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# Mark Scheme (Results)
# January 2008

GCE

## GCE Mathematics (6666/01)



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# January 2008
# 6666 Core Mathematics C4
# Mark Scheme

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **1.** (a) | $x$: 0, $\frac{\pi}{4}$, $\frac{\pi}{2}$, $\frac{3\pi}{4}$, $\pi$ <br> $y$: 0, 1.844321332…, 4.810477381…, 8.87207, 0 | | |
| | | awrt 1.84432 | B1 |
| | | awrt 4.81048 or 4.81047 | B1 |
| | | | **[2]** |
| (b) **Way 1** | [0 can be implied] <br> $\text{Area} \approx \frac{1}{2}\times\frac{\pi}{4}\,;\times\underline{\left\{0+2\left(1.84432+4.81048+8.87207\right)+0\right\}}$ | Outside brackets awrt 0.39 or $\frac{1}{2}\times$ awrt 0.79 <br> $\frac{1}{2}\times\frac{\pi}{4}$ or $\frac{\pi}{8}$ | B1 |
| | | <u>For structure of trapezium rule</u> $\underline{\{............\}}$; | <u>M1</u> √ |
| | | Correct expression <u>inside brackets</u> which all must be multiplied by their "outside constant". | <u>A1</u> √ |
| | $=\frac{\pi}{8}\times 31.05374... \;=\; 12.19477518... = \underline{12.1948}$ (4dp) | <u>12.1948</u> | A1 **cao** |
| | | | **[4]** |
| ***Aliter*** (b) **Way 2** | $\text{Area} \approx \frac{\pi}{4}\times\left\{\frac{0+1.84432}{2}+\frac{1.84432+4.81048}{2}+\frac{4.81048+8.87207}{2}+\frac{8.87207+0}{2}\right\}$ | $\frac{\pi}{4}$ (or awrt 0.79) and a divisor of 2 on all terms inside brackets. | B1 |
| | which is equivalent to: | One of first and last ordinates, two of the middle ordinates inside brackets ignoring the 2. | <u>M1</u> √ |
| | $\text{Area} \approx \frac{1}{2}\times\frac{\pi}{4}\,;\times\underline{\left\{0+2\left(1.84432+4.81048+8.87207\right)+0\right\}}$ | Correct expression inside brackets if $\frac{1}{2}$ was to be factorised out. | <u>A1</u> √ |
| | $=\frac{\pi}{4}\times 15.52687... \;=\; 12.19477518... = \underline{12.1948}$ (4dp) | <u>12.1948</u> | A1 **cao** |
| | | | **[4]** |
| | | | **6 marks** |

Note an expression like $\text{Area} \approx \frac{1}{2}\times\frac{\pi}{4}+2\left(1.84432+4.81048+8.87207\right)$ would score B1M1A0A0

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | ** represents a constant (which must be consistent for first accuracy mark) | | |
| **2.** (a) | $(8-3x)^{\frac{1}{3}} = \underline{(8)^{\frac{1}{3}}}\left(1-\dfrac{3x}{8}\right)^{\frac{1}{3}} = \underline{2}\left(1-\dfrac{3x}{8}\right)^{\frac{1}{3}}$ | Takes 8 outside the bracket to give any of $\underline{(8)^{\frac{1}{3}}}$ or $\underline{2}$. | B1 |
| | $= 2\left\{1+(\tfrac{1}{3})(**x);+\dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3+\ldots\right\}$ **with** $** \neq 1$ | Expands $(1+**x)^{\frac{1}{3}}$ to give a simplified or an un-simplified $1+(\frac{1}{3})(**x)$; A correct simplified or an un-simplified $\{.........\}$ expansion with candidate's followed through $(**x)$ | M1; A1 $\sqrt{}$ |
| | $= 2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-\tfrac{3x}{8})^3+\ldots\right\}$ | Award SC M1 if you see $\dfrac{(\frac{1}{3})(-\frac{2}{3})}{2!}(**x)^2+\dfrac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(**x)^3$ | |
| | $= 2\left\{1-\tfrac{1}{8}x;-\tfrac{1}{64}x^2-\tfrac{5}{1536}x^3-\ldots\right\}$ | Either $2\{1-\frac{1}{8}x\ ........\}$ or anything that cancels to $2-\frac{1}{4}x$; | A1; |
| | $= 2-\dfrac{1}{4}x;-\dfrac{1}{32}x^2-\dfrac{5}{768}x^3-\ldots$ | Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$ | A1 |
| | | | **[5]** |
| (b) | $(7.7)^{\frac{1}{3}} \approx 2-\dfrac{1}{4}(0.1)-\dfrac{1}{32}(0.1)^2-\dfrac{5}{768}(0.1)^3-\ldots$ | ***Attempt to substitute*** $x=0.1$ into a candidate's binomial expansion. | M1 |
| | $= 2 - 0.025 - 0.0003125 - 0.0000065104166\ldots$ | | |
| | $= 1.97468099\ldots$ | awrt 1.9746810 | A1 |
| | | | **[2]** |
| | | | **7 marks** |

You would award B1M1A0 for

$$=2\left\{1+(\tfrac{1}{3})(-\tfrac{3x}{8})+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(-\tfrac{3x}{8})^2+\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(-3x)^3+\ldots\right\}$$

because ** is not consistent.

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822\ldots$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **2.** (a) **Way 2** | $(8-3x)^{\frac{1}{3}}$ | | |
| | | $2$ or $(8)^{\frac{1}{3}}$ **(See note ↓)** | B1 |
| | $=\left\{\begin{array}{l}(8)^{\frac{1}{3}}+(\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x);+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2\\ \quad +\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3+...\end{array}\right\}$ | Expands $(8-3x)^{\frac{1}{3}}$ to give an un-simplified or simplified $(8)^{\frac{1}{3}}+(\tfrac{1}{3})(8)^{-\frac{2}{3}}(**x)$; | M1; |
| | **with** $**\neq 1$ | A correct un-simplified or simplified $\{.........\}$ expansion with candidate's followed through $(**x)$ | A1 $\sqrt{}$ |
| | $=\left\{\begin{array}{l}(8)^{\frac{1}{3}}+(\tfrac{1}{3})(8)^{-\frac{2}{3}}(-3x);+\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(-3x)^2\\ \quad +\frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(-3x)^3+...\end{array}\right\}$ | Award SC M1 if you see $\frac{(\frac{1}{3})(-\frac{2}{3})}{2!}(8)^{-\frac{5}{3}}(**x)^2 + \frac{(\frac{1}{3})(-\frac{2}{3})(-\frac{5}{3})}{3!}(8)^{-\frac{8}{3}}(**x)^3$ | |
| | $=\left\{2+(\tfrac{1}{3})(\tfrac{1}{4})(-3x)+(-\tfrac{1}{9})(\tfrac{1}{32})(9x^2)+(\tfrac{5}{81})(\tfrac{1}{256})(-27x^3)+...\right\}$ | | |
| | $=2-\frac{1}{4}x;-\frac{1}{32}x^2-\frac{5}{768}x^3-...$ | Anything that cancels to $2-\frac{1}{4}x$; or $2\{1-\frac{1}{8}x\ ........\}$ | A1; |
| | | Simplified $-\frac{1}{32}x^2-\frac{5}{768}x^3$ | A1 |
| | | | **[5]** |

Attempts using Maclaurin expansion should be escalated up to your team leader.

Be wary of calculator value of $(7.7)^{\frac{1}{3}} = 1.974680822...$

If you see the constant term "2" in a candidate's final binomial expansion, then you can award B1.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 3. | Volume $= \underline{\pi \int_a^b \left(\frac{1}{2x+1}\right)^2 \mathrm{d}x} = \pi \int_a^b \frac{1}{(2x+1)^2} \mathrm{d}x$ | Use of $V = \underline{\pi \int y^2}\ \mathrm{d}x$. Can be implied. Ignore limits. | B1 |
| | $= \pi \int_a^b (2x+1)^{-2} \mathrm{d}x$ | | |
| | $= (\pi)\left[\frac{(2x+1)^{-1}}{(-1)(2)}\right]_a^b$ | | |
| | $= (\pi)\left[\underline{-\tfrac{1}{2}(2x+1)^{-1}}\right]_a^b$ | Integrating to give $\underline{\pm p(2x+1)^{-1}}$<br>$\underline{-\tfrac{1}{2}(2x+1)^{-1}}$ | M1<br>A1 |
| | $= (\pi)\left[\left(\frac{-1}{2(2b+1)}\right) - \left(\frac{-1}{2(2a+1)}\right)\right]$ | Substitutes limits of $b$ and $a$ and subtracts the correct way round. | dM1 |
| | $= \frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$ | | |
| | $= \frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$ | | |
| | $= \frac{\pi(b-a)}{(2a+1)(2b+1)}$ | $\underline{\frac{\pi(b-a)}{(2a+1)(2b+1)}}$ | A1 aef |
| | | | **[5]** |
| | | | **5 marks** |

Allow other equivalent forms such as

$$\frac{\pi b - \pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b - \pi a}{4ab+2a+2b+1}.$$

Note that $\pi$ is not required for the middle three marks of this question.

| Question Number | Scheme | | Marks |
| --- | --- | --- | --- |
| ***Aliter* 3. Way 2** | Volume $=\pi\int_a^b\left(\frac{1}{2x+1}\right)^2\,\mathrm{d}x = \pi\int_a^b\frac{1}{(2x+1)^2}\,\mathrm{d}x$ | Use of $V=\pi\int y^2\,\mathrm{d}x$. Can be implied. Ignore limits. | B1 |
| | $=\pi\int_a^b(2x+1)^{-2}\,\mathrm{d}x$ | | |
| | Applying substitution $u=2x+1 \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x}=2$ and changing limits $x\to u$ so that $a\to 2a+1$ and $b\to 2b+1$, gives | | |
| | $=(\pi)\int_{2a+1}^{2b+1}\frac{u^{-2}}{2}\,\mathrm{d}u$ | | |
| | $=(\pi)\left[\frac{u^{-1}}{(-1)(2)}\right]_{2a+1}^{2b+1}$ | | |
| | $=(\pi)\left[-\frac{1}{2}u^{-1}\right]_{2a+1}^{2b+1}$ | Integrating to give $\pm pu^{-1}$ | M1 |
| | | $-\frac{1}{2}u^{-1}$ | A1 |
| | $=(\pi)\left[\left(\frac{-1}{2(2b+1)}\right)-\left(\frac{-1}{2(2a+1)}\right)\right]$ | Substitutes limits of $2b+1$ and $2a+1$ and subtracts the correct way round. | dM1 |
| | $=\frac{\pi}{2}\left[\frac{-2a-1+2b+1}{(2a+1)(2b+1)}\right]$ | | |
| | $=\frac{\pi}{2}\left[\frac{2(b-a)}{(2a+1)(2b+1)}\right]$ | | |
| | $=\frac{\pi(b-a)}{(2a+1)(2b+1)}$ | $\frac{\pi(b-a)}{(2a+1)(2b+1)}$ | A1 aef |
| | | | **[5]** |
| | | | **5 marks** |

Note that $\pi$ is not required for the middle three marks of this question.

Allow other equivalent forms such as

$$\frac{\pi b-\pi a}{(2a+1)(2b+1)} \text{ or } \frac{-\pi(a-b)}{(2a+1)(2b+1)} \text{ or } \frac{\pi(b-a)}{4ab+2a+2b+1} \text{ or } \frac{\pi b-\pi a}{4ab+2a+2b+1}.$$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **4.** (i) | $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = \int 1.\ln\left(\frac{x}{2}\right) \mathrm{d}x \Rightarrow \left\{ \begin{array}{lll} u = \ln\left(\frac{x}{2}\right) & \Rightarrow & \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\frac{1}{2}}{\frac{x}{2}} = \frac{1}{x} \\ \frac{\mathrm{d}v}{\mathrm{d}x} = 1 & \Rightarrow & v = x \end{array} \right\}$ | | |
| | $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = x\ln\left(\frac{x}{2}\right) - \int x.\frac{1}{x} \mathrm{d}x$ | Use of 'integration by parts' formula in the correct direction.<br>Correct expression. | M1<br>A1 |
| | $= x\ln\left(\frac{x}{2}\right) - \int \underline{1} \mathrm{d}x$ | An attempt to multiply $x$ by a candidate's $\frac{a}{x}$ or $\frac{1}{bx}$ or $\frac{1}{x}$. | <u>dM1</u> |
| | $= x\ln\left(\frac{x}{2}\right) - x + c$ | Correct integration with $+ c$ | A1 aef<br>**[4]** |
| (ii) | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \mathrm{d}x$ | | |
| | $\left[\text{NB}: \underline{\cos 2x = \pm 1 \pm 2\sin^2 x} \text{ or } \underline{\sin^2 x = \frac{1}{2}(\pm 1 \pm \cos 2x)}\right]$ | Consideration of double angle formula for $\cos 2x$ | M1 |
| | $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{1 - \cos 2x}{2} \mathrm{d}x = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\underline{1 - \cos 2x}) \mathrm{d}x$ | | |
| | $= \frac{1}{2}\left[\underline{x - \frac{1}{2}\sin 2x}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ | <u>Integrating to give</u> <u>$\pm ax \pm b\sin 2x$</u>; $a, b \neq 0$<br>Correct result of anything equivalent to $\frac{1}{2}x - \frac{1}{4}\sin 2x$ | dM1<br>A1 |
| | $= \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{\sin(\pi)}{2}\right) - \left(\frac{\pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right)\right]$<br>$= \frac{1}{2}\left[\left(\frac{\pi}{2} - 0\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)\right]$ | Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. | ddM1 |
| | $= \frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right) = \frac{\pi}{8} + \frac{1}{4}$ | $\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$<br>Candidate must collect their $\pi$ term and constant term together for A1<br>No fluked answers, hence **cso**. | A1 **aef**, **cso**<br>**[5]** |
| | | | **9 marks** |

Note: $\int \ln\left(\frac{x}{2}\right) \mathrm{d}x = (\text{their } v)\ln\left(\frac{x}{2}\right) - \int (\text{their } v).\left(\text{their } \frac{\mathrm{d}u}{\mathrm{d}x}\right) \mathrm{d}x$ for M1 in part (i).

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **4.** (i) **Way 2** | $\int \ln\left(\frac{x}{2}\right)\,dx = \int (\ln x - \ln 2)\,dx = \int \ln x\,dx - \int \ln 2\,dx$ | | |
| | $\int \ln x\,dx = \int 1.\ln x\,dx \Rightarrow \left\{\begin{array}{ll} u = \ln x & \Rightarrow \frac{du}{dx} = \frac{1}{x} \\ \frac{dv}{dx} = 1 & \Rightarrow v = x \end{array}\right\}$ | | |
| | $\int \ln x\,dx = x\ln x - \int x.\frac{1}{x}\,dx$ | Use of 'integration by parts' formula in the correct direction. | M1 |
| | $= x\ln x - x + c$ | Correct integration of $\ln x$ with or without $+ c$ | A1 |
| | $\int \ln 2\,dx = x\ln 2 + c$ | Correct integration of $\ln 2$ with or without $+ c$ | M1 |
| | Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln x - x - x\ln 2 + c$ | Correct integration with $+ c$ | A1 aef |
| | | | **[4]** |

Note: $\int \ln x\,dx = (\text{their } v)\ln x - \int (\text{their } v).(\text{their } \frac{du}{dx})\,dx$ for M1 in part (i).

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **4.** (i) **Way 3** | $\int \ln\left(\frac{x}{2}\right)\,dx$ | | |
| | $u = \frac{x}{2} \Rightarrow \frac{du}{dx} = \frac{1}{2}$ | | |
| | $\int \ln\left(\frac{x}{2}\right)\,dx = 2\int \ln u\,du$ | Applying substitution correctly to give $\int \ln\left(\frac{x}{2}\right)\,dx = 2\int \ln u\,du$ ***Decide to award 2nd M1 here!*** | |
| | $\int \ln u\,dx = \int 1.\ln u\,du$ | | |
| | $\int \ln u\,dx = u\ln u - \int u.\frac{1}{u}\,du$ | Use of 'integration by parts' formula in the correct direction. | M1 |
| | $= u\ln u - u + c$ | Correct integration of $\ln u$ with or without $+ c$ | A1 |
| | | ***Decide to award 2nd M1 here!*** | M1 |
| | $\int \ln\left(\frac{x}{2}\right)dx = 2\left(u\ln u - u\right) + c$ | | |
| | Hence, $\int \ln\left(\frac{x}{2}\right)\,dx = x\ln\left(\frac{x}{2}\right) - x + c$ | Correct integration with $+ c$ | A1 aef |
| | | | **[4]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **4.** (ii) **Way 2** | $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x . \sin x \, dx$ and $I = \int \sin^2 x \, dx$ | | |
| | $\left\{ \begin{array}{l} u = \sin x \Rightarrow \frac{du}{dx} = \cos x \\ \frac{dv}{dx} = \sin x \Rightarrow v = -\cos x \end{array} \right\}$ | | |
| | $\therefore I = \underline{\left\{ -\sin x \cos x + \int \cos^2 x \, dx \right\}}$ | An attempt to use the correct by parts formula. | M1 |
| | $\therefore I = \left\{ -\sin x \cos x + \int (1 - \sin^2 x) dx \right\}$ | | |
| | $\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx - \int \sin^2 x \, dx \right\}$ | | |
| | $2\int \sin^2 x \, dx = \left\{ -\sin x \cos x + \int 1 \, dx \right\}$ | For the LHS becoming $2I$ | dM1 |
| | $2\int \sin^2 x \, dx = \{ -\sin x \cos x + x \}$ | | |
| | $\int \sin^2 x \, dx = \left\{ \underline{-\tfrac{1}{2} \sin x \cos x + \tfrac{x}{2}} \right\}$ | <u>Correct integration</u> | A1 |
| | $\therefore \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^2 x \, dx = \left[ \left( -\tfrac{1}{2}\sin(\tfrac{\pi}{2})\cos(\tfrac{\pi}{2}) + \tfrac{(\frac{\pi}{2})}{2} \right) - \left( -\tfrac{1}{2}\sin(\tfrac{\pi}{4})\cos(\tfrac{\pi}{4}) + \tfrac{(\frac{\pi}{4})}{2} \right) \right]$ <br> $= \left[ (0 + \tfrac{\pi}{4}) - (-\tfrac{1}{4} + \tfrac{\pi}{8}) \right]$ | Substitutes limits of $\frac{\pi}{2}$ and $\frac{\pi}{4}$ and subtracts the correct way round. | ddM1 |
| | $= \frac{\pi}{8} + \frac{1}{4}$ | $\underline{\frac{1}{2}\left(\frac{\pi}{4} + \frac{1}{2}\right)}$ or $\underline{\frac{\pi}{8} + \frac{1}{4}}$ or $\underline{\frac{\pi}{8} + \frac{2}{8}}$ <br> Candidate must collect their $\pi$ term and constant term together for A1 <br> No fluked answers, hence **cso**. | A1 **aef** **cso** <br> **[5]** |

Note $\frac{\pi}{8} + \frac{1}{4} = 0.64269...$

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **5.** (a) | $x^3 - 4y^2 = 12xy \quad (\text{ eqn } *)$ | | |
| | $x = -8 \Rightarrow -512 - 4y^2 = 12(-8)y$<br>$-512 - 4y^2 = -96y$ | Substitutes $x = -8$ (at least once) into * to obtain a three term quadratic in $y$. Condone the loss of $= 0$. | M1 |
| | $4y^2 - 96y + 512 = 0$<br>$y^2 - 24y + 128 = 0$ | | |
| | $(y-16)(y-8) = 0$<br>$y = \dfrac{24 \pm \sqrt{576 - 4(128)}}{2}$ | An attempt to solve the quadratic in $y$ by either factorising or by the formula or by ***completing the square***. | dM1 |
| | $y = 16$ or $y = 8$. | Both $\underline{y = 16}$ and $\underline{y = 8}$. or $\underline{(-8, 8)}$ and $\underline{(-8, 16)}$. | A1 |
| | | | **[3]** |
| (b) | $\left\{\frac{\cancel{dy}}{\cancel{dx}} \times\right\}$ $3x^2 - 8y\frac{dy}{dx};= \left(\underline{12y + 12x\frac{dy}{dx}}\right)$ | Differentiates implicitly to include either $\pm ky\frac{dy}{dx}$ or $12x\frac{dy}{dx}$. Ignore $\frac{dy}{dx} = ...$<br>Correct LHS equation;<br>Correct application of product rule | M1<br>A1;<br>(B1) |
| | $\left\{\dfrac{dy}{dx} = \dfrac{3x^2 - 12y}{12x + 8y}\right\}$ | *not necessarily required.* | |
| | @ $(-8, 8)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(8)}{12(-8) + 8(8)} = \dfrac{96}{-32} = \underline{-3}$,<br>@ $(-8, 16)$, $\dfrac{dy}{dx} = \dfrac{3(64) - 12(16)}{12(-8) + 8(16)} = \dfrac{0}{32} = \underline{0}$. | Substitutes $x = -8$ and *at least one* of their $y$-values to attempt to find any one of $\frac{dy}{dx}$.<br>One gradient found.<br>Both gradients of $\underline{-3}$ and $\underline{0}$ ***correctly*** found. | dM1<br><br>A1<br>A1 **cso** |
| | | | **[6]** |
| | | | **9 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **5.** (b) **Way 2** | $\left\{\frac{\cancel{d}x}{\cancel{d}y}\cancel{\times}\right\}$ $3x^2\frac{dx}{dy}-8y;\ =\underline{\left(12y\frac{dx}{dy}+12x\right)}$ | Differentiates implicitly to include either $\pm kx^2\frac{dx}{dy}$ or $12y\frac{dx}{dy}$. Ignore $\frac{dx}{dy}=\ldots$ | M1 |
| | | Correct LHS equation | A1; |
| | | <u>Correct application of product rule</u> | (B1) |
| | $\left\{\frac{dy}{dx}=\frac{3x^2-12y}{12x+8y}\right\}$ | *not necessarily required.* | |
| | @ $(-8, 8)$, $\frac{dy}{dx}=\frac{3(64)-12(8)}{12(-8)+8(8)}=\frac{96}{-32}=\underline{-3}$, | Substitutes $x=-8$ and *at least one* of their $y$-values to attempt to find any one of $\frac{dy}{dx}$ or $\frac{dx}{dy}$. | dM1 |
| | @ $(-8, 16)$, $\frac{dy}{dx}=\frac{3(64)-12(16)}{12(-8)+8(16)}=\frac{0}{32}=\underline{0}$. | One gradient found. | A1 |
| | | Both gradients of <u>-3</u> and <u>0</u> ***correctly*** found. | A1 **cso** |
| | | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **5.** (b) **Way 3** | $x^3 - 4y^2 = 12xy$ ( eqn $*$ ) | | |
| | $4y^2 + 12xy - x^3 = 0$ | | |
| | $y = \dfrac{-12x \pm \sqrt{144x^2 - 4(4)(-x^3)}}{8}$ | | |
| | $y = \dfrac{-12x \pm \sqrt{144x^2 + 16x^3}}{8}$ | | |
| | $y = \dfrac{-12x \pm 4\sqrt{9x^2 + x^3}}{8}$ | | |
| | $y = -\frac{3}{2}x \pm \frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$ | | |
| | $\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$ | A credible attempt to make $y$ the subject and an attempt to differentiate either $-\frac{3}{2}x$ or $\frac{1}{2}\left(9x^2 + x^3\right)^{\frac{1}{2}}$. | M1 |
| | $\dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18x + 3x^2}{4(9x^2 + x^3)^{\frac{1}{2}}}$ | $\dfrac{dy}{dx} = -\frac{3}{2} \pm k\left(9x^2 + x^3\right)^{-\frac{1}{2}}\left(g(x)\right)$ | A1 |
| | | $\dfrac{dy}{dx} = -\frac{3}{2} \pm \frac{1}{2}\left(\frac{1}{2}\right)\left(9x^2 + x^3\right)^{-\frac{1}{2}};\left(18x + 3x^2\right)$ | A1 |
| | @ $x = -8$ $\quad \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{18(-8) + 3(64)}{4(9(64) + (-512))^{\frac{1}{2}}}$ | Substitutes $x = -8$ find any one of $\frac{dy}{dx}$. | dM1 |
| | $= -\dfrac{3}{2} \pm \dfrac{48}{4\sqrt{(64)}} = -\dfrac{3}{2} \pm \dfrac{48}{32}$ | | |
| | $\therefore \dfrac{dy}{dx} = -\dfrac{3}{2} \pm \dfrac{3}{2} = \underline{-3}, \underline{0}.$ | One gradient correctly found. | A1 |
| | | Both gradients of $\underline{-3}$ and $\underline{0}$ correctly found. | A1 |
| | | | **[6]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** (a) | $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$ | | |
| | $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$ | Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. | M1 ± |
| | | Correct answer. | A1 |
| | | | **[2]** |
| (b) | $l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ | An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$ | M1 |
| | $l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ | $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or<br>$\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or<br>$\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or<br>$\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$<br>($\mathbf{r}$ is needed.) | A1 √<br>aef |
| | | | **[2]** |
| (c) | $l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ | | |
| | $\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & $\theta$ is angle | | |
| | $\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$ | ← Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$. | M1 √ |
| | $\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$ | Correct followed through expression or **equation**. | A1 √ |
| | $\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79}.$ | $\underline{\theta = 45^\circ \text{ or } \frac{\pi}{4} \text{ or awrt } 0.79}$ | A1 **cao** |
| | | | **[3]** |

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **6.** (d) | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$ | | |
| | $\mathbf{i}: \quad 2+\lambda=\mu \quad (1)$<br>$\mathbf{j}: \quad 6-2\lambda=0 \quad (2)$<br>$\mathbf{k}: -1+2\lambda=\mu \quad (3)$ | **Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. | M1 $\sqrt{}$ |
| | (2) yields $\lambda=3$<br>Any two yields $\lambda=3, \quad \mu=5$ | Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …<br>either one of $\lambda$ or $\mu$ correct. | dM1<br>A1 |
| | $l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ | $\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$<br>Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier. | A1 **cso** |
| | | | **[4]** |
| **Aliter 6.** (d) **Way 2** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$ | | |
| | $\mathbf{i}: \quad 3+\lambda=\mu \quad (1)$<br>$\mathbf{j}: \quad 4-2\lambda=0 \quad (2)$<br>$\mathbf{k}: \quad 1+2\lambda=\mu \quad (3)$ | **Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. | M1 $\sqrt{}$ |
| | (2) yields $\lambda=2$<br>Any two yields $\lambda=2, \quad \mu=5$ | Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …<br>either one of $\lambda$ or $\mu$ correct. | dM1<br>A1 |
| | $l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}+2\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ | $\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$<br>Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier. | A1 **cso** |
| | | | **[4]** |
| | | | **11 marks** |

**Note:** Be careful! $\lambda$ and $\mu$ are not defined in the question, so a candidate could interchange these or use different scalar parameters.

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter* 6. (d) Way 3** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$ | | |
| | $\mathbf{i}: \quad 2-\lambda=\mu \quad (1)$<br>$\mathbf{j}: \quad 6+2\lambda=0 \quad (2)$<br>$\mathbf{k}: -1-2\lambda=\mu \quad (3)$ | **Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. | M1 $\sqrt{}$ |
| | (2) yields $\lambda=-3$<br>Any two yields $\lambda=-3,\ \mu=5$ | Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … | dM1 |
| | | either one of $\lambda$ or $\mu$ correct. | A1 |
| | $l_1: \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}-3\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ | $\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$<br>Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier. | A1 **cso** |
| | | | **[4]** |
| **Aliter 6. (d) Way 4** | If $l_1$ and $l_2$ intersect then: $\begin{pmatrix}3\\4\\1\end{pmatrix}+\lambda\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$ | | |
| | $\mathbf{i}: \quad 3-\lambda=\mu \quad (1)$<br>$\mathbf{j}: \quad 4+2\lambda=0 \quad (2)$<br>$\mathbf{k}: \quad 1-2\lambda=\mu \quad (3)$ | **Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly. | M1 $\sqrt{}$ |
| | (2) yields $\lambda=-2$<br>Any two yields $\lambda=-2,\ \mu=5$ | Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find … | dM1 |
| | | either one of $\lambda$ or $\mu$ correct. | A1 |
| | $l_1: \mathbf{r}=\begin{pmatrix}3\\4\\1\end{pmatrix}-2\begin{pmatrix}-1\\2\\-2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ | $\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$<br>Fully correct solution & no incorrect values of $\lambda$ or $\mu$ seen earlier. | A1 **cso** |
| | | | **[4]** |
| | | | **11 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **7.** (a) | $\left[x = \ln(t+2),\ y = \frac{1}{t+1}\right],\quad \Rightarrow \frac{dx}{dt} = \frac{1}{t+2}$ | Must state $\frac{dx}{dt} = \frac{1}{t+2}$ | B1 |
| | $\text{Area}(R) = \int_{\ln 2}^{\ln 4} \frac{1}{t+1}\,dx ;= \int_0^2 \left(\frac{1}{t+1}\right)\left(\frac{1}{t+2}\right)dt$ | Area $= \int \frac{1}{t+1}\,dx$. Ignore limits. | M1; |
| | | $\int \left(\frac{1}{t+1}\right)\times\left(\frac{1}{t+2}\right)dt$. Ignore limits. | A1 **AG** |
| | Changing limits, when:<br>$x = \ln 2 \Rightarrow \ln 2 = \ln(t+2) \Rightarrow 2 = t+2 \Rightarrow t = 0$<br>$x = \ln 4 \Rightarrow \ln 4 = \ln(t+2) \Rightarrow 4 = t+2 \Rightarrow t = 2$ | changes limits $x \to t$ so that $\ln 2 \to 0$ and $\ln 4 \to 2$ | B1 |
| | Hence, $\text{Area}(R) = \int_0^2 \frac{1}{(t+1)(t+2)}\,dt$ | | |
| | | | **[4]** |
| (b) | $\left(\frac{1}{(t+1)(t+2)}\right) = \frac{A}{(t+1)} + \frac{B}{(t+2)}$ | $\frac{A}{(t+1)} + \frac{B}{(t+2)}$ with $A$ and $B$ found | M1 |
| | $1 = A(t+2) + B(t+1)$ | | |
| | Let $t = -1,\ 1 = A(1) \Rightarrow \underline{A = 1}$<br>Let $t = -2,\ 1 = B(-1) \Rightarrow \underline{B = -1}$ | Finds both $A$ and $B$ correctly. Can be implied. (See note below) | A1 |
| | $\int_0^2 \frac{1}{(t+1)(t+2)}\,dt = \int_0^2 \frac{1}{(t+1)} - \frac{1}{(t+2)}\,dt$ | | |
| | $= \left[\ln(t+1) - \ln(t+2)\right]_0^2$ | Either $\pm a\ln(t+1)$ or $\pm b\ln(t+2)$<br>Both ln terms correctly ft. | dM1<br>A1 $\sqrt{\ }$ |
| | $= (\ln 3 - \ln 4) - (\ln 1 - \ln 2)$ | Substitutes ***both*** limits of 2 and 0 and subtracts the correct way round. | ddM1 |
| | $= \ln 3 - \ln 4 + \ln 2 = \ln 3 - \ln 2 = \ln\left(\frac{3}{2}\right)$ | $\underline{\ln 3 - \ln 4 + \ln 2}$ or $\underline{\ln\left(\frac{3}{4}\right) - \ln\left(\frac{1}{2}\right)}$ or $\underline{\ln 3 - \ln 2}$ or $\underline{\ln\left(\frac{3}{2}\right)}$ (must deal with ln 1) | A1 aef isw |
| | | | **[6]** |

Takes out brackets.

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} + \frac{1}{(t+2)}$ means first M1A0 in (b).

Writing down $\frac{1}{(t+1)(t+2)} = \frac{1}{(t+1)} - \frac{1}{(t+2)}$ means first M1A1 in (b).

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $x = \ln(t+2), \quad y = \dfrac{1}{t+1}$ | | |
| **7.** (c) | $e^x = t + 2 \;\Rightarrow\; t = e^x - 2$ | Attempt to make $t = \ldots$ the subject<br>giving $t = e^x - 2$ | M1<br>A1 |
| | $y = \dfrac{1}{e^x - 2 + 1} \;\Rightarrow\; y = \dfrac{1}{e^x - 1}$ | Eliminates $t$ by substituting in $y$<br>giving $y = \dfrac{1}{e^x - 1}$ | dM1<br>A1 |
| | | | **[4]** |
| ***Aliter*** **7.** (c) **Way 2** | $t + 1 = \dfrac{1}{y} \;\Rightarrow\; t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$<br>$y(t+1) = 1 \;\Rightarrow\; yt + y = 1 \;\Rightarrow yt = 1 - y \;\Rightarrow t = \dfrac{1-y}{y}$ | Attempt to make $t = \ldots$ the subject<br>Giving either $t = \dfrac{1}{y} - 1$ or $t = \dfrac{1-y}{y}$ | M1<br>A1 |
| | $x = \ln\left(\dfrac{1}{y} - 1 + 2\right)$ or $x = \ln\left(\dfrac{1-y}{y} + 2\right)$ | Eliminates $t$ by substituting in $x$ | dM1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ | | |
| | $e^x = \dfrac{1}{y} + 1$ | | |
| | $e^x - 1 = \dfrac{1}{y}$ | | |
| | $y = \dfrac{1}{e^x - 1}$ | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |
| (d) | Domain : $\underline{x > 0}$ | $\underline{x > 0}$ or just $> 0$ | B1 |
| | | | **[1]** |
| | | | **15 marks** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| ***Aliter*** **7.** (c) **Way 3** | $e^x = t+2 \Rightarrow t+1 = e^x - 1$ | Attempt to make $t+1 = \ldots$ the subject<br>giving $t+1 = e^x - 1$ | M1<br>A1 |
| | $y = \dfrac{1}{t+1} \Rightarrow y = \dfrac{1}{e^x - 1}$ | Eliminates $t$ by substituting in $y$<br>giving $y = \dfrac{1}{e^x - 1}$ | dM1<br>A1 |
| | | | **[4]** |
| ***Aliter*** **7.** (c) **Way 4** | $t+1 = \dfrac{1}{y} \Rightarrow t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$ | Attempt to make $t+2 = \ldots$ the subject<br>Either $t+2 = \dfrac{1}{y} + 1$ or $t+2 = \dfrac{1+y}{y}$ | M1<br>A1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ or $x = \ln\left(\dfrac{1+y}{y}\right)$ | Eliminates $t$ by substituting in $x$ | dM1 |
| | $x = \ln\left(\dfrac{1}{y} + 1\right)$ | | |
| | $e^x = \dfrac{1}{y} + 1 \Rightarrow e^x - 1 = \dfrac{1}{y}$ | | |
| | $y = \dfrac{1}{e^x - 1}$ | giving $y = \dfrac{1}{e^x - 1}$ | A1 |
| | | | **[4]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** (a) | $\frac{dV}{dt} = 1600 - c\sqrt{h}$ or $\frac{dV}{dt} = 1600 - k\sqrt{h}$, | Either of these statements | M1 |
| | $\left(V = 4000h \Rightarrow\right) \frac{dV}{dh} = 4000$ | $\frac{dV}{dh} = 4000$ or $\frac{dh}{dV} = \frac{1}{4000}$ | M1 |
| | $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt} = \frac{\frac{dV}{dt}}{\frac{dV}{dh}}$ | | |
| | Either, $\frac{dh}{dt} = \frac{1600 - c\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{c\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ <br> or $\frac{dh}{dt} = \frac{1600 - k\sqrt{h}}{4000} = \frac{1600}{4000} - \frac{k\sqrt{h}}{4000} = 0.4 - k\sqrt{h}$ | Convincing proof of $\frac{dh}{dt}$ | A1 **AG** |
| | | | **[3]** |
| (b) | When $h = 25$ water ***leaks out such that*** $\frac{dV}{dt} = 400$ | | |
| | $400 = c\sqrt{h} \Rightarrow 400 = c\sqrt{25} \Rightarrow 400 = c(5) \Rightarrow c = 80$ | | |
| | From above; $k = \frac{c}{4000} = \frac{80}{4000} = 0.02$ as required | Proof that $k = 0.02$ | B1 **AG** |
| | | | **[1]** |
| ***Aliter*** (b) **Way 2** | $400 = 4000k\sqrt{h}$ | | |
| | $\Rightarrow 400 = 4000k\sqrt{25}$ | | |
| | $\Rightarrow 400 = k(20000) \Rightarrow k = \frac{400}{20000} = 0.02$ | Using 400, 4000 and $h = 25$ or $\sqrt{h} = 5$. Proof that $k = 0.02$ | B1 **AG** |
| | | | **[1]** |
| (c) | $\frac{dh}{dt} = 0.4 - k\sqrt{h} \Rightarrow \int \frac{dh}{0.4 - k\sqrt{h}} = \int dt$ | ***Separates the variables*** with $\int \frac{dh}{0.4 - k\sqrt{h}}$ and $\int dt$ on either side with integral signs not necessary. | M1 oe |
| | $\therefore$ time required $= \int_0^{100} \frac{1}{0.4 - 0.02\sqrt{h}}\, dh \quad \frac{\div 0.02}{\div 0.02}$ | | |
| | time required $= \int_0^{100} \frac{50}{20 - \sqrt{h}}\, dh$ | Correct proof | A1 **AG** |
| | | | **[2]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **8.** (d) | $\int_0^{100} \frac{50}{20-\sqrt{h}}\,\mathrm{d}h$ with substitution $h=(20-x)^2$ | | |
| | $\frac{\mathrm{d}h}{\mathrm{d}x}=2(20-x)(-1)$ or $\frac{\mathrm{d}h}{\mathrm{d}x}=-2(20-x)$ | Correct $\frac{\mathrm{d}h}{\mathrm{d}x}$ | B1 aef |
| | $h=(20-x)^2 \Rightarrow \sqrt{h}=20-x \Rightarrow x=20-\sqrt{h}$ | | |
| | $\int \frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \int \frac{50}{x}.-2(20-x)\,\mathrm{d}x$ | $\pm\lambda\int\frac{20-x}{x}\,\mathrm{d}x$ or $\pm\lambda\int\frac{20-x}{20-(20-x)}\,\mathrm{d}x$ where $\lambda$ is a constant | M1 |
| | $=100\int\frac{x-20}{x}\,\mathrm{d}x$ | | |
| | $=100\int\left(1-\frac{20}{x}\right)\mathrm{d}x$ | | |
| | $=100\left(x-20\ln x\right)\ (+c)$ | $\pm\alpha x\pm\beta\ln x$ ; $\alpha,\beta\neq 0$<br>$100x-2000\ln x$ | M1<br>A1 |
| | change limits: when $h=0$ then $x=20$<br>and when $h=100$ then $x=10$ | | |
| | $\int_0^{100}\frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100x-2000\ln x\right]_{20}^{10}$ | | |
| | or $\int_0^{100}\frac{50}{20-\sqrt{h}}\,\mathrm{d}h = \left[100\left(20-\sqrt{h}\right)-2000\ln\left(20-\sqrt{h}\right)\right]_0^{100}$ | | |
| | $=\left(1000-2000\ln 10\right)-\left(2000-2000\ln 20\right)$ | Correct use of limits, ie. putting them in the correct way round<br>Either $x=10$ and $x=20$<br>or $h=100$ and $h=0$ | ddM1 |
| | $=2000\ln 20-2000\ln 10-1000$ | Combining logs to give... | |
| | $=2000\ln 2-1000$ | $2000\ln 2-1000$<br>or $-2000\ln\left(\frac{1}{2}\right)-1000$ | A1 **aef** |
| | | | **[6]** |
| (e) | Time required $=2000\ln 2-1000 = 386.2943611...$ sec | | |
| | $=386$ seconds (nearest second) | | |
| | $=6$ minutes and $26$ seconds (nearest second) | <u>6 minutes, 26 seconds</u> | B1 |
| | | | **[1]** |
| | | | **13 marks** |