f) Create a program to find, for a fixed positive integer  $t$ , all pairs of positive integers  $m$  and  $n$  which satisfy  $T_m = n^2$  where  $m \leq t$  and  $n \leq t$ . Write out your program in the Printed Answer Booklet. [2]
- (g) Use your program to find all pairs of positive integers  $m$  and  $n$  which satisfy  $T_m = n^2$  where  $m \leq 300$  and  $n \leq 300$ . Give the pairs in ascending order of the value of  $m$ . [1]
- (h) By comparing your answers to part (c) and part (g), or otherwise, prove that there are infinitely many triangular numbers which are perfect squares. [5]

3 This question concerns the family of differential equations

$$\frac{dy}{dx} = 1 - x^a y \quad (*)$$

where  $a$  is  $-1$ ,  $0$  or  $1$ .

(a) Determine and describe geometrically the isoclines of  $(*)$  when

(i)  $a = -1$ , [2]

(ii)  $a = 0$ , [2]

(iii)  $a = 1$ . [2]

(b) In this part of the question  $a = 0$ .

(i) Write down the solution to  $(*)$  which passes through the point  $(0, b)$  where  $b \neq 1$ . [1]

(ii) Write down the equation of the asymptote to this solution. [1]

(c) In this part of the question  $a = -1$ .

(i) Write down the solution to  $(*)$  which passes through the point  $(c, d)$  where  $c \neq 0$ . [1]

(ii) Describe the relationship between  $c$  and  $d$  when the solution in part (i) has a stationary point. [4]

(d) In this part of the question  $a = 1$ .

(i) The standard Runge-Kutta method of order 4 for the solution of the differential equation

$$f(x, y) = \frac{dy}{dx} \text{ is as follows.}$$

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

Construct a spreadsheet to solve (\*) in the case  $x_0 = 0$  and  $y_0 = 1.5$ . State the formulae you have used in your spreadsheet. [4]

(ii) Use your spreadsheet with  $h = 0.05$  to find an approximation to the value of  $y$  when  $x = 1$ . [1]

(iii) The solution to (\*) in which  $x_0 = 0$  and  $y_0 = 1.5$  has a maximum point  $(r, s)$  with  $0 < r < 1$ . Use your spreadsheet with suitable values of  $h$  to estimate  $r$  to two decimal places. Justify your answer. [2]

**END OF QUESTION PAPER**

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