

Mark Scheme (Results)

January 2008

GCE

GCE Mathematics (6678/01)

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6678 Mechanics M2
Mark Scheme

| Question Number | Scheme | Marks |
|-----------------|---|--|
| 1. | <p>(a) KE lost is $\frac{1}{2} \times 2.5 \times 8^2 = 80 \text{ (J)}$</p> <p>(b) Work energy $80 = R \times 20$ ft their (a) $R = 4$</p> <p>Alternative to (b) $0^2 = 8^2 - 2 \times a \times 20 \Rightarrow a = (-)1.6$</p> <p>N2L $R = 2.5 \times 1.6$ ft their a $= 4$</p> | M1 A1 (2) M1 A1 ft A1 (3) [5] |
| 2. | <p>(a) $\dot{\mathbf{p}} = (6t - 6)\mathbf{i} + (9t^2 - 4)\mathbf{j} \text{ (ms}^{-1}\text{)}$</p> <p>(b) $9t^2 - 4 = 0$ $t = \frac{2}{3}$</p> <p>(c) $t = 1 \Rightarrow \dot{\mathbf{p}} = 5\mathbf{j}$ ft their \dot{p} $(+/-) 2\mathbf{i} - 6\mathbf{j} = 0.5(\mathbf{v} - 5\mathbf{j})$ $\mathbf{v} = 4\mathbf{i} - 7\mathbf{j} \text{ (ms}^{-1}\text{)}$</p> | M1 A1 (2) M1 DM1 A1 (3) B1 ft M1 M1 A1 (4) [9] |

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| 3. | <p>(a) $20000 = 16F \quad (F = 1250)$ $\Rightarrow F = 550 + 1000 \times 9.8 \sin \theta$ ft their F Leading to $\sin \theta = \frac{1}{14} \quad *$ cso</p> <p>(b) N2L $\Rightarrow 550 + 1000 \times 9.8 \times \sin \theta = 1000a$ $(550 + 1000 \times 9.8 \times \frac{1}{14}) = 1000a$ or $1250 = 1000a$ $(a = (-)1.25)$ $v^2 = u^2 + 2as \Rightarrow 16^2 = 2 \times 1.25 \times y$ $y \approx 102 \quad \text{accept } 102.4, 100$</p> <p>Alternative to (b) Work-Energy $\frac{1}{2} \times 1000 \times 16^2 - 1000 \times 9.8 \times \frac{1}{14} y = 550y$ $y \approx 102 \quad \text{accept } 102.4, 100$</p> | M1 A1 M1 A1ft A1 (5) M1 A1 M1 A1 (4) [9] M1 M1 A1 A1 (4) | | | | | | | | |
| 4. | <p>(a) Mass ratio Triangle 126 Circle 9π S (28.3) (97.7)</p> <table style="margin-left: 100px;"> <tr> <td>\bar{x}</td> <td>7</td> <td>5</td> <td>\bar{x}</td> </tr> <tr> <td>\bar{y}</td> <td>4</td> <td>5</td> <td>\bar{y}</td> </tr> </table> <p>4, 7 seen</p> <p>$126 \times 7 = 9\pi \times 5 + (126 - 9\pi) \times \bar{x}$ ft their table values $\bar{x} \approx 7.58 \quad (\frac{882 - 45\pi}{126 - 9\pi})$ awrt 7.6</p> <p>$126 \times 4 = 9\pi \times 5 + (126 - 9\pi) \times \bar{y}$ ft their table values $\bar{y} \approx 3.71 \quad (\frac{504 - 45\pi}{126 - 9\pi})$ awrt 3.7</p> <p>(b) $\tan \theta = \frac{\bar{y}}{21 - \bar{x}}$ ft their \bar{x}, \bar{y} $\theta \approx 15^\circ$</p> | \bar{x} | 7 | 5 | \bar{x} | \bar{y} | 4 | 5 | \bar{y} | B1 B1ft B1 M1 A1ft A1 M1 A1ft A1 (9) M1 A1ft A1 (3) [12] |
| \bar{x} | 7 | 5 | \bar{x} | | | | | | | |
| \bar{y} | 4 | 5 | \bar{y} | | | | | | | |

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| 5. | <p>(a)</p> <p>M(A) $N \times 4a \cos 30^\circ = 3mg \times a \sin 30^\circ + mg \times 2a \sin 30^\circ$</p> $N = \frac{5}{4}mg \tan 30^\circ \quad (= \frac{5}{4\sqrt{3}}mg = 7.07\dots m)$ $\rightarrow F_r = N, \quad \uparrow R = 4mg$ <p>Using $F_r = \mu R$</p> $\frac{5}{4\sqrt{3}}mg = \mu R \quad \text{for their } R$ $\mu = \frac{5}{16\sqrt{3}}$ <p style="text-align: right;">awrt 0.18</p> <p>Alternative method: M(B): $mg \times 2a \sin 30^\circ + 3mg \times 3a \sin 30^\circ + F \times 4a \cos 30^\circ = R \times 4a \sin 30^\circ$</p> $11mga \sin 30^\circ + F \times 4a \cos 30^\circ = R \times 4a \sin 30^\circ$ $\frac{11mg}{2} + F \frac{4\sqrt{3}}{2} = 2R$ $\uparrow R = 4mg,$ <p>Using $F_r = \mu R$</p> $8\mu\sqrt{3} = \frac{5}{2}, \quad \mu = \frac{5}{16\sqrt{3}}$ | <p>M1 A2(1,0)</p> <p>DM1 A1</p> <p>B1, B1</p> <p>B1</p> <p>M1</p> <p>A1 (10)</p> <p>[10]</p> <p>M1A3(2,1,0)</p> <p>DM1A1</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> |

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| 6. | <p>(a)</p> $\begin{aligned} \rightarrow & \quad 30 = 2ut \\ \uparrow & \quad -47.5 = 5ut - 4.9t^2 \\ & \quad -47.5 = 75 - 4.9t^2 \quad \text{eliminating } u \text{ or } t \\ & \quad t^2 = \frac{75+47.5}{4.9} (= 25) \\ & \quad t = 5 \quad * \end{aligned}$ <p style="text-align: right;">cso</p> | <p>B1 M1 A1 DM1 DM1 A1 (6)</p> |
| | <p>(b)</p> $30 = 2ut \Rightarrow 30 = 10u \Rightarrow u = 3$ | <p>M1 A1 (2)</p> |
| | <p>(c)</p> $\begin{aligned} \uparrow & \quad \dot{y} = 5u - 9.8t = -34 & \text{M1 requires both} \\ \rightarrow & \quad \dot{x} = 2u = 6 & \dot{x} \text{ and } \dot{y} \\ & \quad v^2 = 6^2 + (-34)^2 \\ & \quad v \approx 34.5 \quad (\text{ms}^{-1}) & \text{accept 35} \end{aligned}$ | <p>M1 A1 A1 DM1 A1 (5)</p> |
| | <p>Alternative to (c)</p> $\begin{aligned} \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 &= m \times g \times 47.5 \quad \text{with} \quad v_A^2 = 6^2 + 15^2 = 261 \\ v_B^2 &= 261 + 2 \times 9.8 \times 47.5 \quad (= 1192) \\ v_B &\approx 34.5 \quad (\text{ms}^{-1}) & \text{accept 35} \end{aligned}$ | <p>[13]</p> <p>M1 A(2,1,0) DM1 A1 (5)</p> |
| | <p>BEWARE : Watch out for incorrect use of $v^2 = u^2 + 2as$</p> | |

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| 7. | (a) | |
| | <p style="text-align: center;">$\xrightarrow{2u}$ \xrightarrow{u} $\circlearrowleft 2m$ $\circlearrowright 3m$ \xrightarrow{x} \xrightarrow{y}</p> | |
| | LM $4mu + 3mu = 2mx + 3my$ NEL $y - x = \frac{1}{2}u$ Solving to $y = \frac{8}{5}u *$ | M1 A1 B1 cs0 M1 A1 (5) |
| | (b) $x = \frac{11}{10}u$ or equivalent | B1 |
| | Energy loss $\frac{1}{2} \times 2m \left((2u)^2 - \left(\frac{11}{10}u\right)^2 \right) + \frac{1}{2} \times 3m \left(u^2 - \left(\frac{8}{5}u\right)^2 \right)$ $= \frac{9}{20}mu^2$ | M1 A(2,1,0) A1 (5) |
| | (c) | |
| | <p style="text-align: center;">$\xrightarrow{\frac{8}{5}u}$ \xrightarrow{s} \xrightarrow{t} $\circlearrowright m$</p> | |
| | LM $\frac{24}{5}mu = 3ms + mt$ NEL $t - s = \frac{8}{5}eu$ Solving to $s = \frac{2}{5}u(3 - e)$ | M1 A1 B1 M1 A1 |
| | For a further collision $\frac{11}{10}u > \frac{2}{5}u(3 - e)$ $e > \frac{1}{4}$ ignore $e \leq 1$ | M1 A1 (7) [17] |