OCR
Oxford Cambridge and RSA

Friday 24 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. If additional space is required, you should use the lined page(s) at the end of the Printed Answer Book. The question number(s) must be clearly shown.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

1 Express $\cos\theta - 3\sin\theta$ in the form $R\cos(\theta+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence show that the equation $\cos\theta - 3\sin\theta = 4$ has no solution. **[6]**

2 Given that $\left(1+\frac{x}{p}\right)^q = 1 - x + \frac{3}{4}x^2 + \ldots$, find p and q, and state the set of values of x for which the expansion is valid. **[7]**

3 Fig. 3 shows the curve $y = x^4$ and the line $y = 4$.

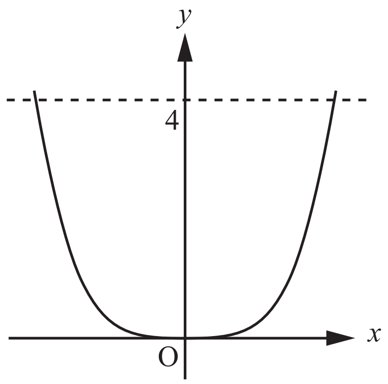

Fig. 3

The finite region enclosed by the curve and the line is rotated through 180° about the y-axis. Find the exact volume of revolution generated. **[4]**

4 Solve the equation $2\sin 2\theta = 1 + \cos 2\theta$ for $0° \leqslant \theta \leqslant 180°$. **[5]**

5 In Fig. 5, triangles ABC, ACD and ADE are all right-angled, and angles BAC, CAD and DAE are all θ.

AB = x and AE = $2x$.

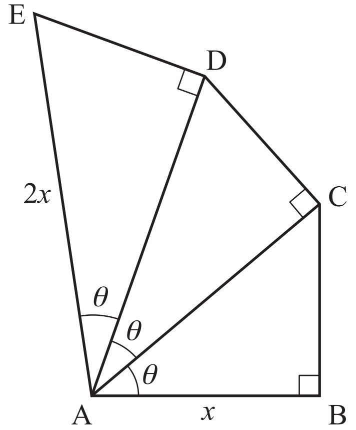

Fig. 5

(i) Show that $\sec^3\theta = 2$. **[3]**

(ii) Hence show the ratio of lengths ED to CB is $2^{\frac{2}{3}} : 1$. **[4]**

6 P is a general point on the curve with parametric equations $x = 2t,\ y = \frac{2}{t}$. This is shown in Fig. 6. The tangent at P intersects the x- and y-axes at the points Q and R respectively.

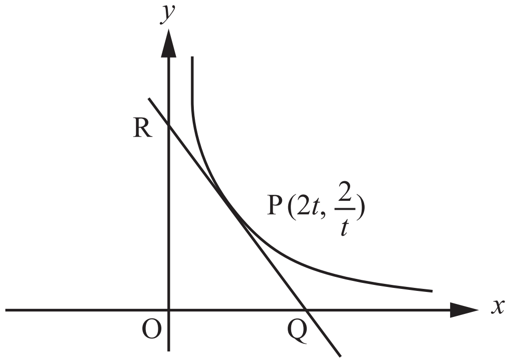

Fig. 6

Show that the area of the triangle OQR, where O is the origin, is independent of t. **[7]**

Section B (36 marks)

7 Fig. 7 shows a cuboid OABCDEFG with coordinates as shown. The point P has coordinates (4, 2, 0).

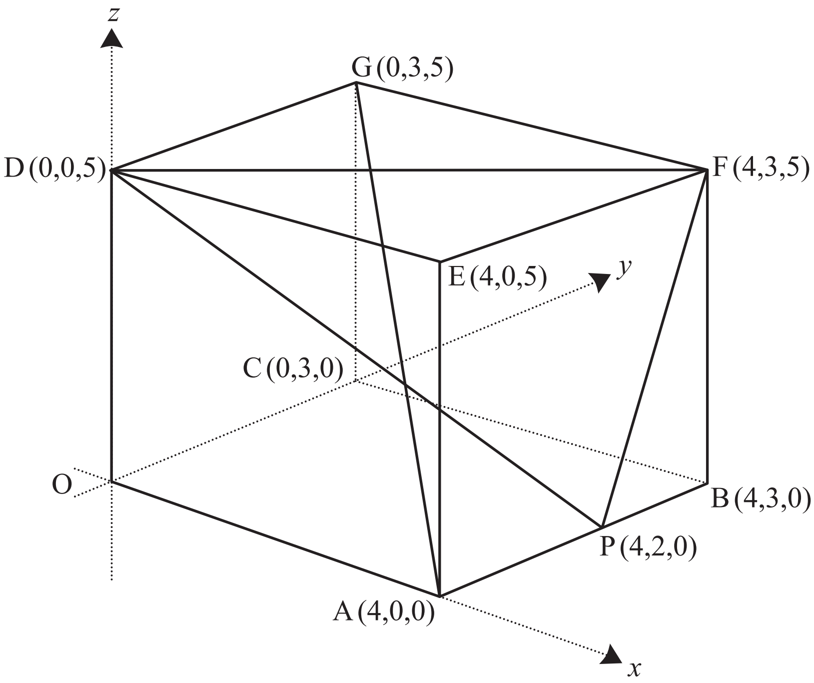

Fig. 7

(i) Find the length of the diagonal AG. **[2]**

(ii) Show that the vector $\mathbf{n} = 15\mathbf{i} - 20\mathbf{j} + 4\mathbf{k}$ is normal to the plane DPF. Hence find the cartesian equation of this plane. **[6]**

The diagonal AG intersects the plane DPF at Q.

(iii) Write down a vector equation of the line AG. Hence find the coordinates of the point Q, and the ratio AQ : QG. **[6]**

(iv) Find the acute angle between the line AG and the plane DPF. **[4]**

8 **(i)** Show that $\dfrac{1}{2+x}+\dfrac{1}{2-x}=\dfrac{4}{(2+x)(2-x)}$. **[1]**

In a chemical reaction, the time t minutes taken for a mass x mg of a substance to be produced is modelled by the equation

$$t=\ln\left(\frac{2+x}{2-x}\right).$$

(ii) Show that when $t = 0$, $x = 0$. **[2]**

(iii) Show that the rate of change of x is proportional to the product of $(2 + x)$ and $(2 - x)$, and find the constant of proportionality. **[4]**

(iv) Show that $x=\dfrac{2\left(1-\mathrm{e}^{-t}\right)}{1+\mathrm{e}^{-t}}$.

Hence determine the long-term mass of the substance predicted by this model. **[4]**

In another chemical reaction, the mass x mg at time t minutes is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t}=k(2+x)(2-x)\mathrm{e}^{-t},$$

where k is a positive constant, and $x = 0$ when $t = 0$.

(v) Show by integration that, for this reaction, $\ln\left(\dfrac{2+x}{2-x}\right)=4k\left(1-\mathrm{e}^{-t}\right)$. **[5]**

(vi) Given that the long-term mass of this substance is 1.85 mg, find the value of k. **[2]**

END OF QUESTION PAPER

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Friday 24 June 2016 – Morning

A2 GCE MATHEMATICS (MEI)

4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

QUESTION PAPER

Candidates answer on the Question Paper.

OCR supplied materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: Up to 1 hour

Candidate forename		Candidate surname	

Centre number						Candidate number				

INSTRUCTIONS TO CANDIDATES

- The Insert will be found inside this document.
- Write your name, centre number and candidate number in the boxes above. Please write clearly and in capital letters.
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the bar codes.
- The Insert contains the text for use with the questions.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **8** pages. Any blank pages are indicated.

1 The blades of a wind turbine sweep out a circle of diameter 90 m. The turbine's blade tip height is 149.5 m.

Calculate the hub height of this turbine. **[1]**

1	

2 In lines 46 and 47, the article says
'So someone at the point of observation would not see the bottom 12 m of the turbine.'

Explain how the figure of 12 m was obtained. **[2]**

2	

3 A wind turbine with a blade tip height of 125 m is seen from a distance of 623 m. The ground is level and horizontal so that the whole of the turbine can be seen.

(i) Calculate the angle of elevation of the tip of a blade when it is pointing vertically upwards. You should assume that the viewer's eye is at the same height as the base of the turbine. **[1]**

The wind turbine is shown on a photomontage; the viewing distance is stated to be 51.4 cm.

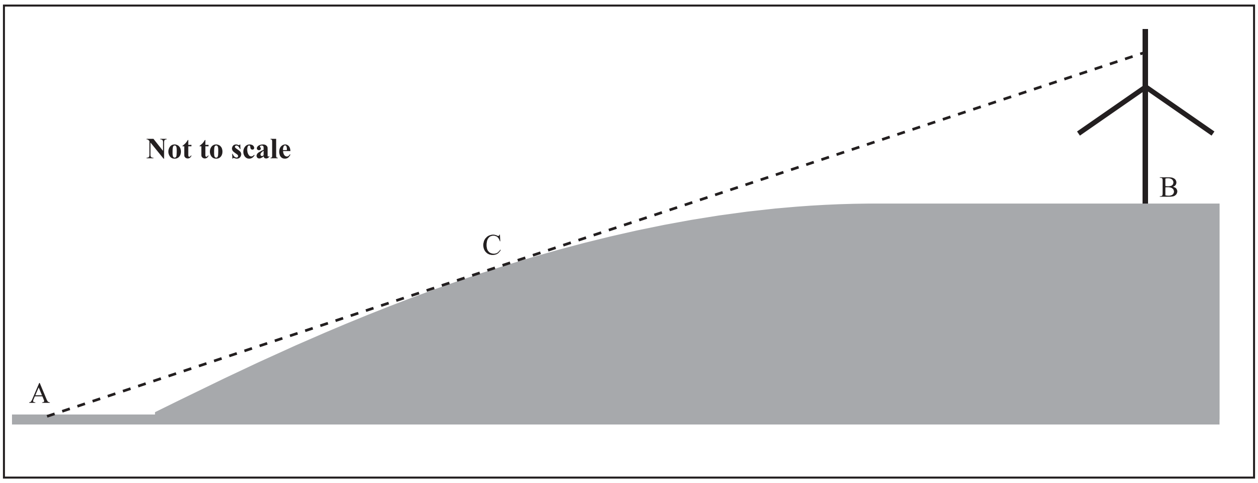
(ii) Calculate the height that the turbine would have on the photomontage if it were seen with the same angle of elevation as that in part **(i)**. **[1]**

The image of the wind turbine is 7.3 cm high when the photomontage is printed on A4 paper.

(iii) Show that when the photomontage is printed on A3 paper, the height of the wind turbine is consistent with the angle of elevation found in part **(i)**. **[2]**

3 (i)	
3 (ii)	

3 (iii)	



4 The diagram illustrates the situation in Fig. 5 of the article.

The blade tip height of the wind turbine is 99.5 m.

The base, B, of the turbine is 120 m higher than A and at a horizontal distance of 320 m from A.

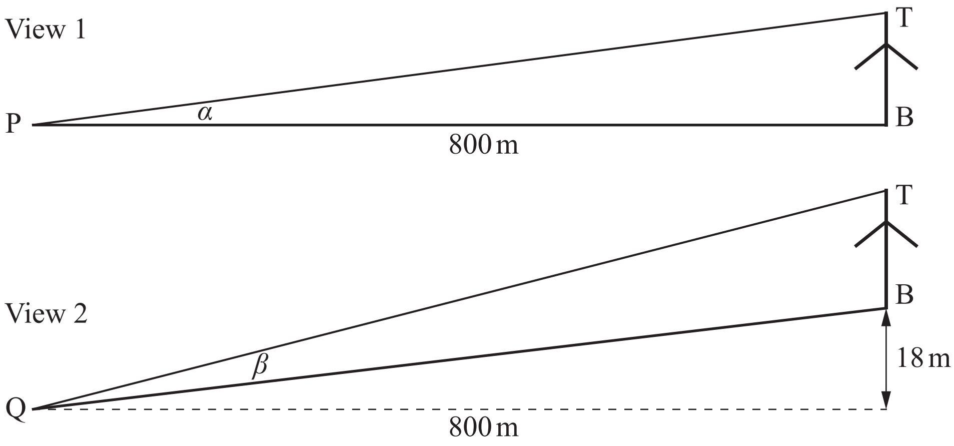
An observer at A can see the top 20 m of a blade when it is pointing vertically upwards, as in the diagram.

The observer's line of vision is a tangent to the hill at C. The horizontal distance from A to C is 140 m.

Find the height of C above A. **[4]**

4	

Not to scale



5 In the diagram, the wind turbine BT is observed from two different positions P and Q. The blade tip height of the turbine is 72 m.

Both P and Q are a horizontal distance of 800 m from the turbine.

P is at the same height as the base, B, of the turbine. Q is 18 m below the level of B.

The angle of elevation from P is α; the angle TQB is β.

Show that the angles α and β, in degrees, are the same to 2 significant figures. **[3]**

5

6 In line 96, the article says

‘As a result of the study, it was recommended that a focal length of 75 mm should be used.’

Make a reasoned estimate of the percentages of participants in Stirling University’s study who would have thought the photomontages made the wind turbines appear ‘Too large’, ‘About right’ and ‘Too small’ if a lens of focal length 75 mm had been used. You must state your assumptions clearly. **[4]**

6	

END OF QUESTION PAPER

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4754/01B Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

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Photomontages

Introduction

When a new building or structure is proposed, planning permission has to be obtained from the authorities. It is common practice to present an image of what the new building or structure is expected to look like. The image is often an artist's impression but it may also be a photomontage.

Photomontages are commonly used to support applications to develop wind farms and this context is used in this article. Electricity at a wind farm is generated by wind turbines. A wind turbine is illustrated in Fig. 1. This diagram also explains the meanings of the terms used throughout the article.

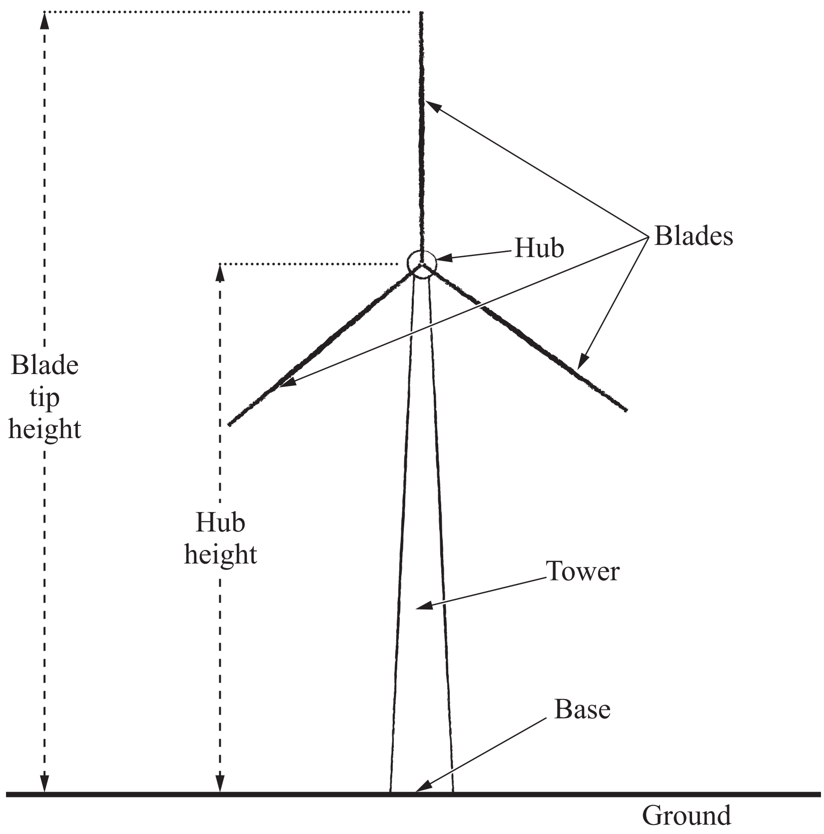

Fig. 1

A photomontage is based on a photograph taken from a particular place. The photograph shows the background before any building work has started. An image of the proposed structure is then superimposed on this. The image may be a photograph or a computer drawing and must be in the right place and to the right scale. Fig. 2 on the next page is typical of many photomontages that have been used for planning applications.

The type of wind turbine in Fig. 2 has blade tip height 99.5 m and hub height 64.5 m. So the blades are 35 m long.

Fig. 2

Recommended viewing distance when viewed with both eyes 51.4 cm; distance to turbine 548 m; included angle 38.6°.

The impression given by photomontages

The purpose of a photomontage is to provide an accurate representation of how the proposed development would appear in reality. A photomontage in which the development is the wrong size, either too large or too small, is misleading.

After a wind farm has been built, local people have sometimes complained that the turbines looked larger than they did in the photomontages. This article looks at three possible explanations.

The angle of elevation

The turbine in Fig. 2 has a blade tip height of 99.5 metres and the photomontage shows the view from a place 548 metres away. So it is possible to work out the angle of elevation of the highest point. This is the angle α in Fig. 3. To simplify the calculations it is assumed that the observer's eye is at the same level as the base of the turbine.

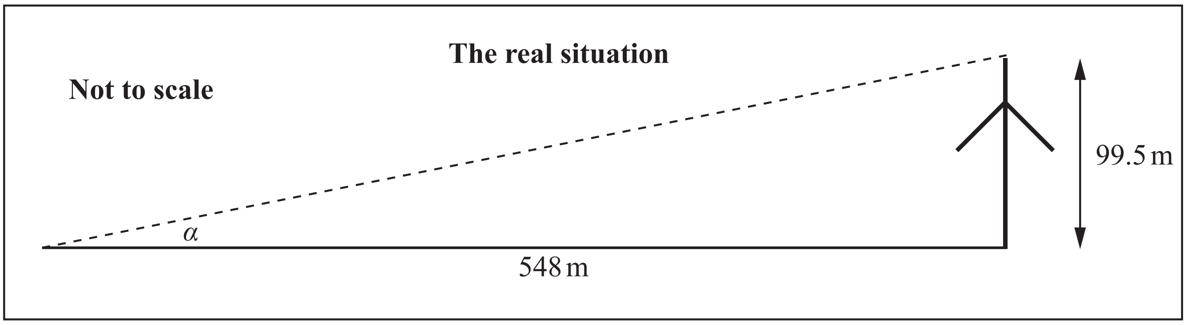

Fig. 3

The information with the photomontage also gives the recommended viewing distance as 51.4 centimetres; this is about an arm's length. So you might expect that, seen from this distance, the turbine in the photomontage should have the same angle of elevation of α if it is to appear the right size to a viewer. This is illustrated in Fig. 4.

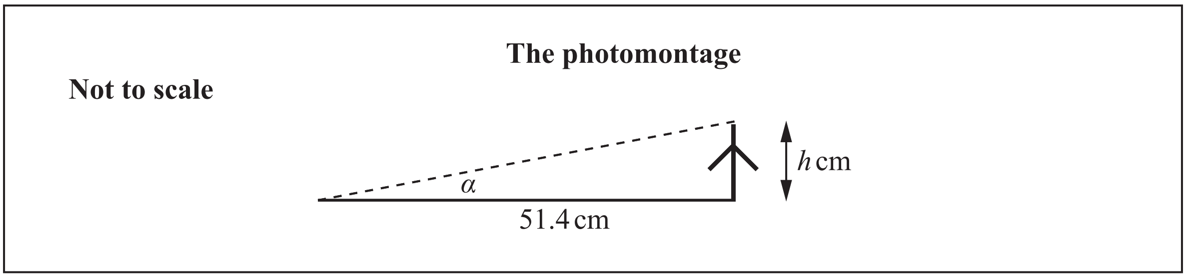

Fig. 4

From the triangles in Figs 3 and 4, you can deduce that the height of the turbine in the photomontage should be 9.3 cm, to 2 significant figures. However, if you measure it in Fig. 2, you obtain the much smaller figure of about 5.9 cm. How can this discrepancy be explained?

Seeing only part of a wind turbine

One possible explanation is that not all of the turbine is being seen.

Wind turbines are often positioned near the tops of hills where the wind is strongest, so you might expect the whole turbine to be visible from nearby. However, that is often not the case. The view may be obstructed as illustrated in Fig. 5. In this case, an observer at point **A** would only ever see the tips of the blades.

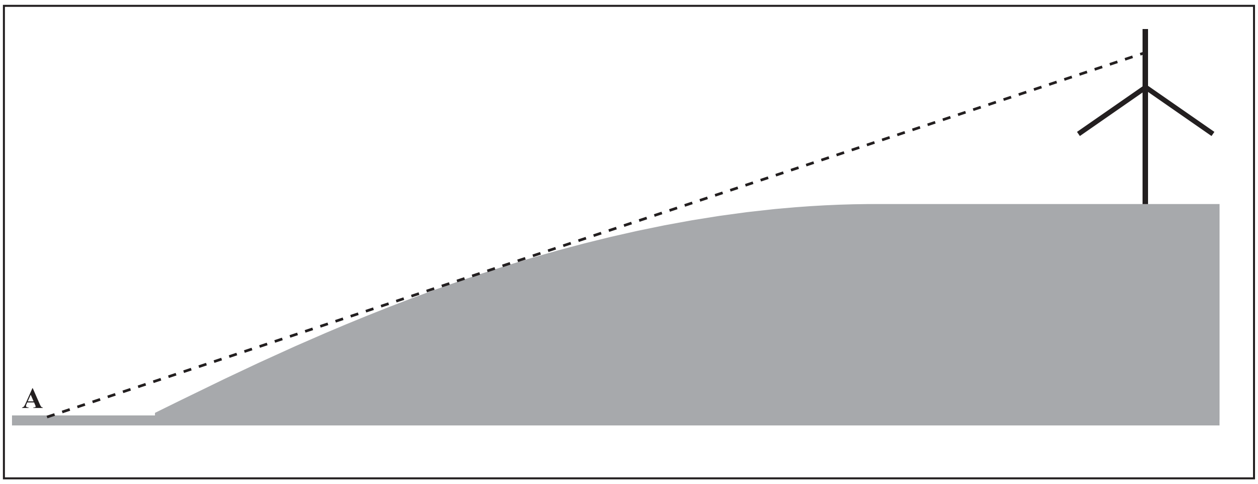

Fig. 5

So a photomontage has to take account of the lie of the land between the observer and the wind turbine. The choice of the observer's position can have a substantial effect on the appearance of the wind turbine. Might this have happened in Fig. 2?

One of the blades in Fig. 2 is upright; its length on the photomontage is about 2.35 cm. As the dimensions of the turbine are known (given on lines 14 and 15), it is possible to work out that the height of the full turbine would be about 6.7 cm on the photomontage. However, as the height seen is only 5.9 cm, this means that about 0.8 cm of the tower must be hidden from view in the photomontage. So someone at the point of observation would not see the bottom 12 m of the turbine.

So the obscuring effect of the land explains some of the discrepancy. However, even though the revised figure of 6.7 cm, is closer than 5.9 cm, it is still much less than the 9.3 cm that is needed for the angle of elevation to be correct.

So other possible causes need to be considered.

Printed size

Another possible explanation relates to the size of paper on which the photomontage is printed.

All of the pages that this article are printed on are A4 size; they are 210 mm wide and 297 mm high, to the nearest mm. When two A4 sheets are laid side by side, the resulting size is 420 mm × 297 mm. This is A3 size and is illustrated in Fig. 6.

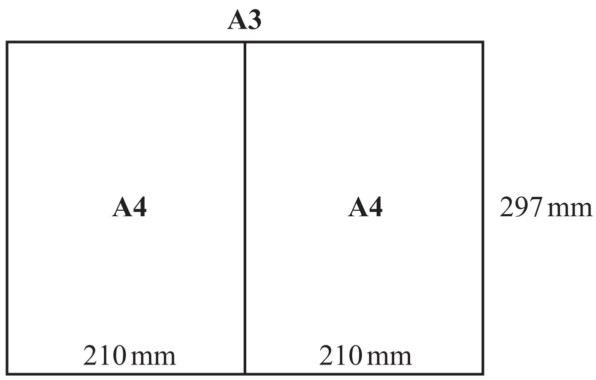

Fig. 6

In the same way, two A3 sheets laid side by side make an A2 sheet, and so on up to A0. The area of an A0 sheet is exactly 1 m^2. For all the A sizes, the ratio *short side* : *long side* is $1 : \sqrt{2}$. Because the ratio is the same, every A-sized sheet of paper is similar to every other sheet.

It follows that a photomontage printed on an A3 sheet would be an exact enlargement of one printed on an A4 sheet, with a scale factor of $\sqrt{2}$ or 1.41, to 3 significant figures. Thus if the photomontage in Fig. 2 were printed on A3 paper, the height of the turbine would be 6.7×1.41 or about 9.45 cm. Given the approximate nature of the measurements, this is very close to 9.3 cm.

So an explanation of the discrepancy is that the photomontage should have been printed on an A3 sheet, rather than on A4. However, it is not clear that this is actually the right explanation.

It is common practice to give the viewing distance at the bottom of a photomontage, as in Fig. 2, but not the recommended printing size. Many people view photomontages on their computers and print them out. A standard printer produces an A4 image and this is also about the size that it appears on many computer monitors. So those people who view photomontages on-screen or from their print-outs would expect the turbines to be smaller than they will appear when they are actually constructed.

However, complaints have also come from some people who only saw the photomontages at public meetings where they were presented at A3 size. These people have also said that the turbines looked too small in the photomontages they were shown.

Visualisation

There are professional people, such as architects and artists, whose expertise includes *visualisation*. They say that arguments about the printed size and viewing distance miss the point, which is that it does not matter whether the size of the photomontage is large or small, as your brain will convert it into an image inside your head. What matters is whether the image in your head from looking at a photomontage is the same as the image from looking at the real thing.

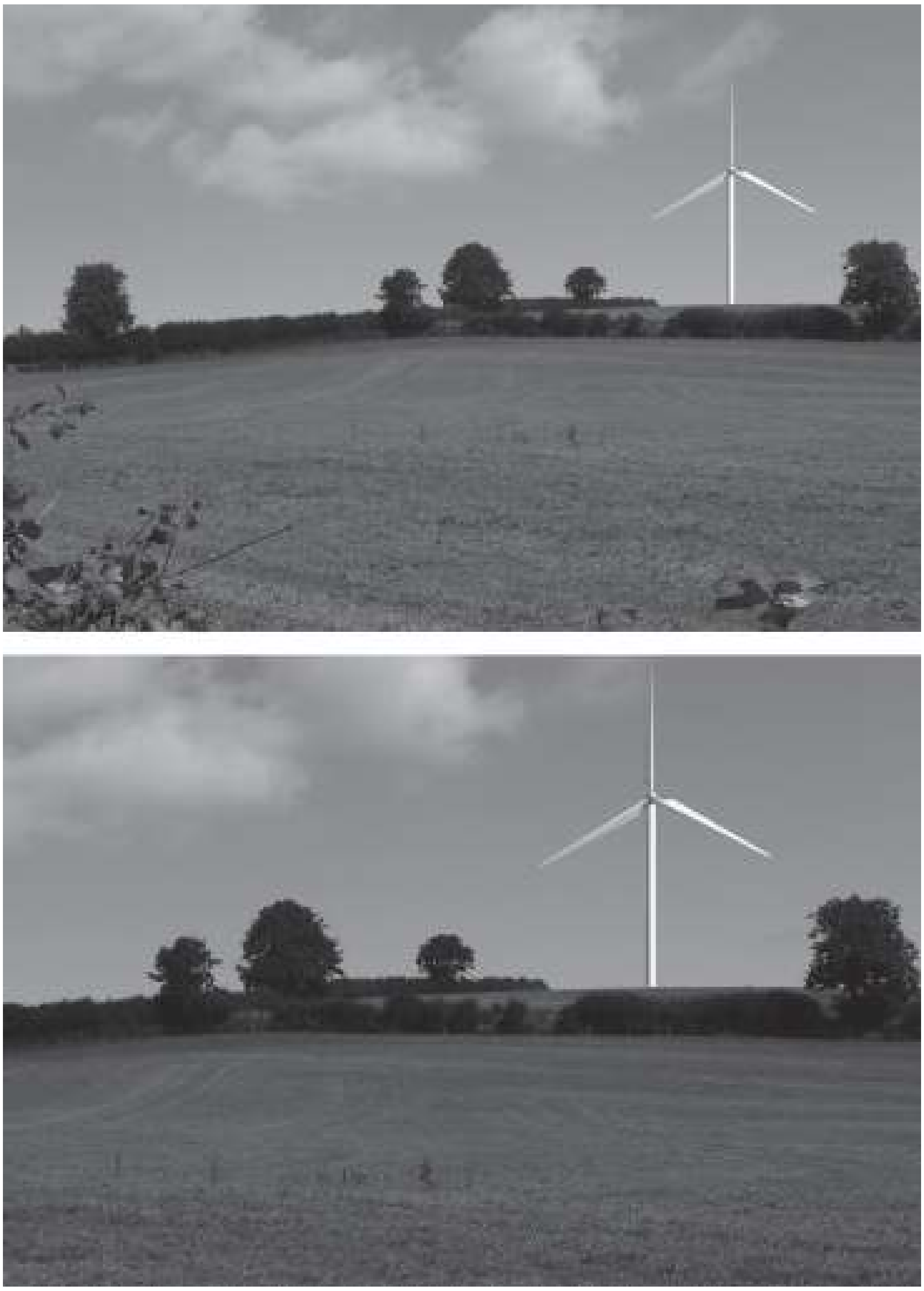

Fig. 7 (top) and Fig. 8 (bottom)

Fig. 7 and Fig. 8 show two views from the same place. Fig. 8 is a rectangular window that has been taken from within Fig. 7 and then enlarged to the same size. It follows that

- Fig. 8 has a narrower field of view than Fig. 7
- objects look larger in Fig. 8 than in Fig. 7.

The key question is which of these two views is the better representation of what a typical person would actually see from that place.

Different fields of view are obtained when photographs are taken with lenses of different focal length. Fig. 9 illustrates the relationship between the focal length and the included angle in the field of view of the photograph. A shorter focal length would give a larger included angle and so a wider field of view.

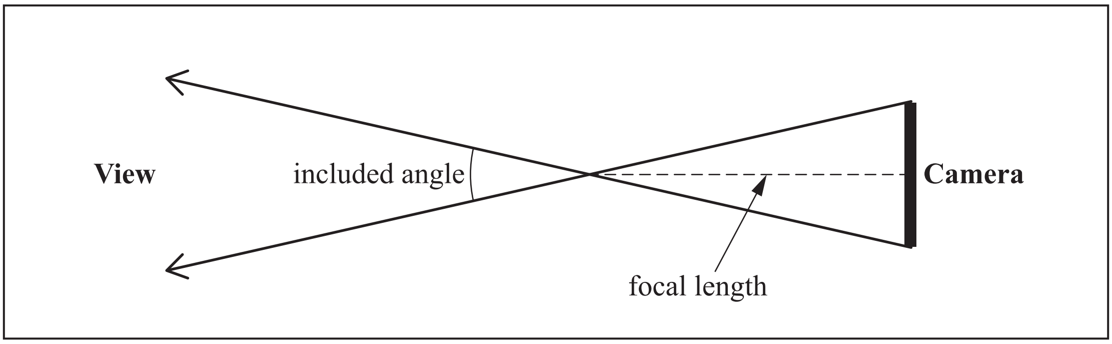

Fig. 9

So photographs taken using a camera with a longer focal length lens have a narrower field of view, resulting in distant objects looking larger.

The question of what is the best focal length was investigated in a study by Stirling University for the Highland Council in 2011 to 2012. Participants were taken to places near existing wind farms and were given 7 photomontages. These used photographs that had been taken with lenses with a range of focal lengths from 50 mm to 110 mm. They were asked to select the one that best represented what they saw.

In total 362 people took part and their responses are given in Table 10.

Focal length (mm)	50	60	70	80	90	100	110
Number of preferences	16	50	85	85	72	37	17

Table 10

As a result of the study, it was recommended that a focal length of 75 mm should be used. This contrasts with the previous common practice of using a focal length of 50 mm.

Fig. 8 is the picture that would have been obtained using a 75 mm lens, which is consistent with this recommendation. If this is enlarged to the same size as Fig. 2, and allowance is made for the part of the tower that is obscured, the height of the turbine would be close to 9.3 cm. So the angle of elevation when an A4 print-out was viewed at arm's length would be about right.

The wide spread of the data in Table 10 indicates that people see things differently. However, for a very large majority of viewers (over 95% of participants in the study) a photomontage based on a photograph taken with a 50 mm lens will under-represent the height of a wind turbine.

Consequently, of the three explanations considered in this article, the use of an unsuitable camera lens has been accepted as the most plausible.