MATHEMATICS   4729

Mechanics 2

| Friday | **27 JANUARY 2006** | Afternoon | 1 hour 30 minutes |

Additional materials:
    8 page answer booklet
    Graph paper
    List of Formulae (MF1)

**TIME**   1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 5 printed pages and 3 blank pages.

1
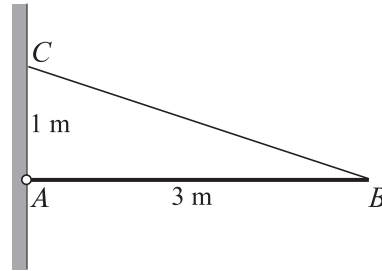
A uniform rod AB has weight $20\,\text{N}$ and length $3\,\text{m}$. The end A is freely hinged to a point on a vertical wall. The rod is held horizontally and in equilibrium by a light inextensible string. One end of the string is attached to the rod at B. The other end of the string is attached to a point C, which is $1\,\text{m}$ directly above A (see diagram). Calculate the tension in the string. [4]

2 A golfer hits a ball from a point O on horizontal ground with a velocity of $50\,\text{m}\,\text{s}^{-1}$ at an angle of $25°$ above the horizontal. The ball first hits the ground at a point A. Assuming that there is no air resistance, calculate

   (i) the time taken for the ball to travel from O to A, [3]

   (ii) the distance OA. [2]

3 A box of mass $50\,\text{kg}$ is dragged along a horizontal floor by a constant force of magnitude $400\,\text{N}$ acting at an angle of α above the horizontal. The total resistance to the motion of the box has magnitude $300\,\text{N}$. The box starts from rest at the point O, and passes the point P, $25\,\text{m}$ from O, with a speed of $2\,\text{m}\,\text{s}^{-1}$.
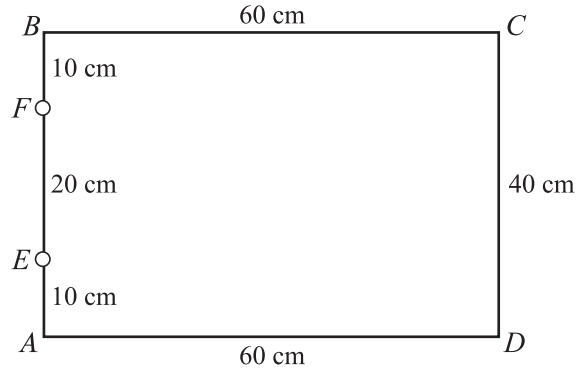
   (i) For the box's motion from O to P, find

      (a) the increase in kinetic energy of the box, [1]

      (b) the work done against the resistance to motion of the box. [1]

   (ii) Hence calculate α. [3]

4

A rectangular frame consists of four uniform metal rods. AB and CD are vertical and each is $40\,\text{cm}$ long and has mass $0.2\,\text{kg}$. AD and BC are horizontal and each is $60\,\text{cm}$ long. AD has mass $0.7\,\text{kg}$ and BC has mass $0.5\,\text{kg}$. The frame is freely hinged at E and F, where E is $10\,\text{cm}$ above A, and F is $10\,\text{cm}$ below B (see diagram).

(i) Sketch a diagram showing the directions of the horizontal components of the forces acting on the frame at E and F. [2]

(ii) Calculate the magnitude of the horizontal component of the force acting on the frame at E. [3]

(iii) Calculate the distance from AD of the centre of mass of the frame. [3]

5 Three smooth spheres A, B and C, of equal radius and of masses $3m\,\text{kg}$, $2m\,\text{kg}$ and $m\,\text{kg}$ respectively, are free to move in a straight line on a smooth horizontal table. Spheres B and C are stationary. Sphere A is moving with speed $2\,\text{m s}^{-1}$ when it collides directly with sphere B. The collision is perfectly elastic.

(i) Find the velocities of A and B after the collision. [6]

(ii) Find, in terms of m, the magnitude of the impulse that A exerts on B, and state the direction of this impulse. [2]

Sphere B continues its motion and hits C. After the collision, B continues in the same direction with speed $1.0\,\text{m s}^{-1}$ and C moves with speed $2.8\,\text{m s}^{-1}$.

(iii) Find the coefficient of restitution between B and C. [2]

6 A stone is projected horizontally with speed $7\,\text{m s}^{-1}$ from a point O on the edge of a vertical cliff. The horizontal and upward vertical displacements of the stone from O at any subsequent time, t seconds, are $x\,\text{m}$ and $y\,\text{m}$ respectively. Assume that there is no air resistance.

(i) Express x and y in terms of t, and hence show that $y = -\frac{1}{10}x^2$. [4]

The stone hits the sea at a point which is $20\,\text{m}$ below the level of O.

(ii) Find the distance between the foot of the cliff and the point where the stone hits the sea. [2]

(iii) Find the speed and direction of motion of the stone immediately before it hits the sea. [6]

[Turn over

7 Marco is riding his bicycle at a constant speed of $12\,\mathrm{m\,s^{-1}}$ along a horizontal road, working at a constant rate of $300\,\mathrm{W}$. Marco and his bicycle have a combined mass of $75\,\mathrm{kg}$.

   (i) Calculate the wind resistance acting on Marco and his bicycle. [2]

Nicolas is riding his bicycle at the same speed as Marco and directly behind him. Nicolas experiences 30% less wind resistance than Marco.

   (ii) Calculate the power output of Nicolas. [2]

The two cyclists arrive at the bottom of a hill which is at an angle of $1°$ to the horizontal. Marco increases his power output to $500\,\mathrm{W}$.

(iii) Assuming Marco's wind resistance is unchanged, calculate his instantaneous acceleration immediately after starting to climb the hill. [5]

Marco reaches the top of the hill at a speed of $13\,\mathrm{m\,s^{-1}}$. He then freewheels down a hill of length $200\,\mathrm{m}$ which is at a constant angle of $10°$ to the horizontal. He experiences a constant wind resistance of $120\,\mathrm{N}$.

(iv) Calculate Marco's speed at the bottom of this hill. [5]

8

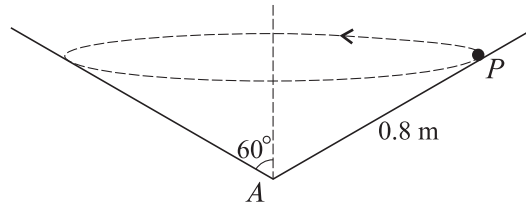**Fig. 1**

A particle P of mass 0.1 kg is moving with constant angular speed ω rad s^{-1} in a horizontal circle on the smooth inner surface of a cone which is fixed with its axis vertical and its vertex A at its lowest point. The semi-vertical angle of the cone is $60°$ and the distance AP is 0.8 m (see Fig. 1).

   (i) Calculate the magnitude of the force exerted by the cone on the particle. [3]
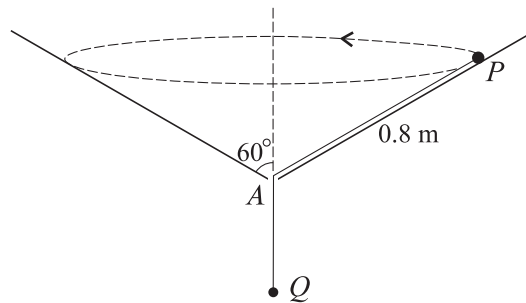
   (ii) Calculate ω. [4]

Fig. 2

The particle P is now attached to one end of a light inextensible string which passes through a small smooth hole at A. The lower end of the string is attached to a particle Q of mass 0.2 kg. Q is in equilibrium with the string taut and $AP = 0.8$ m. P moves in a horizontal circle with constant speed v m s^{-1} (see Fig. 2).

(iii) State the tension in the string. [1]

(iv) Find v. [6]

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