



Oxford Cambridge and RSA

**Monday 05 October 2020 – Afternoon**

**A Level Further Mathematics A**

**Y540/01 Pure Core 1**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further Mathematics A
- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use  $g = 9.8$  unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 Find the mean value of  $f(x) = x^2 + 6x$  over the interval  $[0, 3]$ . [2]

2 Find an expression for  $1 \times 2^2 + 2 \times 3^2 + 3 \times 4^2 + \dots + n(n+1)^2$  in terms of  $n$ . Give your answer in fully factorised form. [3]

3 You are given the matrix  $\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$ .

(a) Find  $\mathbf{A}^4$ . [1]

(b) Describe the transformation that  $\mathbf{A}$  represents. [2]

The matrix  $\mathbf{B}$  represents a reflection in the plane  $x = 0$ .

(c) Write down the matrix  $\mathbf{B}$ . [1]

The point  $P$  has coordinates  $(2, 3, 4)$ . The point  $P'$  is the image of  $P$  under the transformation represented by  $\mathbf{B}$ .

(d) Find the coordinates of  $P'$ . [1]

4 **In this question you must show detailed reasoning.**

(a) Determine the square roots of  $25i$  in the form  $re^{i\theta}$ , where  $0 \leq \theta < 2\pi$ . [3]

(b) Illustrate the number  $25i$  and its square roots on an Argand diagram. [1]

5 By expanding  $\left(z^2 + \frac{1}{z^2}\right)^3$ , where  $z = e^{i\theta}$ , show that  $4 \cos^3 2\theta = \cos 6\theta + 3 \cos 2\theta$ . [5]

- 6 The equations of two non-intersecting lines,  $l_1$  and  $l_2$ , are

$$l_1: \mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 2 \\ 2 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}.$$

Find the shortest distance between lines  $l_1$  and  $l_2$ . [5]

- 7 Prove by induction that the sum of the cubes of three consecutive positive integers is divisible by 9. [5]

- 8 (a) Using exponentials, show that  $\cosh 2u \equiv 2 \sinh^2 u + 1$ . [2]

(b) By differentiating both sides of the identity in part (a) with respect to  $u$ , show that

$$\sinh 2u \equiv 2 \sinh u \cosh u. \quad [1]$$

- (c) Use the substitution  $x = \sinh^2 u$  to find  $\int \sqrt{\frac{x}{x+1}} dx$ . Give your answer in the form  $a \sinh^{-1} b \sqrt{x} + f(x)$  where  $a$  and  $b$  are integers and  $f(x)$  is a function to be determined. [5]

- (d) Hence determine the exact area of the region between the curve  $y = \sqrt{\frac{x}{x+1}}$ , the  $x$ -axis, the line  $x = 1$  and the line  $x = 2$ . Give your answer in the form  $p + q \ln r$  where  $p$ ,  $q$  and  $r$  are numbers to be determined. [2]

- 9 You are given that the cubic equation  $2x^3 + px^2 + qx - 3 = 0$ , where  $p$  and  $q$  are real numbers, has a complex root  $\alpha = 1 + i\sqrt{2}$ .

(a) Write down a second complex root,  $\beta$ . [1]

(b) Determine the third root,  $\gamma$ . [2]

(c) Find the value of  $p$  and the value of  $q$ . [2]

(d) Show that if  $n$  is an integer then  $\alpha^n + \beta^n + \gamma^n = 2 \times 3^{\frac{1}{2}n} \times \cos n\theta + \frac{1}{2^n}$  where  $\tan \theta = \sqrt{2}$ . [4]

- 10** A particle of mass  $0.5\text{ kg}$  is initially at point  $O$ . It moves from rest along the  $x$ -axis under the influence of two forces  $F_1\text{ N}$  and  $F_2\text{ N}$  which act parallel to the  $x$ -axis. At time  $t$  seconds the velocity of the particle is  $v\text{ m s}^{-1}$ .  
 $F_1$  is acting in the direction of motion of the particle and  $F_2$  is resisting motion.

In an initial model

- $F_1$  is proportional to  $t$  with constant of proportionality  $\lambda > 0$ ,
- $F_2$  is proportional to  $v$  with constant of proportionality  $\mu > 0$ .

- (a)** Show that the motion of the particle can be modelled by the following differential equation.

$$\frac{1}{2} \frac{dv}{dt} = \lambda t - \mu v \quad [2]$$

- (b)** Solve the differential equation in part **(a)**, giving the particular solution for  $v$  in terms of  $t$ ,  $\lambda$  and  $\mu$ . [7]

You are now given that  $\lambda = 2$  and  $\mu = 1$ .

- (c)** Find a formula for an approximation for  $v$  in terms of  $t$  when  $t$  is large. [2]

In a refined model

- $F_1$  is constant, acting in the direction of motion with magnitude  $2\text{ N}$ ,
- $F_2$  is as before with  $\mu = 1$ .

- (d)** Write down a differential equation for the refined model. [1]

- (e)** Without solving the differential equation in part **(d)**, write down what will happen to the velocity in the long term according to this refined model. [1]

11 A curve has cartesian equation  $x^3 + y^3 = 2xy$ .

$C$  is the portion of the curve for which  $x \geq 0$  and  $y \geq 0$ . The equation of  $C$  in polar form is given by  $r = f(\theta)$  for  $0 \leq \theta \leq \frac{1}{2}\pi$ .

(a) Find  $f(\theta)$ . [2]

(b) Find an expression for  $f(\frac{1}{2}\pi - \theta)$ , giving your answer in terms of  $\sin \theta$  and  $\cos \theta$ . [2]

(c) Hence find the line of symmetry of  $C$ . [1]

(d) Find the value of  $r$  when  $\theta = \frac{1}{4}\pi$ . [1]

(e) By finding values of  $\theta$  when  $r = 0$ , show that  $C$  has a loop. [2]

12 Show that  $\int_0^{\frac{1}{\sqrt{3}}} \frac{4}{1-x^4} dx = \ln(a + \sqrt{b}) + \frac{\pi}{c}$  where  $a$ ,  $b$  and  $c$  are integers to be determined. [6]

**END OF QUESTION PAPER**

**BLANK PAGE**

**BLANK PAGE**

---

# OCR

Oxford Cambridge and RSA

## Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website ([www.ocr.org.uk](http://www.ocr.org.uk)) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.