OCR
RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4761:** Mechanics 1

Mark Scheme for January 2011

comment

You should expect to follow through from one part to another unless the scheme says otherwise but not follow through within a part unless the scheme specifies this.
Each script must be viewed as a whole at some stage so that
(i) a candidate's writing of letters, digits, symbols on diagrams etc can be better interpreted;
(ii) repeated mistakes can be recognised (e.g. calculator in wrong angle mode throughout – penalty 1 in the script and FT except given answers).

You are advised to 'set width' for most questions but to 'set height' for the following:

Q 1		mark	note
(i)		B1 B1 2	Section from $t = 10$ to $t = 15$ Section from $t = 15$ to $t = 20$. FT connecting from their point when $t = 15$. Ignore graph outside $0 \le t \le 20$.
(ii)	$\frac{-6-14}{10} = -2$ so -2 m s^{-2}	M1 A1 2	Attempt at $\frac{\Delta v}{\Delta t}$
(iii)	**either** Displacement is $\frac{14}{2}\times 7 - \frac{13+5}{2}\times 6$ or $\frac{14}{2}\times 7 - \frac{3\times 6}{2} - 5\times 6 - \frac{5\times 6}{2}$ $= -5$ so 5 m downwards	M1 B1 B1 A1	FT misread from graph or graphing error to all but final A1 cao Attempt at whole area. Condone 'overlap' but not 'gaps'. 'Positive' area expression correct. Condone sign error. 'Negative' area expression correct. Condone overall sign error. Accept –5 m cao

	or Displacement is $14\times10+\frac{1}{2}\times(-2)\times10^2-\ 5\times 6+\ \frac{-6+0}{2}\times5$ $= 140 – 100 – 30 –15 = –5$ so 5 m downwards	 M1 A1 B1 A1 4	 Using *suvat* from 0 to 10 or 15 to 20. Condone 'overlap' but not 'gaps' Subtracting 30 or 15 or 45 Accept –5 m cao
		8	

Q 2		**mark**	**notes**
(i)	$\mathbf{F} = (10 – 8\cos50)\mathbf{i} + 8\sin50\mathbf{j}$ $= 4.85769…\mathbf{i} + 6.128355…\ \mathbf{j}$ so $4.86\mathbf{i} + 6.13\mathbf{j}$ (3 s. f.)	M1 A1 A1 3	Resolution. Accept $s \leftrightarrow c$. Condone resolution in only one direction. Award for a vector with either component correct or consistent $s \leftrightarrow c$ error is only mistake in the vector. Need not be evaluated. cao. Must be in $a\mathbf{i} + b\mathbf{j}$ or column format. Must be correct to 3 s. f.
(ii)	$\lvert\mathbf{F}\rvert = \sqrt{4.85769...^2 + 6.12835...^2} = 7.820101...$ so 7.82 (3 s. f.) angle is $\arctan\frac{4.857...}{6.128...}$ $= 38.40243…$ so 38.4° (3 s. f.)	 B1 M1 F1 3	 FT **their F** Or equivalent. FT **their F.** Accept $\arctan\frac{6.128...}{4.857...}$. Accept complementary angle and ± signs FT only **their** *F*.
		6	

Q 3		mark	notes
(i)	For P: the distance is $8T$	B1	Allow – ve. Allow any form.
	For Q: the distance is $\frac{1}{2}\times 4\times T^2$	B1 2	Allow – ve. Allow any form.
(ii)	Require $8T+\frac{1}{2}\times 4\times T^2=90$	M1	For linking correct expressions or **their** expressions from (i) with 90. Condone sign errors and use of displacement instead of distance. Condone '= 0'implied.
		A1	The expression is correct or correctly derived from **their** (i). Reason not required.
	so $8T + 2T^2 – 90 = 0$ so $T^2 + 4T – 45 = 0$	E1	Must be established. Do not award if **their** 'correct expression' comes from incorrect manipulation.
	This gives $(T – 5)(T + 9) = 0$	M1	Solving to find +ve root. Accept $(T + 5)(T – 9)$.
	so $T = 5$ since $T > 0$	A1 5	Condone 2nd root not found/discussed but not both roots given.
		7	

Q 4		mark	notes
(i)	When $t = 1$, $\mathbf{r} = \begin{pmatrix} 8 \\ 10-2 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$ $[8\mathbf{i} + (10-2)\mathbf{j} = 8\mathbf{i} + 8\mathbf{j}]$ Bearing OP is 045°	 B1 F1 2	Accept column or $a\mathbf{i} + b\mathbf{j}$ notation May be implied Accept 45°. Accept NE and northeast. Condone $\lvert\mathbf{r}\rvert$ given as well.
(ii)	$\mathbf{v} = \begin{pmatrix} 8 \\ 20t - 6t^2 \end{pmatrix}$ $[8\mathbf{i} + (20t - 6t^2)\mathbf{j}]$ The **i** cpt is always 8 so $\mathbf{v} \neq \mathbf{0}$ for any t	M1 A1 E1 3	Differentiating both components. Condone 1 error if clearly attempting differentiation. Must be a vector answer. Accept any correct argument e.g. based on **i** cpt never 0.
(iii)	$\mathbf{a} = \begin{pmatrix} 0 \\ 20-12t \end{pmatrix}$ $[(20 - 12t)\mathbf{j}]$ $\mathbf{a} = \mathbf{0}$ when $t = \frac{20}{12} = \frac{5}{3}$ so $\frac{5}{3}$ s (1.67 s (3 s. f.))	M1 F1 B1 3	Differentiating as a vector. Condone 1 error if clearly attempting differentiation of **their v**. FT **their v**. cao. Condone obtained from scalar equation.
		8	

Q5		mark	notes
(i)	In direction $\rightarrow$ $0^2 = 1.5^2 + 2 \times a \times 0.375$ so $a = -3$ and deceleration is 3 m s^{-2} N2L on both boxes $\rightarrow$ $-2F = (12+6) \times (-3)$ so $F = 27$	M1 A1 M1 A1 4	Use of $v^2 = u^2 + 2as$ or complete sequence of *suvat.* CWO. Accept ± 3 and ignore accel or decal. N2L. Correct mass. Condone $F = mga$. Allow F on LHS. FT **their** a. Accept sign errors. No extra terms. cao Condone this obtained from an equation with consistent signs not justified.
(ii)	Suppose the force in the rod is a tension T N2L gives box A $\rightarrow$ $T - 27 = 12 \times (-3)$ [box B $\rightarrow$ $-T - 27 = 6 \times (-3)$] so $T = -9$ and the force has magnitude 9 N It is a thrust (tension is +ve).	 M1 F1 E1 3	 N2L. $F = ma$. Correct mass. The '27' and the '3' must have the same sign. Ignore the sign of 'T'. FT only for mod(their 27) in place of '27' and/or mod(their 3) in place of '3' in this sign pattern. No extra terms. Accept $T = \pm 9$. FT only for mod(their 27) in place of '27' and/or mod(their 3) in place of '3'. cao Only accept thrust with $T = \pm 9$ and a sound argument.
		7	

Q 6		mark	notes
(i)	Let tension be T N N2L $\rightarrow$ $T - 6 = 4 \times 3$ T = 18 so 18 N	 M1 A1 A1 3	 Condone $F = mga$. Condone resistance omitted or an extra force. Allow only sign error(s). cao
(ii)	Let acceleration be a m s^{-2} $25\cos 40 - 6 = 4a$ a = 3.28777.. so 3.29 m s^{-2} (3 s. f.)	 M1 M1 A1 3	 Attempt at resolution of 25 N. Allow $s \leftrightarrow c$. Allow $F = mga$ and sign error(s). No extra forces. Both forces present. cao
(iii)	Let tension be T N up the slope $T + 6 - 4 \times 9.8 \times \sin 35 = 0$ T = 16.48419… so 16.5 N (3 s. f.)	 M1 B1 A1 3	 Resolving along slope. Allow 6 N omitted. If different direction used all required forces present (except 6 N). Allow $s \leftrightarrow c$. No extra forces. Allow sign errors. Condone g omitted. If resolution is along plane, weight term correct. If resolution in another direction, one resolution correct.
(iv) (A)		 B1 B1 2	 At least two of tension, weight and NR marked correctly with arrows and labels (accept *mg*, *W*, *T* and words etc). All correct. No extra forces. Accept *mg*, *W*, *T* and words etc. Condone resolved parts as well only if clearly indicated as such by e.g. using dotted lines.
(B)	**continued**		

Q6 (iv) (B)	up the slope $25\cos\theta + 6 - 4g\sin 35 = 0$ so $25\cos\theta = 16.48414...$ so $\theta = 48.7483....$ so 48.7° (3 s. f.)	M1 A1 A1 3	No extra forces. Allow $s \leftrightarrow c$. All forces present and required resolutions attempted. Allow sign errors. Condone *g* omitted. Condone *g* omitted. cao [If they use **their** (iii): M1 Equating **their** (iii) to an attempt at resolving 25. Allow $s \leftrightarrow c$. No extra forces. A1 FT **their** *T* from (iii) A1 cao]
(C)	Resolve perp to slope $R + 25\sin\theta - 4\times 9.8\times\cos 35 = 0$ *R* = 13.315248.. so 13.3 N (3 s. f.)	M1 A1 A1 3	All forces present and resolutions attempted. No extra forces. Allow $s \leftrightarrow c$. FT their angle. Condone *g* omitted. FT their angle. Condone *g* omitted. cao
		17	

Q7		**mark**	**notes**
(i) (A)	$x = Ut\cos 68.5°$	B1 1	
(i) (B)	$y = Ut\sin 68.5° - 4.9\times t^2$	M1 A1 2	Allow '*u*' = *U*. Allow $s \leftrightarrow c$. Allow *g* as *g*, ±9.8, ±9.81, ±10. Allow +2. Accept not 'shown'. Do not allow +2. Allow e.g + 0.5 × (- 9.8) × t^2 instead of – 4.9t^2. Accept *g* not evaluated
	continued		

Q7 (ii)	**either** At D, $y = 0$ so $U\sin 68.5°T - 4.9\times T^2 = 0$ $\Rightarrow T\left(U\sin 68.5° - 4.9T\right) = 0$ so $T = 0$ (at C) or $T = \dfrac{U\sin 68.5°}{4.9}$ (at D) **or** Use (i)(A) and put $x = 10$ with $t = T$ to get $UT\cos 68.5° = 10$	 M1 M1 E1 M1 M1 E1 B1 4	 Equating correct y to 0 or **their** y to correct value. Attempting to factorise (or solve). Allow ÷ T without comment. Properly shown. Accept no ref to $T = 0$. Accept $T = 0$ given as well without comment. Find time to top Double time to the top
(iii)	Eliminating T from the results in (ii) gives $U\cos 68.5°\times\dfrac{U\sin 68.5°}{4.9} = 10$ so $U = 11.98729\ldots$ so 12.0 (3 s. f.)	 M1 M1 E1 3	 Substituting, using correct expressions or **their** expressions from (ii). Attempt to solve for U^2 or U. Some evidence seen. e.g. $142.8025.. < U^2 < 145.2025...$ with clear statement, or 11.9… seen with clear statement or 11.98… seen. Accept 11.98… seen for full marks.
(iv)	**continued**		

(iv)	Require $Ut\sin 68.5° - 4.9t^2 = -2$	M1	Equating correct y to -2 or **their** y to correct value. Allow use of U, 11.987… or 12. Allow implicit '= 0'
	Solving $4.9t^2 - Ut\sin 68.5° - 2 = 0$	M1	Dep on 1st M1. Attempt to solve a 3 term quadratic to find at least the +ve root. Allow if two correct roots seen WW.
	$t = -$ 0.1670594541… , 2.4431591… (Using 12: – 0.1669052502.. , 2.445478886..)	A1	Accept only + ve root given
			Alternative method of e.g. finding time to highest point and then time to the ground. M1 all times attempted, at least one by a sound method. M1 both methods sound and complete. A1.
	Require $U\cos 68.5° \times 2.44… - 10$ = 0.7336…. so 0.734 m (3 s. f.) (Using 12 consistently, 0.7552… so 0.755 (3 s. f.))	M1 A1	Dep on first M1. Allow **their** expression for x. Allow ' – 10' omitted. cao. Accept $0.73 \le x \le 0.76$
		5	
(v)	Eliminate t from (i) (B)		
	using $t = \dfrac{x}{U\cos 68.5°}$ from (i)(A)	M1	May be implied. FT their (i).
	so $y = x\tan 68.5° - \dfrac{4.9x^2}{U^2(\cos 68.5°)^2}$	E1	Clearly shown.
	We require $y = 0$ when $x = 10$	M1	Must see attempt to solve. Or use $x = 10.73…$ when $y = -2$.
	so $U = 11.98729…$ so 12.0 (3 s. f.)	E1	Must see evidence of fresh calculation or statement that they have now got the same expression for evaluation.
		4	
		19	