



Oxford Cambridge and RSA

**GCE**

**Mathematics A**

**H240/03: Pure Mathematics and Mechanics**

A Level

**Mark Scheme for June 2023**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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## MARKING INSTRUCTIONS

### PREPARATION FOR MARKING

#### RM ASSESSOR

1. Make sure that you have accessed and completed the relevant training packages for on-screen marking: RM Assessor Online Training; OCR Essential Guide to Marking.
2. Make sure that you have read and understood the mark scheme and the question paper for this unit. These are posted on the RM Cambridge Assessment Support Portal <http://www.rm.com/support/ca>
3. Log-in to RM Assessor and mark the **required number** of practice responses (“scripts”) and the **number of required** standardisation responses.

### MARKING

1. Mark strictly to the mark scheme.
2. Marks awarded must relate directly to the marking criteria.
3. The schedule of dates is very important. It is essential that you meet the RM Assessor 50% and 100% (traditional 40% Batch 1 and 100% Batch 2) deadlines. If you experience problems, you must contact your Team Leader (Supervisor) without delay.

## 4. Annotations

| Annotation   | Meaning                       |
|--------------|-------------------------------|
| ✓ and ✗      |                               |
| BOD          | Benefit of doubt              |
| FT           | Follow through                |
| ISW          | Ignore subsequent working     |
| M0, M1       | Method mark awarded 0, 1      |
| A0, A1       | Accuracy mark awarded 0, 1    |
| B0, B1       | Independent mark awarded 0, 1 |
| SC           | Special case                  |
| ^            | Omission sign                 |
| MR           | Misread                       |
| BP           | Blank Page                    |
| Seen         |                               |
| Highlighting |                               |

| Other abbreviations in mark scheme | Meaning   |
|------------------------------------|---|
| dep*                               | Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark |
| cao                                | Correct answer only   |
| oe                                 | Or equivalent   |
| rot                                | Rounded or truncated  |
| soi                                | Seen or implied   |
| www                                | Without wrong working   |
| AG                                 | Answer given  |
| awrt                               | Anything which rounds to  |
| BC                                 | By Calculator   |
| DR                                 | This question included the instruction: In this question you must show detailed reasoning.          |

## 5. Subject Specific Marking Instructions

- a. Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

### Award NR (No Response)

- if there is nothing written at all in the answer space and no attempt elsewhere in the script
- OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
- OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.

Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

If you are in any doubt whatsoever you should contact your Team Leader.

- c. The following types of marks are available.

**M**

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words “Determine” or “Show that”, or some other indication that the method must be given explicitly.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
  - When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
- NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for  $g$  should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

- g. Rules for replaced work and multiple attempts:
- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
  - If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
  - if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.
- h. For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i. If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j. If in any case the scheme operates with considerable unfairness consult your Team Leader.



| Question  | Answer  | Marks         | AO         | Guidance  |   |
|---|---|---------------|------------|---|---|
| <b>ENSURE THAT PAGE 20 (WHICH APPEARS ABOVE QUESTION 1) IS EITHER LINKED TO THE CORRESPONDING QUESTION OR ANNOTATED AS 'BP' OR 'SEEN'. NOTE 5(b), 7(b) AND 9(b) ARE OVER TWO PAGES AND THEREFORE THE 2<sup>nd</sup> PAGE MUST BE CHECKED AND MARKED IF USED, OR ANNOTATED AS 'SEEN' IF NOT USED</b> |   |               |            |   |   |
| <b>1</b>  | $4^{2x+1} = 5^x \Rightarrow (2x+1)\log 4 = x\log 5$ | <b>M1*</b>    | <b>1.1</b> | Take logs of both sides (any base) correctly and use power law correctly at <b>least once</b> . Common correct answers that score <b>M1</b> are:<br>$2x+1 = \log_4 5^x$ , $\log_5 4^{2x+1} = x$ ,<br>$(2x+1)\ln 4 = \ln(5^x)$   | Condone lack of bracket on the $2x + 1$ term                                    |
|   | $x(2\log 4 - \log 5) = -\log 4$                     | <b>M1dep*</b> | <b>1.1</b> | Re-arrange to get an equation with a single term in $x$ – condone sign errors only e.g. the following score the first two <b>M</b> marks<br>$\frac{1}{x} + 2 = \frac{\log 5}{\log 4}$ , $x\left(2 - \frac{\log 5}{\log 4}\right) = -1$ ,<br>$x(\log_4 5 - 2) = 1$ ,<br>$x(1 - 2\log_5 4) = \log_5 4$<br>$x(2\ln 4 - \ln 5) + \ln 4 = 0$<br>This mark can be implied by a correct answer provided the first <b>M</b> mark was awarded (that is, we must see logs being taken and the power law used at least once) |   |
|   | $x = \frac{\log 4}{\log 5 - 2\log 4} = -1.19$       | <b>A1</b>     | <b>1.1</b> | awrt $-1.19$  | $-1.19184404\dots$<br><br><b>Correct answer with no working scores no marks</b> |

|          |   |           |  |  |   |
|----------|---|-----------|--|--|---|
|          |   | [3]       |  |  |   |
| <b>1</b> | <b>ALTERNATIVE</b>  |           |  |  |   |
|          | $4^{2x+1} = 5^x \Rightarrow \left(\frac{5}{16}\right)^x = 4$    | <b>B1</b> |  | Correctly re-writes the given equation in the form $a^x = b$ with a and b correct (check carefully for other equivalent correct equations)                               |   |
|          | $x = \log_{\frac{5}{16}} 4$ or $x \log_{\frac{5}{16}} = \log 4$ | <b>M1</b> |  | Taking logs correctly of <b>their</b> $a^x = b$ , where a and b are both positive, to obtain either $x \log a = \log b$ (any base) or $x = \log_a b$ (for their a and b) | Not dependent on the <b>B</b> mark  |
|          | $x = -1.19$   | <b>A1</b> |  | awrt -1.19   | -1.19184404...<br><br><b>Correct answer with no working scores no marks</b> |
|          |   | [3]       |  |  |   |

| Question |     | Answer   | Marks     | AO         | Guidance   |   |
|----------|-----|--|-----------|------------|--|---|
| 2        | (a) | $R = 5$  | <b>B1</b> | <b>1.1</b> | <b>B0</b> for $R = \pm 5, \sqrt{25}$ etc. unless replaced with 5   | No working required for this mark. <b>Ignore working</b>  |
|          |     | $R \cos \alpha = 3$<br>$R \sin \alpha = 4 \Rightarrow \tan \alpha = \frac{4}{3}$ | <b>M1</b> | <b>1.1</b> | <b>M1</b> for $\tan \alpha = k$ where $k = \pm \frac{3}{4}, \pm \frac{4}{3}$ or equivalent e.g. $\cos \alpha = \pm \frac{3}{R}, \sin \alpha = \pm \frac{4}{R}$ with their value of R (but not just R and do not allow reciprocals for this mark). 53.1 (or better) with no working implies <b>M1</b> | <b>SC</b> If $\cos \alpha = 3, \sin \alpha = 4 \Rightarrow \tan \alpha = \frac{4}{3}$ explicitly seen then this scores <b>M1 A0</b> but do not penalise again in (b) (if correct answer seen) |
|          |     | $\alpha = 53.13$   | <b>A1</b> | <b>1.1</b> | <b>www</b> awrt 53.13 (at least 4 sf required) so 53.1 (or 53) is <b>A0</b> (but if an awrt 53.13 seen then isw if replaced with a less accurate value)  | 53.13010235... - an answer in radians scores <b>A0</b><br>53.13 from $R \sin(x - \alpha)$ soi   |
|          |     |  | [3]       |            |  |   |
| 2        | (b) | $x = 53.13 + \arcsin\left(\frac{2}{5}\right)$                                    | <b>M1</b> | <b>1.1</b> | <b>M1</b> for $x = \alpha + \arcsin\left(\frac{2}{R}\right)$ or $x - \alpha = \arcsin\left(\frac{2}{R}\right)$ with their R and $\alpha$ substituted   | <b>SC B1</b> for 76.7 only (in the given range) from using an alternative method e.g. $9 \sin^2 x = (2 + 4 \cos x)^2$   |
|          |     | $x = 76.7$   | <b>A1</b> | <b>1.1</b> | awrt 76.7 (at least 3 sf required) – ignore any answers given outside the range $0 < x < 90$ but do not award this mark if any other values in this range are given – <b>www but see SC in (a)</b>   | <b>Correct answer with no working seen scores SC B1</b><br>Answer in radians scores <b>A0</b>   |
|          |     |  | [2]       |            |  |   |

| Question |     |      | Answer   | Marks                  | AO                       | Guidance  |   |
|----------|-----|------|--|------------------------|--------------------------|---|---|
| 3        | (a) | (i)  | $f(x) = x^3 + px + q \Rightarrow f'(x) = 3x^2 + p$                           | <b>M1</b>              | <b>1.1</b>               | Attempt at differentiating $f(x)$ with at least one non-zero term correct   | <b>M0</b> for $f'(x) = x^2 + p + \frac{q}{x}$   |
|          |     |      | $f'(2) = 13 \Rightarrow p = 1$   | <b>A1</b>              | <b>1.1</b>               | Correct value for $p$   |   |
|          |     |      |  | <b>[2]</b>             |                          |   |   |
| 3        | (a) | (ii) | $2^3 + 2p + q = 0$   | <b>M1</b>              | <b>1.1a</b>              | Substituting $x = 2$ into $f(x)$ and equating to 0 <b>or</b> for correctly re-writing as $(x-2)(x^2 + 2x + 4 + p)$ (with $p$ or their value of $p$ from <b>(a)(i)</b> ) | Could be in terms of $q$ only e.g., $2^3 + 2 \times \text{their}(p) + q = 0$<br><br>Possibly seen in an attempt at long division                        |
|          |     |      | $q = -10$  | <b>A1</b>              | <b>1.1</b>               | Correct value for $q$   |   |
|          |     |      |  | <b>[2]</b>             |                          |   |   |
| 3        | (b) |      | $y = (x-2)^3 + p(x-2) + q - 3$   | <b>B1</b><br><b>B1</b> | <b>1.1</b><br><b>1.1</b> | Substituting $(x \pm 2)$ into both $x$ terms of $y = f(x)$<br>Subtracting 3 (oe) at some stage  | Allow with $p$ and $q$ or with incorrect values of $p$ and/or $q$ – <b>note that using 2 vertically and/or 3 horizontally cannot be treated as a MR</b> |
|          |     |      | $y = x^3 + 3x^2(-2) + 3x(-2)^2 + (-2)^3 + px - 2p + q - 3$                   | <b>M1</b>              | <b>1.1</b>               | Correct expansion of $(x \pm 2)^3$ . Can be unsimplified for <b>M1</b> . If correct bracketing not seen then must be implied by later working                           |   |
|          |     |      | $y = (x^3 - 6x^2 + 12x - 8) + x - 2 - 10 - 3$<br>$y = x^3 - 6x^2 + 13x - 23$ | <b>A1</b>              | <b>2.2a</b>              | cao – condone just the expression $x^3 - 6x^2 + 13x - 23$ (so do not need to see $y = \dots$ )  | For reference:<br>$a = -6$ , $b = 13$ , $c = -23$   |
|          |     |      |  | <b>[4]</b>             |                          |   |   |

| Question |     | Answer                            | Marks | AO   | Guidance   |                                      |
|----------|-----|-----------------------------------|-------|------|--|--------------------------------------|
| 4        | (a) | $(x-3)^2 + (y+5)^2 = -k + 9 + 25$ | M1    | 3.1a | Complete the square (for x and y) to obtain $(x \pm 3)^2 + (y \pm 5)^2 + \dots$  | or for $\dots \pm k \pm 3^2 \pm 5^2$ |
|          |     | $-k + 34 > 0 \Rightarrow k < 34$  | A1    | 2.3  | cao – www<br><br>allow equivalent in either set notation e.g. $\{k : k < 34\}$ or interval notation e.g. $(-\infty, 34)$ or $(-\infty, 34]$ but not $[-\infty, 34)$ or $[-\infty, 34]$ unless $k < 34$ already seen<br><br>If implying a lower limit then A0 | Allow $k \leq 34$                    |
|          |     |                                   | [2]   |      |  |                                      |

| Question |     | Answer   | Marks         | AO          | Guidance  |  |
|----------|-----|--|---------------|-------------|---|--|
| 4        | (b) | $x^2 + y^2 - 6x + 10y - 46 = 0$  |               |             | For reference   |  |
|          |     | $2x + 2y \frac{dy}{dx} - 6 + 10 \frac{dy}{dx} (= 0)$                           | <b>M1*</b>    | <b>3.1a</b> | Attempt to differentiate the equation for C implicitly – must be four terms including a $2y \frac{dy}{dx}$ term and two other terms correct (condone either $-46$ <b>or</b> $k$ appearing in their deriv. as a 5 <sup>th</sup> term) but if the derivative of $x^2 + y^2 - 6x + 10y - 46 = 0$ is put equal to $\frac{1}{2}$ or $\frac{dy}{dx}$ (and used subsequently) then <b>M0</b> | <b>or</b> applying the chain rule to an expression of the form $\pm 5 \pm \sqrt{\lambda \pm (x \pm 3)^2}$ for some non-zero $\lambda$ , so must be of the form $\frac{1}{2}(f(x))^{-\frac{1}{2}}g(x)$ where $f(x)$ is quadratic and $g(x)$ is linear |
|          |     | $\frac{dy}{dx} = \frac{6-2x}{2y+10} \Rightarrow \frac{3-x}{y+5} = \frac{1}{2}$ | <b>M1dep*</b> | <b>1.1</b>  | Sets derivative equal to $\frac{1}{2}$  | <b>or</b> substitutes $\frac{1}{2}$ for $\frac{dy}{dx}$  |
|          |     | $2x + y = 1$<br>$\Rightarrow x^2 + (1 - 2x)^2 - 6x + 10(1 - 2x) - 46 (= 0)$    | <b>M1</b>     | <b>2.1</b>  | Substitutes their linear expression into the given equation of C with $k = -46$ (must be five terms with two quadratic terms and two linear terms in $x$ or if using completing the square form from (a)) to obtain an expression/equation in $x$ (or $y$ ) <b>only</b>   | <b>Dependent on first two M marks</b> – if chain rule used this mark is implied by setting $\frac{dy}{dx}$ equal to $\frac{1}{2}$  |
|          |     | $5x^2 - 30x - 35 (= 0)$  | <b>M1</b>     | <b>1.1</b>  | Simplify to a 3TQ in $x$ (or $y$ ) (allow sign errors <b>only</b> when simplifying from <b>their</b> five term equation/expression <b>or</b> completing the square equation/expression)   | <b>Dependent on all M marks</b><br>For $y$ if correct:<br>$y^2 + 10y - 39 (= 0)$   |
|          |     | $(7, -13), (-1, 3)$  | <b>A1</b>     | <b>2.2a</b> | <b>BC</b> - do not need to be stated as coordinates   | <b>Two values of x and y only</b>  |

| Question |  | Answer | Marks | AO | Guidance |  |
|----------|--|--------|-------|----|----------|--|
|          |  |        | [5]   |    |          |  |

| Question |            | Answer  | Marks         | AO | Guidance   |  |
|----------|------------|---|---------------|----|--|--|
|          |            | <b>Alternative Method for first two marks</b> |               |    |  |  |
| <b>4</b> | <b>(b)</b> | $m_1 = \frac{y+5}{x-3}$ or $y+5 = m_1(x-3)$   | <b>M1*</b>    |    | Finding an expression for the gradient of the line segment that passes through the centre and any point (x, y) on the circumference of C | Follow through their centre from <b>(a)</b> – gradient expression must be correct for their centre |
|          |            | $\frac{y+5}{x-3} = -2$ or $y+5 = -2(x-3)$     | <b>M1dep*</b> |    | Equating gradient with $-2$ (oe)   |  |



| Question |     | Answer   | Marks         | AO          | Guidance  |   |
|----------|-----|--|---------------|-------------|---|---|
| 5        | (a) | $\frac{dx}{dt} = 0.2 + \sin t$   | <b>B1*</b>    | <b>1.1</b>  | <b>B1</b> for $\frac{dx}{dt} = 0.2 \pm \sin t$  |   |
|          |     | $\int_0^\pi k \sin^2 t (0.2 + \sin t) dt \Rightarrow$<br>$\int_0^\pi k(1 - \cos^2 t)(0.2 + \sin t) dt$ | <b>M1dep*</b> | <b>3.1a</b> | Uses $\int y \frac{dx}{dt} dt$ <b>and</b> replaces $\sin^2 t$ with $1 - \cos^2 t$ to obtain an expression involving $\cos^2 t$ and $\sin t$   | <b>No limits required for this mark</b>   |
|          |     | $k \int_0^\pi (0.2 + \sin t - 0.2 \cos^2 t - \sin t \cos^2 t) dt (= 1)$                                | <b>A1</b>     | <b>2.2a</b> | <b>AG</b> (so must be checked carefully for any errors e.g. must contain relevant brackets around the $1 - \cos^2 t$ term(s))<br>A correct expression e.g.<br>$\int_0^\pi k(1 - \cos^2 t)(0.2 + \sin t) dt$<br>followed by the correct given answer can score this mark – <b>limits and dt must be seen at least once (but need not be on the final integral)</b> | Do not need to see = 1 anywhere in their solution (and condone lack of brackets around the integrand) |
|          |     |  | [3]           |             |   |   |

| Question |     | Answer  | Marks         | AO         | Guidance   |  |
|----------|-----|---|---------------|------------|--|--|
| 5        | (b) | $\int (0.2 + \sin t - 0.2(\frac{1}{2})(1 + \cos 2t) - \sin t \cos^2 t) dt$  | <b>B1</b>     | <b>1.2</b> | Correctly <b>applies</b> $2\cos^2 t \equiv 1 + \cos 2t$ (so <b>not</b> just stating this identity) - implied by seeing $-0.1t - 0.05\sin 2t$ after integration   | e.g. <b>applies</b> could be for an attempt to integrate $\frac{1}{2}(1 + \cos 2t)$ <b>or</b> stating this identity in an integral |
|          |     | $= 0.2t - \cos t$   | <b>B1</b>     | <b>1.1</b> | <b>First two terms</b> integrated correctly (Look out for those that have $0.1t$ only from combining $0.2t$ with $-0.1t$ )   | <b>This mark should be awarded if 0.2 is combined with another constant term and integrated correctly</b>                          |
|          |     |   | <b>M1*</b>    | <b>1.1</b> | <b>M1</b> for an answer of the form $\pm pt \pm q \sin 2t \pm r \cos^3 t$ <b>or</b> an answer of the form $\pm pt \pm q \sin 2t \pm r \sin t \sin 2t \pm u \cos t \cos 2t$ <b>or</b> an answer of the form $\pm pt \pm q \cos t \pm r \sin 2t \pm u \sin t \sin 2t \pm v \cos t \cos 2t$ for non-zero p, q, r (and u, v) from integrating $-0.2\cos^2 t - \sin t \cos^2 t$ |  |
|          |     | $-0.1t - 0.05 \sin 2t + \frac{1}{3} \cos^3 t$   | <b>A1</b>     | <b>1.1</b> | <b>A1</b> for the correct remaining three/four terms (Alternatives: $-0.1t - 0.05 \sin 2t + \frac{1}{6} \sin t \sin 2t + \frac{1}{3} \cos t \cos 2t$ <b>or</b> $-0.1t + 0.5 \cos t - 0.05 \sin 2t - \frac{1}{3} \sin t \sin 2t - \frac{1}{6} \cos t \cos 2t$ )   |  |
|          |     | $\left[ 0.1t - \cos t - 0.05 \sin 2t + \frac{1}{3} \cos^3 t \right]_0^\pi$<br>$= (0.1\pi - (-1) - 0 + \frac{1}{3}(-1)^3) - (0 - 1 - 0 + \frac{1}{3})$ | <b>M1dep*</b> | <b>1.1</b> | Uses correct limits correctly $F(\pi) - F(0)$ - condone limits the wrong way round only if the sign of their answer is subsequently changed  | Must be using exact values (so must include term(s) in $\pi$ )   |
|          |     | $k(0.1\pi + 2 - \frac{2}{3}) = 1 \Rightarrow k = \frac{30}{3\pi + 40}$  | <b>A1</b>     | <b>1.1</b> | <b>www</b> oe e.g. $k = \frac{1}{0.1\pi + \frac{4}{3}}$ - isw once a correct expression for k seen   | Correct exact value must be seen at some stage for the final mark – <b>check first A mark carefully</b>                            |
|          |     |   | <b>[6]</b>    |            |  |  |

| Question |     | Answer  | Marks         | AO          | Guidance   |   |
|----------|-----|---|---------------|-------------|--|---|
| 6        | (a) | $u_1 = a = 2\sin\theta, u_3 = a + 2d = -\sqrt{3}\cos\theta$ and<br>$u_4 = a + 3d = \frac{7}{2}\sin\theta$                             |               |             | <b>For reference</b>   |   |
|          |     | $d = \frac{7}{2}\sin\theta + \sqrt{3}\cos\theta$  | <b>B1*</b>    | <b>2.1</b>  | Forming a correct expression for d (or a correct equation containing d) e.g.<br>$(d =) \frac{1}{2}(-\sqrt{3}\cos\theta - 2\sin\theta)$ ,<br>$(d =) \frac{1}{3}(\frac{7}{2}\sin\theta - 2\sin\theta) (= 0.5\sin\theta)$<br>Can be implied e.g.<br>$\frac{7}{2}\sin\theta = 2\sin\theta + 3(\dots)$ seen | Can be implied by a correct equation for $\theta$ |
|          |     | $-\sqrt{3}\cos\theta = 2\sin\theta + 2(\frac{7}{2}\sin\theta + \sqrt{3}\cos\theta) \Rightarrow$<br>$\tan\theta = -\frac{\sqrt{3}}{3}$ | <b>M1dep*</b> | <b>3.1a</b> | Obtaining an equation of the form $\tan\theta = k$ from a trigonometric equation which initially had 3 sine and 1 cosine terms <b>or</b> 2 sine and 2 cosine terms e.g. <b>if</b> correct<br>$\frac{7}{2}\sin\theta = 2\sin\theta + 3(\frac{7}{2}\sin\theta + \sqrt{3}\cos\theta)$                     |   |
|          |     | $\theta = \frac{5}{6}\pi$   | <b>A1</b>     | <b>2.2a</b> | Condone $-\frac{\pi}{6}$ stated too but <b>A0</b> if any other value given in the interval $\frac{1}{2}\pi < \theta < \pi$ (but ignore any values that are given outside this range)   | Exact answer must be seen at some stage           |
|          |     |   | <b>[3]</b>    |             |  |   |

| Question |     | Answer   | Marks       | AO          | Guidance  |   |
|----------|-----|--|-------------|-------------|---|---|
| 6        | (b) | $S_{100} = \frac{100}{2}[2(2 \sin \theta) + (100 - 1)d]$                               | <b>B1ft</b> | <b>1.2</b>  | Correct formula for the sum of an AP with $a = 2 \sin \theta$ (with either $\theta$ or their value of $\theta$ substituted) <b>and</b> either $d$ <b>or</b> their value of $d$ substituted <b>or</b> their expression for $d$ | Follow through their values of $\theta$ and $d$ if used provided $\frac{100}{2}[2(2 \sin \theta) + (100 - 1)d]$ implied |
|          |     | $d = \frac{7}{2} \sin(\frac{5}{6}\pi) + \sqrt{3} \cos(\frac{5}{6}\pi) (= \frac{1}{4})$ | <b>B1ft</b> | <b>1.1</b>  | Correct expression for $d$ <b>using their</b> $\theta$ (e.g. $d = \frac{1}{2}(-\sqrt{3} \cos \theta - 2 \sin \theta)$ ,<br>$d = \frac{1}{3}(\frac{7}{2} \sin \theta - 2 \sin \theta)$ )                                       | Follow through their value of $\theta$ <b>only</b>  |
|          |     | $S_{100} = 1337.5$   | <b>B1</b>   | <b>2.2a</b> | <b>www</b> – must have come from $\theta = \frac{5}{6}\pi$ correctly derived in (a)<br>oe (not for 1338 or 1340 unless 1337.5 seen so isw once 1337.5 (oe e.g. $\frac{2675}{2}$ ) seen)                                       | <b>Correct answer with no working scores all 3 marks</b>  |
|          |     |  | <b>[3]</b>  |             |   |   |

| Question |     | Answer  | Marks         | AO         | Guidance   |  |  |
|----------|-----|---|---------------|------------|--|--|--|
| 7        | (a) | <b>DR</b><br>$\frac{8t}{7+4t^2} - \frac{1}{2} = 0 \Rightarrow 2(8t) - (7+4t^2) = 0$ | <b>M1*</b>    | <b>1.1</b> | Setting equation for a equal to zero and removing $t^2$ correctly from the denominator e.g. $8t - \frac{1}{2}(7+4t^2) = 0$ to obtain the equivalent of a 3TQ in t <b>only</b>  | This mark can be implied by a correct 3TQ in t                     |  |
|          |     | $4t^2 - 16t + 7 = 0 \Rightarrow (2t-1)(2t-7) = 0$                                   | <b>M1dep*</b> | <b>1.1</b> | Correct method for solving their 3TQ in t<br>If factorising:<br>$at^2 + bt + c \Rightarrow (mt+n)(pt+q)$ where $a = mp$ <b>and</b> one of $mq + np = b$ <b>or</b> $c = nq$ so note that<br>$4t^2 - 16t + 7 = (t-0.5)(t-3.5)$ is <b>M0</b> but e.g. $(-2t+7)(2t-1)$ is <b>M1 bod</b><br>If using the formula:<br>must apply the correct formula for their three-term quadratic in t (no errors)<br>If completing the square:<br>The <b>M</b> mark is not awarded until correctly getting to the stage<br>$t-2 = \pm\sqrt{\frac{9}{4}}$ for their 3TQ in t (must include $\pm$ so implying two roots) with no errors (so consistent with applying the formula correctly) | <b>1.1</b>   | <b>Must see the method</b> – the correct answers <b>do not</b> imply this mark therefore<br>$4t^2 - 16t + 7 = 0$<br>$\Rightarrow t = 0.5$ and $3.5$ scores <b>M1 M0 B1</b><br><br>As a minimum must see (if correct) $\frac{16 \pm \sqrt{144}}{8}$ |
|          |     | $t = 0.5$ or $t = 3.5$  | <b>B1</b>     | <b>1.1</b> | This mark is not dependent on the previous <b>M</b> mark(s)  | So <b>M1 M0 B1</b> is common or <b>M0 M0 B1</b> if no working seen |  |

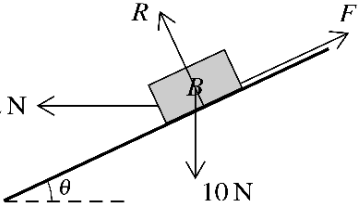
| Question |  | Answer | Marks | AO | Guidance |  |
|----------|--|--------|-------|----|----------|--|
|          |  |        | [3]   |    |          |  |

| Question |     | Answer   | Marks                          | AO                       | Guidance  |   |
|----------|-----|--|--------------------------------|--------------------------|---|---|
| 7        | (b) | $\frac{dv}{dt} = v \left( \frac{8t}{7+4t^2} - \frac{1}{2} \right)$   | <b>B1*</b>                     | <b>3.1b</b>              | Stating the correct differential equation   | Possibly implied by correct separation of variables   |
|          |     | $\ln v = \ln(7+4t^2) - \frac{1}{2}t(+c)$   | <b>B1dep*</b><br><b>B1dep*</b> | <b>1.1</b><br><b>1.1</b> | Correct lhs<br>Correct rhs – not multiplied by v or 5 (or any other constant)   | Condone lack of +c for both <b>B</b> marks  |
|          |     | $t = 0, v = 17.5 \Rightarrow c = \ln 17.5 - \ln 7$   | <b>M1*</b>                     | <b>3.1a</b>              | Uses correct initial conditions to find c from an equation of the form $k_1 \ln v = k_2 \ln(7+4t^2) + k_3 t + c$ (note that e.g. + c may appear on the lhs) | With non-zero values of $k_i$ - accept any equivalent form e.g. $v = A(7+4t^2)e^{-0.5t}$ and then use initial conditions to find A ( <b>if</b> correct then $A = 2.5$ ) |
|          |     | $\ln 5 = \ln(7+4T^2) - \frac{1}{2}T + \ln\left(\frac{5}{2}\right)$   | <b>M1dep*</b>                  | <b>2.1</b>               | Uses $t = T, v = 5$ to obtain an equation in T only – <b>dependent on previous M mark</b>   | <b>Condone use of t for T throughout the remainder of the question</b>  |
|          |     | $\frac{1}{2}T = \ln\left[\frac{5(7+4T^2)}{2 \times 5}\right] \Rightarrow T = 2 \ln\left(\frac{7+4T^2}{2}\right)$ | <b>A1</b>                      | <b>2.2a</b>              | <b>AG</b> so at least one step of intermediate working from substitution of $t = T$ and $v = 5$   | Condone<br>$T = 2 \ln\left \frac{7+4T^2}{2}\right $   |
|          |     |  | <b>[6]</b>                     |                          |   |   |

| Question |     | Answer   | Marks     | AO          | Guidance  |  |
|----------|-----|--|-----------|-------------|---|--|
| 7        | (c) | $T_{n+1} = 2 \ln \left( \frac{7 + 4T_n^2}{2} \right)$ $T_0 = 11.25$ $T_1 = 11.09523175\dots$ $T_2 = 11.04058716\dots$ $T_3 = 11.02111643\dots$ $T_4 = 11.01415608\dots$ $T_5 = 11.011665\dots$ | <b>B1</b> | <b>1.1</b>  | Uses given result and given starting value to obtain correct $T_1$ and $T_2$ (so the first two iterations after the initial value of 11.25) to at least 4 sf (rot) – but all stated values in these two terms must be correct                   |  |
|          |     | $T = 11.01$  | <b>B1</b> | <b>1.1</b>  | Must be stated to 4 sf only – <b>not dependent on the first B mark</b> – can be awarded if either of $T_2$ and/or $T_3$ incorrect (assume that the iterative process corrected itself or a slip in the candidate writing down an earlier value) | <b>Must be clear that T is 11.01 (and not the final term shown in the iterative process)</b> – this mark can be awarded from using alternative iterative methods e.g. Newton-Raphson |
|          |     |  | [2]       |             |   |  |
| 7        | (d) | $11.01 - 3.5 = 7.51$ (s)   | <b>B1</b> | <b>2.2a</b> | awrt 7.51   | No follow through from incorrect earlier values  |
|          |     |  | [1]       |             |   |  |



| Question | Answer  | Marks  | AO  | Guidance  |   |
|----------|---|--------|-----|---|---|
| 8        | $6\mathbf{i} = \mathbf{u} + 4(3\mathbf{i} - 2\mathbf{j})$ | M1*    | 3.3 | Applying $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ correctly - working must imply that $\mathbf{v}$ and $\mathbf{a}$ are vectors<br><br>Or for $\mathbf{v} = 3t\mathbf{i} - 2t\mathbf{j} + \mathbf{c}$ and using $t = 4, \mathbf{v} = 6\mathbf{i}$ to find $\mathbf{c}$ | M0 if $\mathbf{u} = -6\mathbf{i} \pm 14\mathbf{j}$  |
|          | $\mathbf{u} = -6\mathbf{i} + 8\mathbf{j}$                 | A1     | 2.5 | or for $\mathbf{v} = (3t - 6)\mathbf{i} + (-2t + 8)\mathbf{j}$ and setting $t = 0$ to obtain correct $\mathbf{u}$   |   |
|          | $u = \sqrt{(-6)^2 + 8^2}$                                 | M1dep* | 1.1 | Correctly taking the magnitude of their $\mathbf{u}$ but condone<br>$\sqrt{-6^2 + 8^2} = \sqrt{\pm 36 + 64}$  | Correct answer following $-6\mathbf{i} + 8\mathbf{j}$ (with no wrong working) scores full marks |
|          | $u = 10 \text{ (ms}^{-1}\text{)}$                         | A1     | 1.1 | www   |   |
|          |   | [4]    |     |   |   |

| Question |     | Answer  | Marks      | AO         | Guidance   |  |
|----------|-----|---|------------|------------|--|--|
| 9        | (a) |  | <b>B1</b>  | <b>1.2</b> | Correct force diagram showing the weight, normal contact, and frictional contact forces only (but ignore labelling of these forces even if labelled incorrectly) but must include all arrows pointing in the correct direction – the line of action of all forces must pass through the block (or imply passing through the block) but do not need to be attached to the block | Ignore components of any of the three forces if shown – if <b>only</b> components shown then <b>B0</b> |
|          |     |   | <b>[1]</b> |            |  |  |

| Question |     | Answer  | Marks    | AO         | Guidance   |   |
|----------|-----|---|----------|------------|--|---|
| 9        | (b) |   | M1*      | 3.3        | Resolving parallel <b>or</b> perpendicular to the plane – correct number of terms - <b>must</b> be using 10 for the weight so no marks until this value used/applied (however, see <b>SC</b> in the final <b>A</b> mark) | Allow sin/cos confusion and sign errors – <b>but the 2 and 10 must be resolved and the R and F should not</b><br>(if considering $\perp$ & $\parallel$ )  |
|          |     | $R + 2\sin\theta = 10\cos\theta$<br>$F = 2\cos\theta + 10\sin\theta$  | A1<br>A1 | 1.1<br>1.1 | Condone $\leq$ or $\geq$ with F for this mark but withhold the final <b>A</b> mark   | Where R is the normal contact force and F is the frictional contact force   |
|          |     | $2\cos\theta + 10\sin\theta \leq 0.8(10\cos\theta - 2\sin\theta)$   | M1dep*   | 3.4        | Applying $F \leq \mu R$ or $F = \mu R$ to obtain an equation/inequality in $\theta$ only – where R and F are a linear combination of the correct number of relevant resolved terms                                       |   |
|          |     | $10\cos\theta + 50\sin\theta \leq 40\cos\theta - 8\sin\theta$<br>( $\tan\theta \leq \frac{30}{58} \Rightarrow$ greatest value of) $\tan\theta$ is $\frac{15}{29}$ | A1       | 2.2a       | oe e.g. $\frac{30}{58}$ but <b>A0</b> for $\frac{6}{11.6}$ as a final answer. Allow awrt 0.517. Isw if candidates go on to work out $\theta$ .<br>Allow $\tan\theta \leq \frac{15}{29}$ (oe) as a final answer           | Can use equals throughout (no justification required)<br>But <b>not</b> $\tan\theta < \frac{15}{29}$<br><b>If using 10g for the weight, then SC M1* A1 A0 M1dep* A0 max</b> (where the first A mark is for both equations correct using 10g rather than 10) |
|          |     |   | [5]      |            |  |   |
|          |     | <b>Alternative for first three marks</b>  | M1*      |            | Resolving vertically and horizontally  | <b>M1 conditions as above</b>   |
|          |     | $R\cos\theta + F\sin\theta = 10$  | A1       |            |  |   |
|          |     | $F\cos\theta = 2 + R\sin\theta$   | A1       |            | <b>Final M1dep* and A marks as above</b>   |   |

| Question |     | Answer   | Marks  | AO   | Guidance   |   |
|----------|-----|--|--------|------|--|---|
| 10       | (a) | $\mathbf{R} = (-4 + a)\mathbf{i} + (2 + b)\mathbf{j}$                        | B1     | 1.1  | oe e.g. $\begin{pmatrix} -4 \\ 2 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$   |   |
|          |     | $(-4 + a)\mathbf{i} + (2 + b)\mathbf{j} = k(-\mathbf{i} + 3\mathbf{j})$      | M1     | 3.1b | Sets their $\mathbf{R}$ equal to k times $-\mathbf{i} + 3\mathbf{j}$ (or k times $\mathbf{R}$ ) where k is non-numerical/unknown | Implied by a correct equation in a and b e.g. $\frac{a-4}{2+b} = -\frac{1}{3}$ <b>but not from stating</b> $-4 + a = -1$ and $2 + b = 3$ (so must have come from a 'gradient' approach if k not seen) |
|          |     | $k = 4 - a$ therefore $2 + b = 3(4 - a)$<br>so $3a + b = 10$                 | A1     | 2.2a | AG Eliminate k and derive given result   | Using an assumed value of k is A0   |
|          |     |  | [3]    |      |  |   |
| 10       | (b) | $a = 6 \Rightarrow b = -8 \therefore \mathbf{R} = 2\mathbf{i} - 6\mathbf{j}$ | B1     | 1.1  |  |   |
|          |     | $ \mathbf{R}  = \sqrt{2^2 + (-6)^2}$   | M1*    | 3.3  | Calculate $ \mathbf{R} $ or $ \mathbf{R} ^2$ for their $\mathbf{R}$  |   |
|          |     | $\sqrt{2^2 + (-6)^2} = m(5\sqrt{10})$  | M1dep* | 3.4  | N2L applied to the magnitude of their $\mathbf{R}$ with $5\sqrt{10}$   |   |
|          |     | $m = \frac{\sqrt{40}}{5\sqrt{10}} = 0.4$                                     | A1     | 2.2a | www 0.4 (oe) but square roots cancelled  |   |
|          |     |  | [4]    |      |  |   |

| Question |     | Answer   | Marks | AO   | Guidance  |  |
|----------|-----|--|-------|------|---|--|
| 11       | (a) |  | M1    | 3.1b | Moments about A to form an equation with the correct number of terms and both the weight and contact force at B resolved - condone 20g for the weight of the rod. All relevant values must have been substituted. May take moments about another point (and resolve) but must end up after elimination with an equation in $R_B$ only | <b>Resolved</b> for the weight means either sin or cos with an angle of either 30 or 60 <b>only</b> , and for the contact force at B either sin or cos with an angle of 25 or 35 or 55 or 65 <b>only</b>                               |
|          |     | $1.4(20 \cos 30) = R_B(2.8 \cos 25)$   | M1    | 1.1  | One of the two moment terms correct   | <b>Must be as part of a two-term moment equation</b> (so dimensionally correct so <b>M0</b> if 20g for the weight) but the other term need not be resolved (or resolved correctly) e.g. $1.4(20 \cos 30) = 2.8R_B$ scores <b>M0 M1</b> |
|          |     | $R_B = \frac{28 \cos 30}{2.8 \cos 25} = 9.5555330\dots$<br>= 9.56 (N) (correct to 3 significant figures) | A1    | 1.1  | <b>AG</b> - correct equation followed by 9.56 scores all 3 marks  | Allow awrt 9.56  |
|          |     |  | [3]   |      |   |  |

For reference in parts (a) and (b):

$$\text{Moments about B: } R_A(2.8 \cos 30) = F_A(2.8 \sin 30) + 20(1.4 \cos 30)$$

$$\text{Moments about the mid-point: } R_A(1.4 \cos 30) = F_A(1.4 \sin 30) + R_B(1.4 \cos 25)$$

| Question |     | Answer  | Marks                  | AO                       | Guidance  |  |
|----------|-----|---|------------------------|--------------------------|---|--|
| 11       | (b) |   | <b>M1*</b>             | <b>3.3</b>               | Attempt at resolving vertically <b>or</b> horizontally – correct number of terms with the contact force at B resolved (angle must be one of 25, 35, 55 or 65 only) – or taking moments again (see below) (same conditions for taking moments as in part (a)).<br><b>M0</b> if using W or mg only for the weight of the rod (must substitute in the given value before awarding this mark – see second guidance column if using 20g) | Allow sign errors, sin/cos confusion and 20g used as the weight for the <b>M</b> mark.<br>Allow $R_B$ instead of the given value of 9.56 for the <b>M</b> mark |
|          |     | $R_A + 9.56 \sin 35 = 20$<br>$F_A = 9.56 \cos 35$ | <b>A1</b><br><b>A1</b> | <b>1.1</b><br><b>1.1</b> | Correct expression/equation for $R_A$<br>Correct expression/equation for $F_A$  | $R_A = 20 - 5.48... = 14.5...$<br>$F_A = 7.83...$  |
|          |     | $\sqrt{(9.56 \cos 35)^2 + (20 - 9.56 \sin 35)^2}$ | <b>M1dep*</b>          | <b>1.1</b>               | Correct method for calculating the magnitude of the contact force at A where $R_A$ and $F_A$ are a linear combination of the correct number of terms with 9.56 resolved in both equations (angle must be one of 25, 35, 55 or 65 only)  |  |
|          |     | 16.5 (N)  | <b>A1</b>              | <b>2.2a</b>              | <b>www</b> awrt 16.5. For reference if using 9.56 for $R_B$ then 16.494179...   | If using more accurate value for $R_B$ then 16.494698...   |
|          |     |   | <b>[5]</b>             |                          |   |  |

| Question  | Answer | Marks | AO | Guidance |  |
|---|--------|-------|----|----------|--|
| <p>For reference in parts (a) and (b):</p> <p>Moments about B: <math>R_A(2.8 \cos 30) = F_A(2.8 \sin 30) + 20(1.4 \cos 30)</math></p> <p>Moments about the mid-point: <math>R_A(1.4 \cos 30) = F_A(1.4 \sin 30) + R_B(1.4 \cos 25)</math></p> |        |       |    |          |  |

| Question |     | Answer  | Marks | AO   | Guidance  |   |
|----------|-----|---|-------|------|---|---|
| 12       | (a) | $0 = (39 \sin \theta)^2 + 2(-10)h$                    | M1    | 3.3  | Using $v^2 = u^2 + 2as$ with $v = 0$ ,<br>$a = \pm 10$ or $\pm 9.8$ or $\pm g$ , and $u = 39 \sin \theta$<br>or $u = 39 \cos \theta$ .<br>Accept any other complete method (using<br>correct suvat equations) to find the<br>maximum height e.g. $0 = 39 \sin \theta - 10t$<br><b>and</b> with $t$ then substituted into<br>$s = (39 \sin \theta)t + 0.5(-10)t^2$ | Condone $g = 9.8$ for full<br>marks in (a) and (b)                      |
|          |     | $0 = \left(39 \times \frac{5}{13}\right)^2 + 2(-10)h$ | A1    | 1.1  | A correct equation with the correct value<br>of $\sin$ substituted <b>or</b> for<br>$0 = (39 \sin(22.6\dots))^2 + 2(-10)h$  | Allow using $g$ (and not<br>replaced with 10 or 9.8) for<br>this A mark |
|          |     | Max. height = $20 + h = 31.25$ (m)                    | A1    | 2.2a | <b>www</b> <ul style="list-style-type: none"> <li>• accept awrt 31.2 (using 22.6... as<br/> the angle and <math>g = 10</math>)</li> <li>• accept awrt 31.5 (coming from<br/> exact value of <math>\sin</math> or 22.6... and <math>g</math><br/> = 9.8)</li> <li>• condone 31.3 (3 sf)</li> </ul>   |   |
|          |     |   | [3]   |      |   |   |



| Question |     | Answer   | Marks | AO  | Guidance   |   |
|----------|-----|--|-------|-----|--|---|
| 12       | (b) | $-20 = (39 \sin \theta)T + \frac{1}{2}(-10)T^2$                    | M1    | 3.3 | Applying $s = ut + \frac{1}{2}at^2$ with $s = \pm 20$ ,<br>$a = \pm 10$ or $\pm 9.8$ or $\pm g$ and $u = 39 \sin \theta$<br><b>or</b> $u = 39 \cos \theta$   | <b>Condone <math>g = 9.8</math> for full marks in (a) and (b)</b><br><br>Accept any other complete method to find T |
|          |     | $-20 = \left(39 \times \frac{5}{13}\right)T + \frac{1}{2}(-10)T^2$ | A1    | 1.1 | With correct value of sine or $\sin(22.6)$<br>Allow $-20 = \left(39 \times \frac{5}{13}\right)T + \frac{1}{2}(-9.8)T^2$  | Allow using $g$ (and not replaced with 10 or 9.8) for this A mark   |
|          |     | T = 4 <b>only</b>  | A1    | 1.1 | <b>BC</b> <ul style="list-style-type: none"> <li>accept awrt 4.07 (using <math>g = 9.8</math> and exact value of sine)</li> <li>accept awrt 4.06 (using <math>g = 9.8</math> and 22.6...)</li> <li>a value of 4(.00...) coming from 3.998... (using <math>g = 10</math> and 22.6... for sine)</li> </ul> | Condone $t = 4$   |
|          |     |  | [3]   |     |  |   |

| Question |     | Answer  | Marks      | AO          | Guidance  |  |
|----------|-----|---|------------|-------------|---|--|
| 12       | (c) | <p>Examples of possible limitations</p> <ul style="list-style-type: none"> <li>• The ball will have dimensions/volume</li> <li>• The spin/rotational forces of the ball</li> <li>• A 1sf approx. to <math>g</math> was used</li> <li>• Other weather conditions (ignore wind and air resistance)</li> <li>• The ball is not a particle</li> <li>• <math>g</math> is modelled as a (universal) constant</li> </ul> | <b>B1</b>  | <b>3.5b</b> | <p>Allow any correct limitation</p> <p><b>B0</b> if referring to</p> <ul style="list-style-type: none"> <li>• Air resistance and/or wind (only)</li> <li>• The ground is unlikely to be horizontal (only)</li> <li>• The mass or weight or shape of P (only)</li> <li>• The angle/heights/speeds may not be as quoted (only)</li> <li>• Modelling the problem as 3D rather than in 2D (only)</li> </ul> <p>If multiple limitations given, and any are incorrect, then <b>B0</b></p> |  |
|          |     |   | <b>[1]</b> |             |   |  |

| Question |     | Answer   | Marks  | AO   | Guidance  |   |
|----------|-----|--|--------|------|---|---|
| 12       | (d) | $a = 3kt^2 + 12t + \frac{3}{2}$                        | M1     | 3.4  | Differentiate given v (at least two terms correct)  |   |
|          |     | $BC = \left(39 \times \frac{12}{13}\right) 'T' (=144)$ | M1*    | 3.1b | Applying $s = ut$ horizontally to find distance BC with correct value of cos (or cos(22.6...)) and their value of T from (b)  | Allow $g = 9.8$ which <b>if</b> correct leads to 146.34...  |
|          |     | $s = \frac{1}{4}kt^4 + 2t^3 + \frac{3}{4}t^2 (+c)$     | M1*    | 2.1  | Integrate given v (at least two terms correct)  |   |
|          |     | $\frac{1}{4}k(4)^4 + 2(4)^3 + \frac{3}{4}(4)^2 = 144$  | M1dep* | 3.4  | Puts their integrated expression for s, with $t =$ their T from (b), equal to their distance for BC to form an equation in k only – <b>dependent on the two previous M marks only</b> | Must not include a +c unless dealt with as part of a definite integral. However, if a +c is included then subsequently ignored/set equal to zero without justification then give <b>bod</b> for this and any subsequent A marks (if earned) |
|          |     | $k = \frac{1}{16}$                                     | A1     | 1.1  | Correct exact value for k (oe e.g. 0.0625)  | <b>Final two marks can only be awarded if <math>g = 10</math> used</b>  |
|          |     | $a = 52.5 \text{ (m s}^{-2}\text{)}$                   | A1     | 2.2a | oe  |   |
|          |     |  | [6]    |      |   |   |

| Question   | Answer | Marks  | AO            | Guidance   |  |   |
|--|--------|--|---------------|------------|--|---|
| <b>SEE APPENDIX AT THE END OF THE MS FOR IMPORTANT INFORMATION REGARDING THIS PART</b> |        |  |               |            |  |   |
| 13   | (a)    |  | <b>M1*</b>    | <b>2.1</b> | Applying N2L parallel to the plane for P – correct number of terms and weight component resolved – allow sign errors and sin/cos confusion. Allow g missing but <b>M0</b> if 4ga in N2L  | <b>For the first five marks condone:</b> $\mu$ used as the coefficient of friction for both P and B, or implying that $\mu_B = 2\mu_P$ rather than the correct $\mu_P = 2\mu_B$ |
|  |        | $4g \sin 60 - F_p - T = 4a$  | <b>A1</b>     | <b>1.1</b> | NB $T + F_p - 4g \sin 60 = 4a$ is <b>A1</b> (taking up the plane as +ve dir.)  | Where $F_p$ is the frictional force for P   |
|  |        | $R_p = 4g \cos 60$   | <b>B1</b>     | <b>3.3</b> | Resolving <b>correctly</b> perpendicular to the plane for P  | Where $R_p$ is the normal contact force for P   |
|  |        | $4g \sin 60 - \mu_p(4g \cos 60) - T = 4a$<br>$\Rightarrow 2\sqrt{3}g - 2\mu_p g - T = 4a$                      | <b>M1dep*</b> | <b>3.4</b> | Use of $F = \mu R$ in the attempt at N2L for P with their R which must be a component of 4g only   | Where $\mu_p$ is the coefficient of friction between P and the plane  |
|  |        | $T - \mu_B(2g) = 2a$   | <b>M1</b>     | <b>3.3</b> | Applying N2L parallel to the surface for B – correct number of terms – allow sign errors but note that $\mu_B(2g) - T = 2a$ is consistent with $T + F_p - 4g \sin 60 = 4a$ and therefore gives the correct answer (and is not incorrect working) | Where $\mu_B$ is the coefficient of friction between B and the plane<br><br>Allow g missing but <b>M0</b> if 2ga in N2L or for $T - F_B = 2a$ <b>only</b>                       |
|  |        | $2\sqrt{3}g - 2\mu_p g - T = 4a$ and $2T - 4\mu_B g = 4a$<br>with $\mu_p = 2\mu_B$ gives $2\sqrt{3}g - T = 2T$ | <b>A1</b>     | <b>3.3</b> | Solving simultaneously with $\mu_p = 2\mu_B$ (soi) to obtain a correct equation in T only  |   |

| Question | Answer   | Marks | AO   | Guidance                   |                   |
|----------|--|-------|------|----------------------------|-------------------|
|          | $2\sqrt{3}g - T = 2T \Rightarrow T = \frac{2\sqrt{3}}{3}g$ | A1    | 2.2a | Must be seen in terms of g | Accept awrt 1.15g |
|          |  | [7]   |      |                            |                   |

| Question | Answer  | Marks      | AO          | Guidance   |  |
|----------|---|------------|-------------|--|--|
| 13(b)    | $1.9 = 2(0.5) + \frac{1}{2}a_p(0.5)^2$                                  | <b>B1</b>  | <b>3.4</b>  | Applying $s = ut + \frac{1}{2}at^2$ correctly to find $a_p$  |  |
|          | $a_p = 7.2$   | <b>B1</b>  | <b>1.1</b>  |  |  |
|          | $4g \sin 60 - \mu_p(4g \cos 60) = 4a_p$                                 | <b>M1</b>  | <b>3.1b</b> | Set $T = 0$ (or apply N2L) to obtain an expression for the acceleration of P when the string breaks  | Correct number of terms, dimensionally correct (so g not missing), must be using the correct mass of 4 – <b>allow sin/cos mix and sign errors only</b> |
|          | $\mu_p = 0.26266... \Rightarrow \mu_B = 0.13(1...)$                     | <b>A1</b>  | <b>3.4</b>  | Using their acceleration of P to correctly calculate the coefficient of friction for B. Can be implied from a correct deceleration of B. Accept 0.13 (so <b>2 sf</b> ) or better | For reference: exact value is $\frac{-72 + 49\sqrt{3}}{98}$ (which scores <b>A1</b> )  |
|          | $a_B = -\mu_B g \Rightarrow$ deceleration is 1.29 ( $\text{m s}^{-2}$ ) | <b>A1</b>  | <b>3.2a</b> | Accept awrt 1.29 or $-1.29$  | 1.28704895... or, for reference, exact value is $\frac{-72 + 49\sqrt{3}}{10}$ (which scores <b>A1</b> )  |
|          |   | <b>[5]</b> |             |  |  |

## APPENDIX (To assist with 13a)

Exemplar responses for Q13(a) – see main MS for Guidance for the requirements/conditions to award each of these marks

| Response   | Max. Mark   |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
|--|---|--|--------------------------------------|--------------------------------------|---|---|---------------------------------------|--------------------------------------|---|---|----------|
| <p>Case 1: candidates who immediately set <math>\mu</math> as the coefficient of friction for B and <math>2\mu</math> as the coefficient of friction for P (correct)</p> $4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}$ $R_p = 4g \cos 60 \quad \mathbf{B1}$ $4g \sin 60 - 2\mu(4g \cos 60) - T = 4a \quad \mathbf{M1}$ $T - \mu(2g) = 2a \quad \mathbf{M1}$ $2\sqrt{3}g - T = 2T \quad \mathbf{A1}$ $T = \frac{2\sqrt{3}}{3}g \quad \mathbf{A1}$ <p>Or equivalent e.g. replace <math>2\mu</math> above with <math>\mu</math> and <math>\mu</math> above with <math>0.5\mu</math></p>  | <b>7</b>  |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| <p>Case 2: candidates who immediately set <math>2\mu</math> as the coefficient of friction for B and <math>\mu</math> as the coefficient of friction for P or set both coefficients of frictions equal to <math>\mu</math> (both of these are incorrect)</p> <table style="width: 100%; border: none;"> <tr> <td style="width: 50%; border: none;"><math>4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}</math></td> <td style="width: 50%; border: none;"><math>4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}</math></td> </tr> <tr> <td style="border: none;"><math>R_p = 4g \cos 60 \quad \mathbf{B1}</math></td> <td style="border: none;"><math>R_p = 4g \cos 60 \quad \mathbf{B1}</math></td> </tr> <tr> <td style="border: none;"><math>4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}</math></td> <td style="border: none;"><math>4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}</math></td> </tr> <tr> <td style="border: none;"><math>T - 2\mu(2g) = 2a \quad \mathbf{M1}</math></td> <td style="border: none;"><math>T - \mu(2g) = 2a \quad \mathbf{M1}</math></td> </tr> <tr> <td style="border: none;"><math>4g \sin 60 - 4g\mu \cos 60 - T = 2T - 8\mu g \quad \mathbf{A0 A0}</math></td> <td style="border: none;"><math>4g \sin 60 - 4g\mu \cos 60 - T = 2T - 4\mu g \quad \mathbf{A0 A0}</math></td> </tr> </table> | $4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}$                    | $4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}$ | $R_p = 4g \cos 60 \quad \mathbf{B1}$ | $R_p = 4g \cos 60 \quad \mathbf{B1}$ | $4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}$ | $4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}$ | $T - 2\mu(2g) = 2a \quad \mathbf{M1}$ | $T - \mu(2g) = 2a \quad \mathbf{M1}$ | $4g \sin 60 - 4g\mu \cos 60 - T = 2T - 8\mu g \quad \mathbf{A0 A0}$ | $4g \sin 60 - 4g\mu \cos 60 - T = 2T - 4\mu g \quad \mathbf{A0 A0}$ | <b>5</b> |
| $4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}$   | $4g \sin 60 - F_p - T = 4a \quad \mathbf{M1 A1}$                    |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| $R_p = 4g \cos 60 \quad \mathbf{B1}$   | $R_p = 4g \cos 60 \quad \mathbf{B1}$                                |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| $4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}$  | $4g \sin 60 - \mu(4g \cos 60) - T = 4a \quad \mathbf{M1}$           |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| $T - 2\mu(2g) = 2a \quad \mathbf{M1}$  | $T - \mu(2g) = 2a \quad \mathbf{M1}$                                |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| $4g \sin 60 - 4g\mu \cos 60 - T = 2T - 8\mu g \quad \mathbf{A0 A0}$  | $4g \sin 60 - 4g\mu \cos 60 - T = 2T - 4\mu g \quad \mathbf{A0 A0}$ |  |                                      |                                      |   |   |                                       |                                      |   |   |          |
| <p>Case 3: assuming that as <math>\mu_p = 2\mu_B</math> then this implies that <math>F_p = 2F_B</math> without justification (e.g. no calculation of the normal contact forces for P and B considered – so while it turns out to be correct in this case it is not true in general) – if this is assumed then</p> $4g \sin 60 - 2F - T = 4a \quad \mathbf{M1 A1}$ $T - F = 2a \quad \mathbf{M1} \text{ (so now allow this M mark even though in the main MS this would have been M0)}$ <p>leading to <math>2g\sqrt{3} - 2(T - 2a) - T = 4a</math> and therefore <math>T = \frac{2\sqrt{3}}{3}g</math> no further marks (3 marks max. also if assuming that <math>2F_p = F_B</math>)</p>  | <b>3</b>  |  |                                      |                                      |   |   |                                       |                                      |   |   |          |

|  |          |
|--|----------|
| Case 4: if assuming that $a = 0$ or $g$ or any other value then they can score the <b>B</b> mark only for $R_p = 4g \cos 60$ | <b>1</b> |
|--|----------|



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