Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics MPC3

Unit Pure Core 3

Tuesday 10 June 2014 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 Use Simpson's rule, with five ordinates (four strips), to calculate an estimate for

$$\int_0^{\pi} x^{\frac{1}{2}} \sin x \, \mathrm{d}x$$

Give your answer to four significant figures.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 A curve has equation $y = 2\ln(2\mathrm{e} - x)$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

[2 marks]

(b) Find an equation of the normal to the curve $y = 2\ln(2\mathrm{e} - x)$ at the point on the curve where $x = \mathrm{e}$.

[4 marks]

(c) The curve $y = 2\ln(2\mathrm{e} - x)$ intersects the line $y = x$ at a single point, where $x = \alpha$.

(i) Show that α lies between 1 and 3.

[2 marks]

(ii) Use the recurrence relation

$$x_{n+1} = 2\ln(2\mathrm{e} - x_n)$$

with $x_1 = 1$ to find the values of x_2 and x_3, giving your answers to three decimal places.

[2 marks]

(iii) **Figure 1**, on the opposite page, shows a sketch of parts of the graphs of $y = 2\ln(2\mathrm{e} - x)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis.

[2 marks]

QUESTION PART REFERENCE

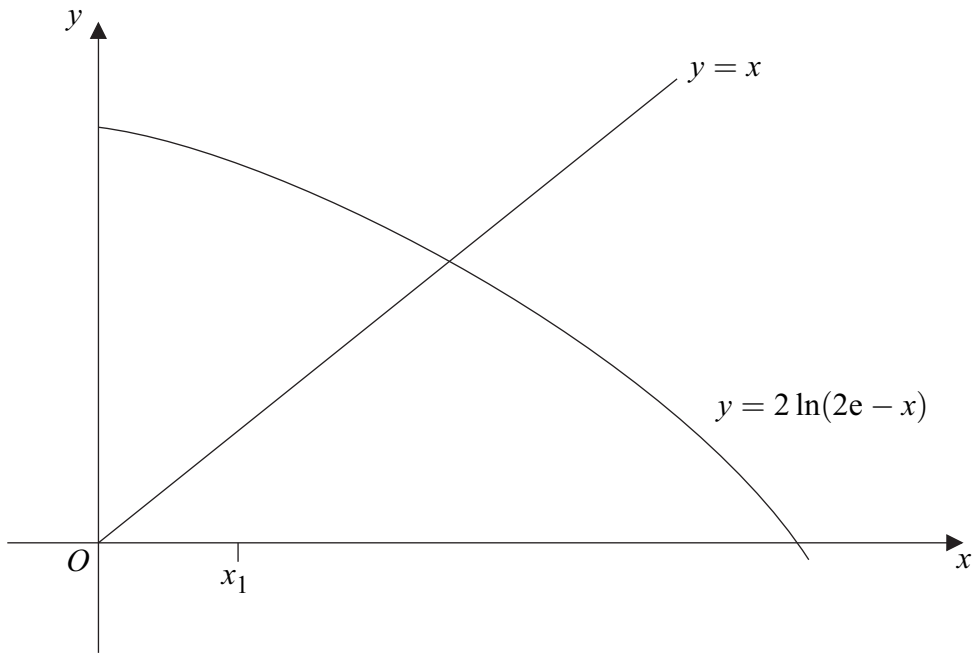
Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

(c)(iii)

Figure 1



3 (a) (i) Differentiate $(x^2 + 1)^{\frac{5}{2}}$ with respect to x.

[2 marks]

(ii) Given that $y = e^{2x}(x^2 + 1)^{\frac{5}{2}}$, find the value of $\frac{dy}{dx}$ when $x = 0$.

[3 marks]

(b) A curve has equation $y = \frac{4x - 3}{x^2 + 1}$. Use the quotient rule to find the x-coordinates of the stationary points of the curve.

[5 marks]

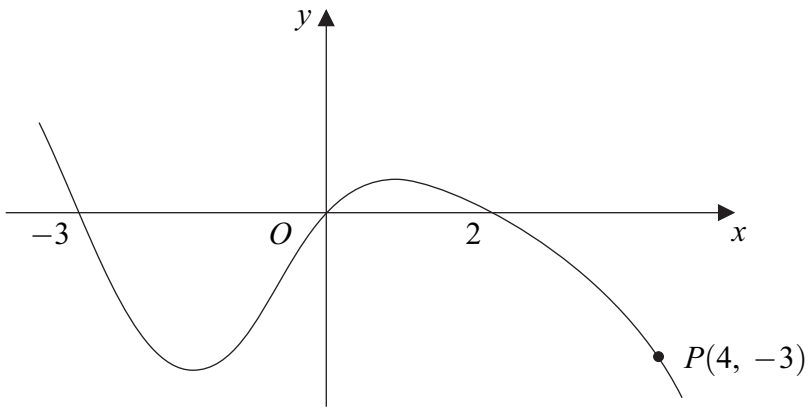
QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 The sketch shows part of the curve with equation $y = \mathrm{f}(x)$.



(a) On **Figure 2** below, sketch the curve with equation $y = -|\mathrm{f}(x)|$.

[3 marks]

(b) On **Figure 3** on the page opposite, sketch the curve with equation $y = \mathrm{f}(|2x|)$.

[2 marks]

(c) (i) Describe a sequence of two geometrical transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{f}(2x + 2)$.

[4 marks]

(ii) Find the coordinates of the image of the point $P(4, -3)$ under the sequence of transformations given in part **(c)(i)**.

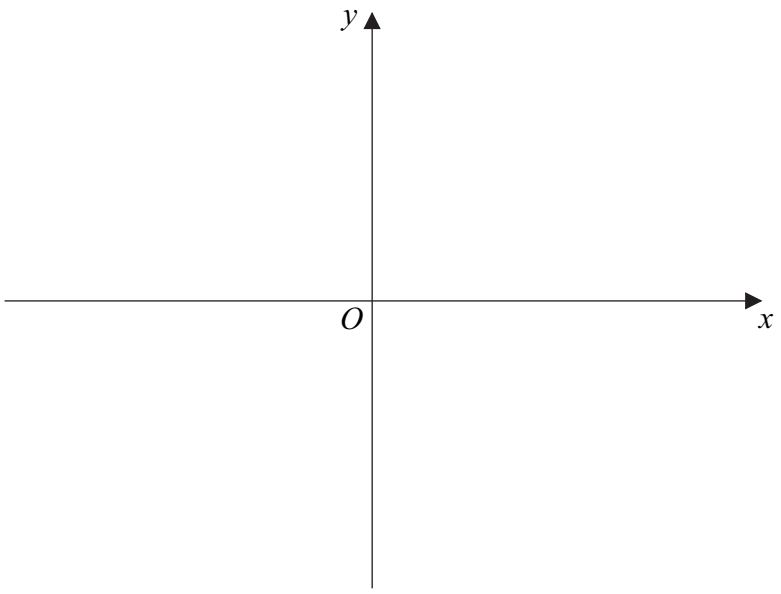
[2 marks]

QUESTION PART REFERENCE

(a)

Answer space for question 4

Figure 2

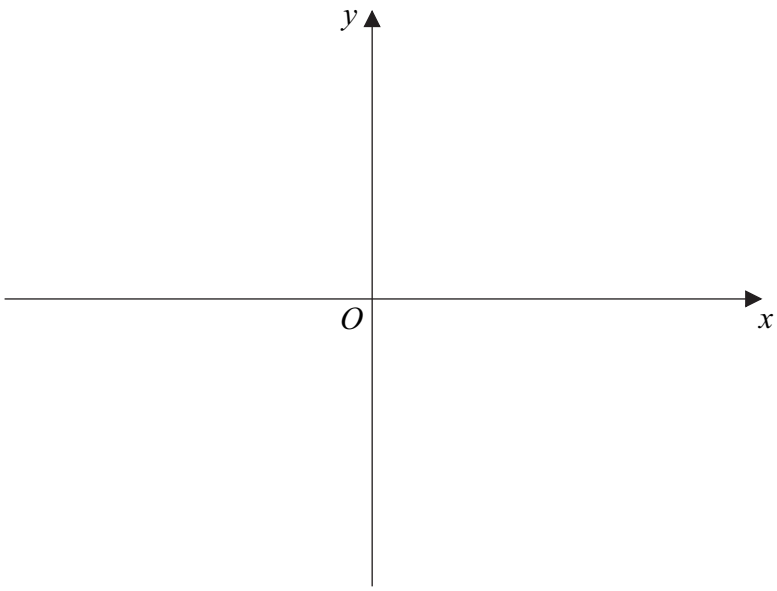


QUESTION PART REFERENCE

Answer space for question 4

(b)

Figure 3



5 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = x^2 - 6x + 5, \quad \text{for } x \geqslant 3$$

$$\mathrm{g}(x) = |x - 6|, \quad \text{for all real values of } x$$

(a) Find the range of f.

[2 marks]

(b) The inverse of f is f^{-1}.

Find $\mathrm{f}^{-1}(x)$. Give your answer in its simplest form.

[4 marks]

(c) (i) Find $\mathrm{gf}(x)$.

[1 mark]

(ii) Solve the equation $\mathrm{gf}(x) = 6$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

6 (a) By using integration by parts twice, find

$$\int x^2 \sin 2x \, \mathrm{d}x$$

[6 marks]

(b) A curve has equation $y = x\sqrt{\sin 2x}$, for $0 \leqslant x \leqslant \frac{\pi}{2}$.

The region bounded by the curve and the x-axis is rotated through 2π radians about the x-axis to generate a solid.

Find the exact value of the volume of the solid generated.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

7 Use the substitution $u = 3 - x^3$ to find the exact value of $\int_0^1 \frac{x^5}{3 - x^3}\,\mathrm{d}x$.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE

Answer space for question 7

8 (a) Show that the expression $\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}$ can be written as $2\sec x$.

[4 marks]

(b) Hence solve the equation

$$\frac{1-\sin x}{\cos x}+\frac{\cos x}{1-\sin x}=\tan^2 x-2$$

giving the values of x to the nearest degree in the interval $0° \leqslant x < 360°$.

[6 marks]

(c) Hence solve the equation

$$\frac{1-\sin(2\theta-30°)}{\cos(2\theta-30°)}+\frac{\cos(2\theta-30°)}{1-\sin(2\theta-30°)}=\tan^2(2\theta-30°)-2$$

giving the values of θ to the nearest degree in the interval $0° \leqslant \theta \leqslant 180°$.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 8

END OF QUESTIONS

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