GCE

Further Mathematics A

Y544/01: Discrete Mathematics

Advanced GCE

Mark Scheme for November 2020

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Text Instructions

1. **Annotations and abbreviations**

Annotation in RM assessor	Meaning
✔and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank Page
Seen	
Highlighting	
Other abbreviations in mark scheme	**Meaning**
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only one previous M mark
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

2. **Subject-specific Marking Instructions for A Level Mathematics A**

a   Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

   For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

   It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

   Award NR (No Response)
   -    if there is nothing written at all in the answer space and no attempt elsewhere in the script
   -    OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
   -    OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
   Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

   If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
   If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.
A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value **is given** in the paper only accept an answer correct to at least as many significant figures as the given value.
- When a value **is not given** in the paper accept any answer that agrees with the correct value to **3 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.
  NB for Specification B (MEI) the rubric is not specific about the level of accuracy required, so this statement reads "2 s.f".

Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.
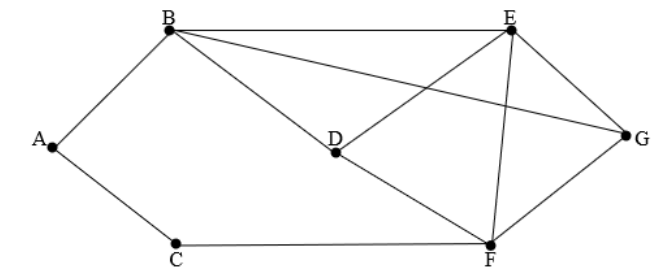
g   Rules for replaced work and multiple attempts:

- If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.
- If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.
- if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h   For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors.

If a candidate corrects the misread in a later part, do not continue to follow through. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
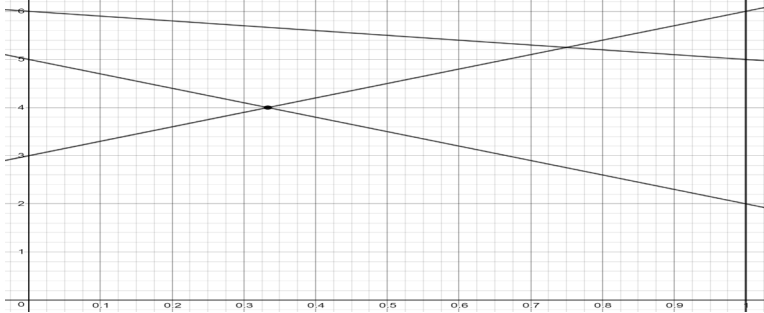
i   If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers, provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" or "Determine". Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j   If in any case the scheme operates with considerable unfairness consult your Team Leader.

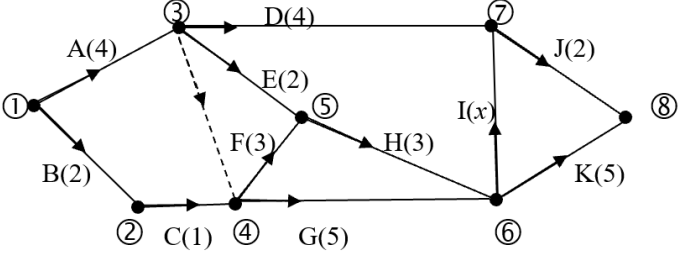| Question			Answer	Mark	Guidance
1					For reference:
1	(a)	(i)	(a sub-division of) $K_{3,3}$ is not a subgraph since there are only 5 vertices with degree ≥ 3	B1	<u>Explaining</u> why $K_{3,3}$ is not a subgraph (not just stating it)
			(a sub-division of) K_5 is not a subgraph since there are only 3 vertices with degree $(\geq)$ 4	B1	<u>Explaining</u> why K_5 is not a subgraph (not just stating it)
			Hence graph is planar		Conclusion may be implied SC B1 for saying 2 vertices with degree 2 but not referring to total number of vertices Kuratowski is asked for in question, so just describing e.g. 'moving arc BG so that it is outside' gets no credit
				[2]	
1	(a)	(ii)	$7 + R = 11 + 2$ $\Rightarrow R = 6$	B1	Correct use of $V + R = E + 2$ leading to answer 6 Answer 6 with no evidence of Euler $\Rightarrow$ B0
				[1]	
1	(b)	(i)	e.g. A – B – D – E – G – F – C – A e.g. A – B – G – E – D – F – C – A	B1	One of these (or in reverse) with any starting point Cycle must be closed and must pass through all 7 vertices
				[1]	
1	(b)	(ii)	e.g. A and G are non-adjacent deg(A) + deg (G) = 2 + 3 = 5 < 7 (= n)	M1 A1	Any two **non-adjacent** vertices <u>and</u> their degrees (or sum) provided degrees sum to 5 or 6 (but not greater than 6) Showing that degree sum is not ≥ 7 (compare with 7)
				[2]	

1	**(c)**	**(i)**		**B1** **[1]**	Graph with AB and CF contracted (and labelled using this notation)
1	**(c)**	**(ii)**	Only non-adjacent vertices are D and G deg(D) + deg(G) = 3 + 3 = 6 $n = 5$ vertices and 6 is greater than 5 Hence result	**M1** **A1** **[2]**	D and G have degree sum = 6 Showing that degree sum is > 5 (compare with 5) $6 \geq 5$ or $6 > 5$, or equivalent, or in words
2	**(a)**	**(i)**	<table><tr><td></td><td>R</td><td>S</td><td>T</td><td>Row min for A</td></tr><tr><td>K</td><td>(7, 3)</td><td>(2, 6)</td><td>(5, 3)</td><td>2</td></tr><tr><td>L</td><td>(1, 5)</td><td>(8, 2)</td><td>(2, 5)</td><td>1</td></tr><tr><td>M</td><td>(3, 2)</td><td>(1, 5)</td><td>(4, 6)</td><td>1</td></tr></table> Play-safe strategy for Annie is K	 **B1** **B1** **[2]**	May reduce <u>every</u> a value by a constant and/or every b value by a constant, but not e.g. reducing rows and not calculating differences, $a - b$ Row minima 2, 1, 1 (o.e.) seen, on table or written But not just stating row maximin = 2 K identified as play-safe
2	**(a)**	**(ii)**	<table><tr><td></td><td>R</td><td>S</td><td>T</td></tr><tr><td>K</td><td>(7, 3)</td><td>(2, 6)</td><td>(5, 3)</td></tr><tr><td>L</td><td>(1, 5)</td><td>(8, 2)</td><td>(2, 5)</td></tr><tr><td>M</td><td>(3, 2)</td><td>(1, 5)</td><td>(4, 6)</td></tr><tr><td>Col <u>min</u> for B</td><td>2</td><td>2</td><td>3</td></tr></table> T has largest minimum Maximin = max {2, 2, 3} = 3 (so play-safe for Brett is T, as given)	 **M1** **A1** **[2]**	May reduce <u>every</u> a value by a constant and/or every b value by a constant, but not e.g. reducing rows and not calculating differences, $a - b$ Col minina 2, 2, 3 (o.e.) seen, on table or written Col maximin = 3 'max' or 'largest'

Question			Answer	Mark	Guidance
2	(b)	(i)	K	B1 [1]	K
2	(b)	(ii)	S	B1 FT [1]	Follow through to col with max b value in row from (b)(i)
2	(c)		(Column R is weakly dominated by) column T $3 = 3,\ 5 = 5$ and $2 < 6$	M1 A1 [2]	T (only) $3, 3\ \ 5, 5\ $ and $\ 2 < 6$
2	(d)		Let Brett play S with probability p and T with probability $1 - p$ If Annie plays X, Brett expects to win $6p + 3(1 - p) = 3 + 3p$ If Annie plays Y, Brett expects to win $2p + 5(1 - p) = 5 - 3p$ If Annie plays Z, Brett expects to win $5p + 6(1 - p) = 6 - p$ Optimum at $p = \frac{1}{3}$ Play S with probability $\frac{1}{3}$ and T with probability $\frac{2}{3}$	B1 B1 M1 A1FT [4]	Three correct expressions (in any form) for expected winnings in terms of a <u>single</u> variable (which need not be called p) Graph showing lines (0, 3) to (1, 6) (0, 5) to (1, 2) and (0, 6) to (1, 5) Horizontal axis from 0 to 1 using at least half width of grid (values 0 and 1 may be implied from lines) Vertical axis with a scale (at least 2 values marked on axis, or origin and at least 1 value marked) $\frac{1}{3}$ or prob corresponding to max point of their lower boundary Interpretation of their p in context, or implied if meaning of p (or $1 - p$) was given when setting up equations
2	(e)		Best outcome in each col for Annie: (K, R), (L, S), (K, T) Best outcome in each row for Brett: (K, S), (L, R), (L, T), (M, T) No common cell so no Nash equilibrium	B1 B1 [2]	(K, R), (L, S) and (K, T) identified as a set in any way (K, S), (L, R), (L, T) and (M, T) identified as a set in any way Or one set and verify none of these are optimal for other player

Question			Answer	Mark	Guidance
3					For reference:

For reference:

P	x	y	z	s	t	RHS
1	−3	1	0	0	0	0
0	2	0	1	1	0	18
0	−1	2	3	0	1	20

Question			Answer	Mark	Guidance
3	(a)		Maximise $P = 3x - y$	**B1** **B1** **[2]**	$3x - y$ Max P
3	(b)		Slack variables are added to $\leq$ inequalities to form equations $-x + 2y + 3z \leq 20$ becomes $-x + 2y + 3z + t = 20$, where $t \geq 0$	**M1** **A1** **[2]**	An appropriate description about removing inequalities $-x + 2y + 3z \leq 20$ or $-x + 2y + 3z + t = 20$ (allow a different letter is used for slack variable)
3	(c)		Pivot on 2 in x column	**B1**	Pivot choice stated or indicated on tableau (not just implied) (x column and value 2 or x column and middle row)

Table for part (c):

P	x	y	z	s	t	RHS
1	0	1	1.5	1.5	0	27
0	1	0	0.5	0.5	0	9
0	0	2	3.5	0.5	1	29

M1   Their pivot row divided through by their positive pivot value
M1   A tableau with basis cols P, their pivot col and either s or t in which the values in the final col are non-negative
A1   A correct tableau

$P = 27$ **B1**   27
$x = 9, y = 0, z = 0$ **M1**   Reading off ≥ 0 x, y, z values from their final tableau, not all 0
     A1   (9, 0, 0) oe

Moves along an edge of a convex polyhedron (or 3-dimensional convex polygon) or along the x-axis in 3-dimensional space (from the origin to (9, 0, 0))   **B1**   'edge' or 'x-axis' (not just implied from coordinates) <u>and</u> 3 dimensions (which may be implied from coordinates stated)

[8]

Question			Answer	Mark	Guidance
4	(a)		$2^8 = 256$ ways to assign 8 items to 2 sets		
			Order does not matter so divide by 2	M1	2^8 (or 256) and later dividing by 2
			Cannot have a set of size 0 and a set of size 8	M1	Dealing with the case(s) where one set is empty
			Total = 127 (as given)	A1	To achieve 127 from valid reasoning
			Alternative method		
			Set of 1 and set of 7 = $^8C_1 = 8$ ways	M1	8 ways to partition as a set of size 1 and a set of size 7
			Set of 2 and set of 6 = $^8C_2 = 28$ ways		(or any other ^8C_r evaluated, where $r = 2, 3, 4, 5$ or 6)
			Set of 3 and set of 5 = $^8C_3 = 56$ ways		
			Two sets of 4 = $^8C_4 \div 2 = 35$ ways	M1	Dividing by 2, including $^8C_4 \div 2 = 35$
			Total = 127 (as given)	A1	To achieve 127 from valid reasoning
				[3]	
4	(b)		$8 \times 7 \times 6 \times 5$	M1	Or attempt at $^8P_4$ or $(4! \times {}^8C_4)$
			=1680	A1	1680
				[2]	
4	(c)		<table><tr><td>Seat 5</td><td>F</td><td>H</td><td>G</td><td>F</td><td>G</td><td>H</td><td>F</td><td>G</td><td>H</td></tr><tr><td>Seat 6</td><td>E</td><td>E</td><td>E</td><td>H</td><td>H</td><td>G</td><td>G</td><td>H</td><td>G</td></tr><tr><td>Seat 7</td><td>H</td><td>F</td><td>H</td><td>E</td><td>E</td><td>E</td><td>H</td><td>F</td><td>F</td></tr><tr><td>Seat 8</td><td>G</td><td>G</td><td>F</td><td>G</td><td>F</td><td>F</td><td>E</td><td>E</td><td>E</td></tr></table>	M1	All nine columns completed Each column contains all four letters, even if people are in their designated seat(s) or there are repeated columns At least three different columns
				M1	At least three different correct derangements
				A1	All nine correct (in any order)
				[3]	
4	(d)		$^8C_4 = 70$ ways to choose four people to be in correct seats	M1	Appropriate use of $^8C_4$
			9 derangements of the other four people		
			$70 \times 9 = 630$	A1	630
				[2]	

Question			Answer	Mark	Guidance
5					For reference: <table><tr><td>AB</td><td>AC</td><td>AD</td><td>AE</td><td>BC</td><td>BD</td><td>BE</td><td>CD</td><td>CE</td><td>DE</td></tr><tr><td>300</td><td>500</td><td>900</td><td>700</td><td>200</td><td>600</td><td>400</td><td>500</td><td>-</td><td>100</td></tr></table>

5	(a)		D – E – B – C
			     |
			      A
			
			£1000
				M1
				
				A1
				[2]
					Any spanning tree for the 5 vertices
					May list arcs included rather than drawing tree, oe
					1000 (cao)

5	(b)	(i)	Lower bound for TSP Deleting A: MST = 700 700 + 300 + 500 = 1500 Deleting B: MST = 1100 1100 + 200 + 300 = 1600 Deleting C: MST = 800 800 + 200 + 500 = 1500 Deleting D: MST = 900 900 + 100 +500 = 1500 Deleting E: MST = 1000 1000 + 100 + 400 = 1500 Best lower bound = £1600	M1 A1 A1 B1 **[4]**	Weight of any one of these reduced MST's seen A correct calculation of 'MST + two least weight arcs from deleted vertex' Their best (greatest) lower bound (with all 5 totals given) 1600 (cao)
5		(ii)	Nearest neighbour method A – B – C – D – E – A = 1800 B – C – D – E – A – B = 1800 C – B – A – E – D – C = 1800 D – E – B – C – A – D = 2100 E – D – C – B – A – E = 1800 Best upper bound = £1800	M1 A1 A1 B1 **[4]**	Nearest neighbour (even if not closed, but visiting all 5 vertices) for at least one starting point, or implied from a correct total At least three correct totals Their best (least) upper bound (with all 5 totals given) 1800 (cao)
5		(iii)	A – B – C – D – E – A	M1 A1FT **[2]**	First three vertices correct, for any starting point and direction Correct cycle, or in reverse, with any starting point Or FT cycle seen in answer to (b)(ii) if UB ≠ 1800
5	(c)		Need to pave a route from the shop to the first field	B1 **[1]**	Connect to shop, fields are not points, or other valid reason
5	(d)	(i)	A – C – E – D (or A–B–C–E–D) < 900 so cost of CE < £300	M1 A1 **[2]**	Shortest route including CE, or implied from cost www 300 (or 299.99, 299, 290)
5	(d)	(ii)	MST = CE + ED + CB + BA (CE replaces BE) £600 + cost of CE < £900	M1 A1FT **[2]**	MST including CE, or implied from cost www 900 (or 899.99, 899, 890), follow through their 300 from (d)(i)

	Question		Answer	Mark	Guidance
6					For reference:

	Question		Answer	Mark	Guidance
6	**(a)**	**(i)**	<table><tr><td>Event</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td><td>⑥</td><td>⑦</td><td>⑧</td></tr><tr><td>Early time</td><td>0</td><td>2</td><td>4</td><td>4</td><td>7</td><td>10</td><td>10+x</td><td>15</td></tr></table> $0 \leq x \leq 3$	**M1** **A1** **B1** **[3]**	Forward pass, early event times at every vertex, shown on network or listed (increasing from 0) All correct, apart possibly from ⑦ ≤ 3
6	**(a)**	**(ii)**	<table><tr><td>Event</td><td>①</td><td>②</td><td>③</td><td>④</td><td>⑤</td><td>⑥</td><td>⑦</td><td>⑧</td></tr><tr><td>Late time</td><td>0</td><td>3</td><td>4</td><td>4</td><td>7</td><td>10</td><td>13</td><td>15</td></tr></table> A, F, H, K	**M1** **A1** **B1** **[3]**	Backward pass, late event times at every vertex (decreasing, with start and finish matching the forward pass) All correct, may have max{12+x, 15} or 12+x, 15 at ⑧ A, F, H, K Allow A, F, H, I, J, K (with both I and J) but no others
6	**(b)**		Max value for *n* is 2 2 do A while another does B then C 4 workers do K while the other does I then J D, (E), F, G and H must be completed between time 4 and time 10 G needs 2 workers so 3 workers are available for D, (E), F and H G takes 5 minutes F followed by H uses 1 worker for 6 minutes E can be fitted in before D so only 2 workers are needed	**B1** **M1*** **M1dep*** **A1** **[4]**	2 For reference: <table><tr><td>Activity</td><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td><td>F</td><td>G</td><td>H</td><td>I</td><td>J</td><td>K</td></tr><tr><td>Workers</td><td>2</td><td>1</td><td>1</td><td>2</td><td>n</td><td>1</td><td>2</td><td>1</td><td>1</td><td>1</td><td>4</td></tr></table> Reasoning may be seen on grids D, F, G and H must be done between time 4 and time 10 3 workers available for D, F and H (or implied from E followed by D or vice versa) E done before D

Question	Answer	Mark	Guidance
(c)	18 minutes | Time	0	2	4	6	8	11	16 | |---|---|---|---|---|---|---|---| | Start | A, B | C | E, F, D | G | H | K | I, J | | | | | $(n = 1)$ | | | | $(x = 0)$ |	**B1** **[1]**	18 or $18 + x$ Working is not necessary but may be seen on grids
(d)	0 minutes	**B1FT** **[1]**	Their answer to part c minus 18 Working is not necessary but may be seen on grids
(e)	1 worker	**B1** **[1]**	1 Working is not necessary but may be seen on grids

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