4754 (C4) Applications of Advanced Mathematics

Section A

1 $4\cos\theta - \sin\theta = R\cos(\theta+\alpha)$ $= R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$ $\Rightarrow R\cos\alpha = 4,\ R\sin\alpha = 1$ $\Rightarrow\ R^2 = 1^2 + 4^2 = 17,\ R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ $\Rightarrow\ \alpha = 0.245$ $\sqrt{17}\cos(\theta + 0.245) = 3$ $\Rightarrow$ $\cos(\theta + 0.245) = 3/\sqrt{17}$ $\Rightarrow$ $\theta + 0.245 = 0.756,\ 5.527$ $\Rightarrow$ $\theta = 0.511,\ 5.282$	M1 B1 M1 A1 M1 A1A1 [7]	correct pairs $R = \sqrt{17} = 4.123$ $\tan\alpha = ¼$ o.e. $\alpha = 0.245$ $\theta + 0.245 = \arccos 3/\sqrt{17}$ ft their R, α for method (penalise extra solutions in the range (-1))
2 $\dfrac{x}{(x+1)(2x+1)} = \dfrac{A}{x+1} + \dfrac{B}{(2x+1)}$ $\Rightarrow$ $x = A(2x + 1) + B(x + 1)$ $x = -1 \Rightarrow -1 = -A \Rightarrow A = 1$ $x = -½ \Rightarrow -½ = ½B \Rightarrow B = -1$ $\Rightarrow$ $\dfrac{x}{(x+1)(2x+1)} = \dfrac{1}{x+1} - \dfrac{1}{(2x+1)}$ $\Rightarrow$ $\int \dfrac{x}{(x+1)(2x+1)}dx = \int \dfrac{1}{x+1} - \dfrac{1}{(2x+1)}dx$ $= \ln(x + 1) - ½\ln(2x + 1) + c$	M1 M1 A1 A1 B1 B1 A1 [7]	correct partial fractions substituting, equating coeffts or cover-up $A = 1$ $B = -1$ $\ln(x + 1)$ ft their A $-½\ln(2x + 1)$ ft their B cao – must have c
3 $\dfrac{dy}{dx} = 3x^2y$ $\Rightarrow$ $\int \dfrac{dy}{y} = \int 3x^2\, dx$ $\Rightarrow$ $\ln y = x^3 + c$ When $x = 1$, $y = 1$, $\Rightarrow \ln 1 = 1 + c \Rightarrow c = -1$ $\Rightarrow$ $\ln y = x^3 - 1$ $\Rightarrow$ $y = e^{x^3-1}$	M1 A1 B1 A1 [4]	separating variables condone absence of c $c = -1$ oe o.e.
4 When $x = 0$, $y = 4$ $\Rightarrow$ $V = \pi\int_0^4 x^2 dy$ $= \pi\int_0^4 (4-y)dy$ $= \pi\left[4y - \frac{1}{2}y^2\right]_0^4$ $= \pi(16 - 8) = 8\pi$	B1 M1 M1 B1 A1 [5]	must have integral, π, x^2 and dy soi must have π ,their (4-y), their numerical y limits $\left[4y - \frac{1}{2}y^2\right]$

5 $\frac{dy}{dt} = -a(1+t^2)^{-2}.2t$ $\frac{dx}{dt} = 3at^2$ $\Rightarrow$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2at}{3at^2(1+t^2)^2}$ $= \frac{-2}{3t(1+t^2)^2}$ * At $(a, ½\, a)$, $t = 1$ $\Rightarrow$ gradient = $\frac{-2}{3\times 2^2}$ = –1/6	M1 A1 B1 M1 E1 M1 A1 [7]	$(1+t^2)^{-2} \times kt$ for method ft finding t
6 $\text{cosec}^2\theta = 1 + \cot^2\theta$ $\Rightarrow$ $1 + \cot^2\theta - \cot\theta = 3$ * $\Rightarrow$ $\cot^2\theta - \cot\theta - 2 = 0$ $\Rightarrow$ $(\cot\theta - 2)(\cot\theta + 1) = 0$ $\Rightarrow$ $\cot\theta = 2$, $\tan\theta = ½$, $\theta = 26.57°$ $\cot\theta = -1$, $\tan\theta = -1$, $\theta = 135°$	E1 M1 A1 M1 A1 A1 [6]	clear use of $1+\cot^2\theta = \text{cosec}^2\theta$ factorising or formula roots 2, –1 cot = 1/tan used $\theta = 26.57°$ $\theta = 135°$ (penalise extra solutions in the range (-1))

Section B

7(i) $\vec{AB} = \begin{pmatrix} -1 \\ -2 \\ 0 \end{pmatrix}$	B1	
$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$	B1 [2]	or equivalent alternative
(ii) $\mathbf{n} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$	B1	
$\cos\theta = \dfrac{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} . \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}}{\sqrt{2}\sqrt{5}} = \dfrac{1}{\sqrt{10}}$ $\Rightarrow$ $\theta = 71.57°$	B1 M1 M1 A1 [5]	correct vectors (any multiples) scalar product used finding invcos of scalar product divided by two modulae 72° or better
(iii) $\cos\phi = \dfrac{\begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} . \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}}{\sqrt{2}\sqrt{9}} = \dfrac{2+1}{3\sqrt{2}} = \dfrac{1}{\sqrt{2}}$ $\Rightarrow$ $\phi = 45°$ *	M1 A1 E1 [3]	ft their **n** for method $\pm 1/\sqrt{2}$ oe exact
(iv) $\sin 71.57° = k \sin 45°$ $\Rightarrow$ $k = \sin 71.57° / \sin 45° = 1.34$	M1 A1 [2]	ft on their 71.57° oe
(v) $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} -2 \\ -2 \\ -1 \end{pmatrix}$ $x = -2\mu$, $z = 2-\mu$ $x + z = -1$ $\Rightarrow$ $-2\mu + 2 - \mu = -1$ $\Rightarrow$ $3\mu = 3$, $\mu = 1$ $\Rightarrow$ point of intersection is $(-2, -2, 1)$ distance travelled through glass = distance between (0, 0, 2) and (–2, –2, 1) $= \sqrt{(2^2 + 2^2 + 1^2)} = 3$ cm	M1 M1 A1 A1 B1 [5]	soi subst in $x+z = -1$ www dep on $\mu=1$

8(i)	(A) $360° \div 24 = 15°$ CB/OB = sin 15° $\Rightarrow$ CB = 1 sin 15° $\Rightarrow$ AB = 2CB = 2 sin 15°*	 M1 E1 [2]	AB=2AC or 2CB $\angle$AOC= 15° oe
	(B) $\cos 30° = 1 - 2\sin^2 15°$ $\cos 30° = \sqrt{3}/2$ $\Rightarrow$ $\sqrt{3}/2 = 1 - 2\sin^2 15°$ $\Rightarrow$ $2\sin^2 15° = 1 - \sqrt{3}/2 = (2 - \sqrt{3})/2$ $\Rightarrow$ $\sin^2 15° = (2 - \sqrt{3})/4$ $\Rightarrow$ $\sin 15° = \sqrt{\dfrac{2-\sqrt{3}}{4}} = \dfrac{1}{2}\sqrt{2-\sqrt{3}}$ *	B1 B1 M1 E1 [4]	 simplifying
	(C) Perimeter = $12 \times$ AB = $24 \times \frac{1}{2}\sqrt{(2-\sqrt{3})}$ $= 12\sqrt{(2-\sqrt{3})}$ circumference of circle > perimeter of polygon $\Rightarrow$ $2\pi > 12\sqrt{(2-\sqrt{3})}$ $\Rightarrow$ $\pi > 6\sqrt{(2-\sqrt{3})}$	M1 E1 [2]	
(ii)	(A) tan 15° = FE/OF $\Rightarrow$ FE = tan 15° $\Rightarrow$ DE = 2FE = 2tan 15°	M1 E1 [2]	
	(B) $\tan 30 = \dfrac{2\tan 15}{1-\tan^2 15} = \dfrac{2t}{1-t^2}$ tan 30 = $1/\sqrt{3}$ $\Rightarrow$ $\dfrac{2t}{1-t^2} = \dfrac{1}{\sqrt{3}} \Rightarrow 2\sqrt{3}t = 1 - t^2$ $\Rightarrow$ $t^2 + 2\sqrt{3}\,t - 1 = 0$ *	B1 M1 E1 [3]	
	(C) $t = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = 2 - \sqrt{3}$ circumference < perimeter $\Rightarrow$ $2\pi < 24(2-\sqrt{3})$ $\Rightarrow$ $\pi < 12(2-\sqrt{3})$ *	M1 A1 M1 E1 [4]	 using positive root from exact working
(iii)	$6\sqrt{(2-\sqrt{3})} < \pi < 12(2-\sqrt{3})$ $\Rightarrow$ $3.106 < \pi < 3.215$	B1 B1 [2]	3.106, 3.215

Comprehension

1. $\frac{1}{4}\times[3+1+(-1)+(-2)]=0.25$ * **M1, E1**

2. (i) *b* is the benefit of shooting some soldiers from the other side while none of yours are shot**.** *w* is the benefit of having some of your own soldiers shot while not shooting any from the other side.

   Since it is more beneficial to shoot some of the soldiers on the other side than it is to have your own soldiers shot, $b > w$. **E1**

   (ii) *c* is the benefit from mutual co-operation (i.e. no shooting).
   d is the benefit from mutual defection (soldiers on both sides are shot).
   With mutual co-operation people don't get shot, while they do with mutual defection.
   So $c > d$. **E1**

3. $\frac{1\times 2+(-2)\times(n-2)}{n}=-1.999$ **or equivalent** (allow n,n+2) **M1, A1**

   $n=6000$ so you have played 6000 rounds. **A1**

4. No. The inequality on line 132, $b+w<2c$, would not be satisfied since $6+(-3)>2\times 1$. **M1** *b*+*w*<2c and subst **A1** No,3>2oe

5. (i)

Round	You	Opponent	Your score	Opponent's score
1	C	D	-2	3
2	D	C	3	-2
3	C	D	-2	3
4	D	C	3	-2
5	C	D	-2	3
6	D	C	3	-2
7	C	D	-2	3
8	D	C	3	-2
…	…	…	…	…

M1 Cs and Ds in correct places**, A1** C=-2**, A1** D=3

   (ii) $\frac{1}{2}\times[3+(-2)]=0.5$ DM1 A1ft their 3,-2

6. (i) All scores are increased by two points per round **B1**

   (ii) The same player wins. No difference/change. The rank order of the players remains the same. **B1**

7. (i) They would agree to co-operate by spending less on advertising or by sharing equally. **B1**

   (ii) Increased market share (or more money or more customers). **DB1**