A-LEVEL

MATHEMATICS

Mechanics 1B – MM1B
Mark scheme

6360
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Version/Stage V1.0 Final

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.
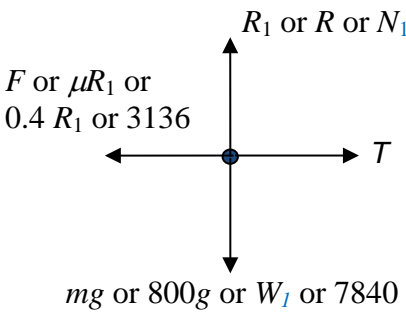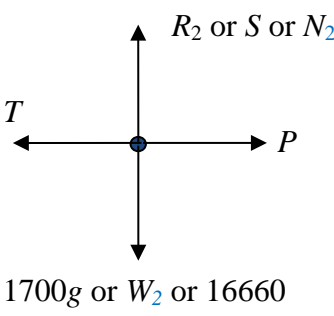
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
1(a)	$v = 14 + 0.8 \times 12$ $= 23.6\,\mathrm{m\,s^{-1}}$	M1A1 A1	 3	M1: Using constant acceleration equation(s) to find v. A1: Correct equation for v. A1: Correct answer. CAO NMS – Full marks for correct answer. Do not accept 24.
(b)	$s = 14 \times 12 + \dfrac{1}{2} \times 0.8 \times 12^2$ $= 225.6 = 226\,\mathrm{m}\ (\text{to 3 sf})$ OR $s = \dfrac{1}{2}(14 + 23.6) \times 12$ $= 225.6 = 226\,\mathrm{m}\ (\text{to 3 sf})$ OR $s = 23.6 \times 12 - \dfrac{1}{2} \times 0.8 \times 12^2$ $= 225.6 = 226\,\mathrm{m}\ (\text{to 3 sf})$ OR $23.6^2 = 14^2 + 2 \times 0.8s$ $s = 225.6 = 226\,\mathrm{m}\ (\text{to 3 sf})$	M1A1 A1 (M1A1F) (A1) (M1A1F) (A1) (M1A1F) (A1)	 3 (3) (3) (3)	M1: Using a constant acceleration equation or area under a velocity-time graph to find s. A1(F): Correct equation using their answer to part (a) if their equation contains v. A1: Correct answer. Accept 225.6 or 226 Do not accept 225 unless 225.6 seen first. Do not accept 230.
(c)	$1600 - R = 1400 \times 0.8$ $R = 480\,\mathrm{N}$	M1A1 A1	 3	M1: Three term equation of motion with correct terms. Allow sign errors. A1: Correct equation. A1: Correct R. CAO NMS – Full marks for correct answer.
	Total		9	

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
2(a)	$F = \sqrt{60^2 + 40^2}$	M1		M1: Equation or expression to find F or F^2 based on Pythagoras. Must have a +.
	$= \sqrt{5200}$	A1	2	A1: Correct force. Accept AWRT 72.1.
	$= 72.1\,\text{N}$			Accept $20\sqrt{13}$. Do not accept 72.
				Note that just $F^2 = 60^2 + 40^2$ Scores M1A0.
	Alternative if angle found first.			
	$F = \dfrac{40}{\sin 33.69°} = 72.1$	(M1) (A1)		M1:Using 40 and sin of their angle OR 60 and cos of their angle.
	OR			A1: Correct force. Accept AWRT 72.1.
	$F = \dfrac{60}{\cos 33.69°} = 72.1$	(M1) (A1)		Do not accept 72.
(b)	$(F \sin\theta = 40)$			
	$(F \cos\theta = 60)$	M1A1		M1: Seeing tan with 40 and 60. can be upside down.
	$\left(\dfrac{F\sin\theta}{F\cos\theta} = \right)\tan\theta = \dfrac{40}{60}$	A1	3	A1: Correct expression for $\tan\theta$. A1: Correct angle. Accept 33.6 or AWRT 33.7.
	$\theta = 33.7°$			
	Or	(M1) (A1F)		Do not accept 34°.
	$\sin\theta = \dfrac{40}{\sqrt{5200}}$	(A1)	(3)	M1: Use of sin with 40 or 60 in the numerator and their answer to (a) as the denominator. A1F: Use of sin and 40 in numerator, provided expression satisfies $-1\le \sin\theta \le 1$.
	$\theta = 33.7°$			
	Or	(M1) (A1F)		A1: Correct angle. Accept 33.6 or AWRT 33.7.
	$\cos\theta = \dfrac{60}{\sqrt{5200}}$	(A1)	(3)	M1: Use of cos with 40 or 60 in the numerator and their answer to (a) as the denominator.
	$\theta = 33.7°$			A1F: Use of cos and 60 in the numerator, provided expression satisfies $-1\le \cos\theta \le 1$. A1: Correct angle.
				Note that a final answer of 56.3° scores M1A0A0.
				Accept answers in radians (0.588).
	Total		**5**	

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
3(a)	$v^2 = 0^2 + 2 \times 0.05 \times 6$ $v = 0.775 \text{ m s}^{-1}$	M1A1 A1	3	M1: Using constant acceleration equation(s) to find v, with $u = 0$. A1: Correct equation. A1: Correct answer. Accept 0.774 or AWRT 0.775 or $\frac{\sqrt{15}}{5}$ Do not accept 0.77 unless AWRT 0.775 seen first. Note that if using two equations the time is 15.49s.
(b)	R_1 or R or N_1 F or μR_1 or $0.4\,R_1$ or 3136 T mg or $800g$ or W_1 or 7840	B1	1	B1: Correct diagram with exactly four forces showing arrow heads and labelled. Note: Do not accept 800 kg for the weight.
(c)	R_2 or S or N_2 T P Mg or $1700g$ or W_2 or 16660	B1		B1: Correct diagram with exactly four forces showing arrow heads and labelled. Do not award if a different letter is used for the tension in this part. Condone the use of the same letter for the normal reaction and the weight / mass in both parts. Condone assumption that normal reaction is equal to the weight when labelling diagrams. Condone diagrams with two or four normal reactions if they are seen acting at the wheels. Note: Do not accept 1700 kg for the weight.
	Only one **normal reaction** force / Don't need to consider where the normal reaction forces act.	B1	2	B1: Any comment about the number of normal reaction forces or where they act.
(d)	$R = 800 \times 9.8$ $(= 7840 \text{ N})$ $F = 0.4 \times 800 \times 9.8$ $= 3136 = 3140 \text{ N (to 3sf)}$	M1 dM1 A1 3		M1: Correct reaction force. dM1: Use of $F = \mu R$. Accept $800g$ for R. Condone negative signs in working. A1: Correct friction. Only accept 3140 or 3136. Do not award for negative final answers.

Q	Solution	Mark	Total	Comment
(e)	$T - 3136 = 800 \times 0.05$ $T = 3176 = 3180\,\text{N (to 3 sf)}$	M1A1F A1	3	M1: Three term equation of motion for the skip with correct terms. Allow sign errors and their value of F from part (d). Must use mass of 800 kg. A1F: Correct equation with their value for F. A1: Correct T. Only accept 3176 or 3180.
(f)	$P - 3176 = 1700 \times 0.05$ $P = 3261 = 3260\,\text{N (to 3sf)}$ OR $P - 3136 = 2500 \times 0.05$ $P = 3261 = 3260\,\text{N (to 3sf)}$	M1A1F A1 (M1A1F) (A1)	3 (3)	M1: Three term equation of motion for the van with correct terms. Allow sign errors and their value of T from part (e). Must use mass of 1700 kg. A1F: Correct equation. Follow through their value for T. A1: Correct P. Accept 3261. Allow 3265 or 3260 or 3270 from use of 3180 for T. M1: Three term equation of motion for the van and skip with correct terms. Allow sign errors and their value of F from part (d). Must use mass of 2500 kg. A1F: Correct equation. Follow through their value for F. A1: Correct P. Accept 3261. Allow 3265 or 3260 or 3270 from use of 3140 for F.
	Total		**15**	
				Use of $g = 9.81$ gives: (d) 3139.2 (e) 3179.2 (f) 3264.2 Do not penalise use of 9.81.

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
4(a)	$t = \dfrac{20}{2\sin 70°}$ $= 10.6$ seconds	M1A1 A1	3	M1: Use of distance divided by speed (20 $\div$ 2sin70° or 2cos70° or 2sin20° or 2cos20° **OR** $\sqrt{409} \div \dfrac{2\sin 70°}{\sin \alpha}$). Value for α does not have to be seen. A1: Correct expression for t, with $\alpha = 81.5$ if included. A1: Correct time. Accept AWRT 10.6.
(b)	$\tan \alpha = \dfrac{20}{3}$ $\alpha = 81.5$	M1 A1	2	M1: Use of $\tan \alpha = \dfrac{20}{3}$ or $\dfrac{3}{20}$. Allow use of sin or cos with $\sqrt{409}$ in the denominator. A1: Correct angle. Allow AWRT 81.5 or 81, but not 81.0.
(c)	$180 - 70 - 81.46 = 28.53$ $\dfrac{V}{\sin 28.53°} = \dfrac{2}{\sin 81.47°}$ $V = \dfrac{2\sin 28.53°}{\sin 81.47°} = 0.966$ **OR** $(V - 2\cos 70°) \times 10.64 = 3$ $V = 0.966$ **OR** $20\tan 20° + 3 = 10.64V$ $V = 0.966$ **OR** $V = 2\cos 70° + 2\sin 70° \tan 8.53°$ $= 0.966$	M1A1 A1 dM1A1 (M1A1) (A1) (dM1A1) (M1A1) (A1) (dM1A1) (M1A1) (A1) (dM1A1)	5	M1: Forming an equation to find V. A1, A1: Correct equation award A2, if one error award A1. dM1: Solving equation for V A1: Correct V. Accept AWRT 0.97 from correct working, particularly from working with 10.6 or 8.5° etc.
	Total		10	

Q	Solution	Mark	Total	Comment
				Follow through mis-reads in this question.
5	$m(4\mathbf{i} + 2\mathbf{j}) + km(6\mathbf{i} - 2\mathbf{j}) = (m + km)(5.2\mathbf{i} - 0.4\mathbf{j})$ $4m + 6km = 5.2(m + km)$ $4 + 6k = 5.2 + 5.2k$ $0.8k = 1.2$ $k = \dfrac{1.2}{0.8} = 1.5$ **OR** $2m - 2km = -0.4(m + km)$ $2 - 2k = -0.4 - 0.4k$ $2.4 = 1.6k$ $k = \dfrac{2.4}{1.6} = 1.5$	B1 M1A1 dM1 A1 (M1A1) (dM1) (A1)	5	B1: Correct vector equation for conservation of momentum. Can be implied in later working by correct equation for one component. Condone missing brackets if later working correct. M1: Conservation of momentum equation for one component with correct terms, but allow sign errors. A1: Correct equation. Allow inclusion of **i** or **j** in equations. dM1: Valid method for solving for k. Working like $k = \dfrac{1.2\mathbf{i}}{0.8\mathbf{i}} = 1.5$ scores dM0. A1: Correct value for k. CAO Candidates who work with both components and get two different values for k, should be treated as two solutions. (Mark both and take mean, rounding down.)
	OR $-1.2\mathbf{i} + 2.4\mathbf{j} = k(-0.8\mathbf{i} + 1.6\mathbf{j})$ $1.2^2 + 2.4^2 = k^2(0.8^2 + 1.6^2)$ $k^2 = \dfrac{7.2}{3.2}$ $k = \pm 1.5$ $k = 1.5$	(M1A1) (dM1) (A1)		M1: Forming equation for k using magnitudes of vectors. A1: Correct equation. dM1: Obtaining ± 1.5 A1: Selecting 1.5. Consistent use of weight instead of mass deduct 1 mark.
	Total		**5**	

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
6(a)	$V\cos 2° \times 1.8 = 420$ $$V = \frac{420}{1.8\cos 2°} = 233.47 = 233 \text{ m s}^{-1} \text{ (to 3sf)}$$	M1A1 A1	3	M1: Equation to find V with $420 = V \times 1.8$ (cos2° or sin2°). A1: Correct equation. A1: Correct V. Allow 233.5 or AWRT233 from correct working.
(b)	$$y = 233.47\sin 2° \times 1.8 - \frac{1}{2} \times 9.8 \times 1.8^2$$ $$= -1.209 = -1.21 \text{ m (to 3sf)}$$ Distance between A and C is 1.21 metres.	M1A1F A1 A1	4	M1: Use of constant acceleration equation to find y (vertical displacement). Allow cos2° and ±g. A1F: Correct equation. Follow through their V from part (a). A1: Correct y. Accept AWRT -1.21. Accept-1.23 or -1.24 from use of 233, provided correct working seen in part (a). Accept-1.20 or -1.21 from use of 233.5, provided correct working seen in part (a). A1: Correct distance. Accept modulus of answers as above.
(c)	No air resistance.	B1	1	B1: No (air) resistance or only force is weight. Do not allow 'only force acting is g'.
	Total		8	
				Use of $g = 9.81$ gives: (b) -1.2258 Use of 233 gives -1.2555. Use of 233.5 gives -1.2239. Do not penalise use of 9.81.

Q	Solution	Mark	Total	Comment
				Candidates who use $\mathbf{a} = -0.4\mathbf{i} + 0.2\mathbf{j}$ for both particles can gain all method marks but no A or B marks. Otherwise no MR or MC in this question.
7(a)	$$\mathbf{r} = (4\mathbf{i} + 2\mathbf{j}) \times 10 + \frac{1}{2}(-0.4\mathbf{i}) \times 10^2$$ $$= 20\mathbf{i} + 20\mathbf{j}$$	M1 A1	2	M1: Use of $\mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with a non-zero acceleration. A1: Correct position vector.
(b)	$\mathbf{r}_A = (4\mathbf{i} + 2\mathbf{j})t - 0.2t^2\mathbf{i}$ $\mathbf{r}_B = (0.4\mathbf{i} + 0.6\mathbf{j})t + 0.1t^2\mathbf{j} + 11.2\mathbf{i}$ $(4\mathbf{i} + 2\mathbf{j})t - 0.2t^2\mathbf{i} = (0.4\mathbf{i} + 0.6\mathbf{j})t + 0.1t^2\mathbf{j} + 11.2\mathbf{i}$ $2t = 0.6t + 0.1t^2$ $t^2 - 14t = 0$ $(t = 0)$ or $t = 14$ $4t - 0.2t^2 = 0.4t + 11.2$ $t^2 - 18t + 56 = 0$ $t = 4$ or $t = 14$ $\therefore t = 14$ **OR** $4 \times 14 - 0.2 \times 14^2 = 16.8$ $0.4 \times 14 + 11.2 = 16.8$ $\therefore t = 14$ **THEN** $\mathbf{r} = (4\mathbf{i} + 2\mathbf{j}) \times 14 - 0.2 \times 14^2\mathbf{i} = 16.8\mathbf{i} + 28\mathbf{j}$	B1 B1 M1A1 A1 dM1A1 A1 (dM1) (A1) (A1) B1	9	B1: Seeing position vector for *A*, may be implied by later working if not seen explicitly. B1: Seeing position vector for *B*, may be implied by later working if not seen explicitly. M1: Forming an equation by equating either **i** or **j** components to form a quadratic equation. A1: Correct equation A1: Correct solution(s). dM1: Forming an equation by equating other components to form a quadratic equation. A1: Correct equation A1: Correct solutions and selection of common solution. (dM1): Substituting one non-zero value from their two solutions into the other component for both *A* and *B*. (A1): Obtaining two identical values. (A1): Concluding that $t = 14$. B1: Correct position vector.
	Total		11	

Q	Solution	Mark	Total	Comment
				No MR or MC in this question.
8(a)	$F = 40 \times 9.8 \sin 30° = 196$ or $F = mg \sin 30°$ $R = 40 \times 9.8 \cos 30° = 339.48$ or $R = mg \cos 30°$ $40 \times 9.8 \sin 30° = \mu \times 40 \times 9.8 \cos 30°$ $\mu = \dfrac{\sin 30°}{\cos 30°} = \tan 30° = 0.577$ OR $\mu = \dfrac{196}{339.48} = 0.577$	M1A1 A1 dM1 A1	5	M1: Resolving parallel and perpendicular to the slope to obtain expressions for F **and** R. Allow consistent mixing of sin and cos. A1: Correct F. A1: Correct R. dM1: Use of $F = \mu R$ A1: Correct μ. Accept $\dfrac{1}{\sqrt{3}}$ or $\dfrac{\sqrt{3}}{3}$ or AWRT 0.577 or AWRT 0.578. NMS gives full marks. Complete solution with omission of g throughout with correct final answer scores SC4.
(b)(i)	$R = X \sin 30° + 40 \times 9.8 \cos 30°$ $(= 0.5X + 339.5)$	M1A1	2	M1: Resolving perpendicular to the plane with three terms; R, $X\sin 30°$ or $X\cos 30°$ and $40g\cos 30°$ or $40g\sin 30°$, with consistent mixing of sin and cos. Allow sign errors. A1: Correct expression for R, may include m and g.
(b)(ii)	$X \cos 30° - 40 \times 9.8 \sin 30° - F = 40 \times 0.2$ $X \cos 30° - 196 - \tan 30°(0.5X + 392 \cos 30°) = 8$ $X = \dfrac{8 + 196 + 392 \cos 30° \tan 30°}{\cos 30° - 0.5 \tan 30°} = 692.8 = 693 \text{ N (to 3sf)}$ Alternative format $X \cos 30° - 40g \sin 30° - F = 40 \times 0.2$ $X \cos 30° - 40g \sin 30° - \tan 30°(0.5X + 40g \cos 30°) = 8$ $X = \dfrac{8 + 40g \sin 30° + 40g \cos 30° \tan 30°}{\cos 30° - 0.5 \tan 30°} = 692.8 = 693 \text{ N (to 3sf)}$	M1A1 dM1 dM1 A1	5	M1: Equation of motion with correct terms. Allow sign errors. A1: Correct equation. May be in terms of m and g. dM1: Substituting for F using $F = \mu R$, where R is in the form $a + bX$ where a and b are non-zero constants. dM1: Solving for X. A1: Correct value for X. Accept 692 or AWRT 693.
	Total		12	
	TOTAL		75	
				Use of $g = 9.81$ gives: (b) (i) $0.5X + 339.8$ (b) (ii) $693.5 = 694$ (to 3sf) Do not penalise use of 9.81.