General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MM2B

Unit Mechanics 2B

Wednesday 20 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

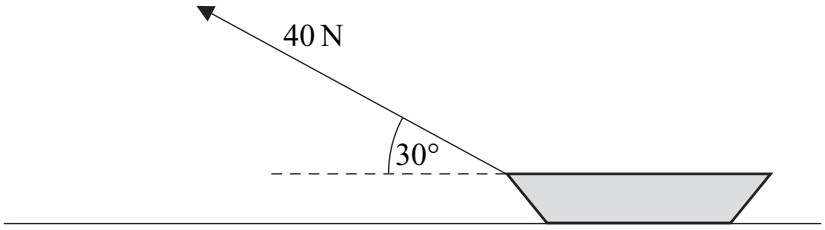
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

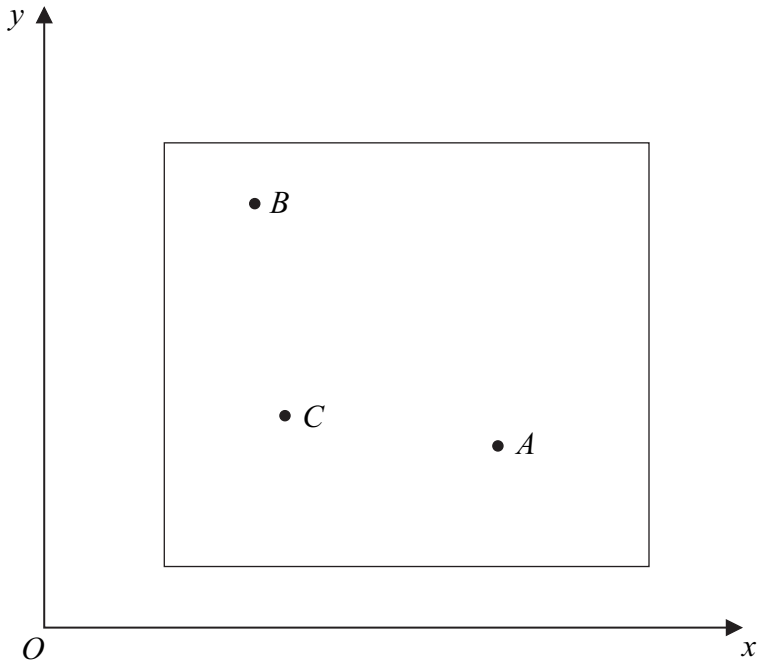
1 An inextensible rope is attached to a sledge which is at rest on a horizontal surface. A constant force of magnitude 40 newtons at an angle of 30° to the horizontal is applied to the sledge, as shown in the diagram.



Calculate the work done by the force as the sledge is moved 5 metres along the surface.

(3 marks)

2 A piece of modern art is modelled as a uniform lamina and three particles. The diagram shows the lamina, the three particles A, B and C, and the x- and y-axes.

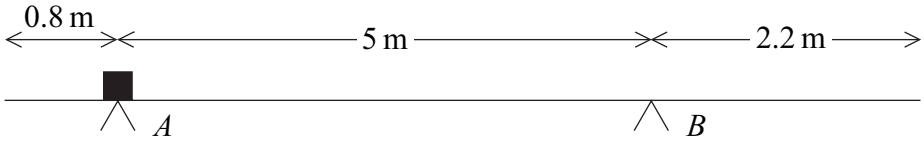


The lamina, which is fixed in the x–y plane, has mass 10 kg and its centre of mass is at the point $(12, 9)$.

The three particles are attached to the lamina.
Particle A has mass 3 kg and is at the point $(15, 6)$.
Particle B has mass 1 kg and is at the point $(7, 14)$.
Particle C has mass 6 kg and is at the point $(8, 7)$.

Find the coordinates of the centre of mass of the piece of modern art. *(6 marks)*

3 A uniform plank, of length 8 metres, has mass 30 kg. The plank is supported in equilibrium in a horizontal position by two smooth supports at the points A and B, as shown in the diagram. A block, of mass 20 kg, is placed on the plank at point A.



(a) Draw a diagram to show the forces acting on the plank. *(2 marks)*

(b) Show that the magnitude of the force exerted on the plank by the support at B is $19.2g$ newtons. *(3 marks)*

(c) Find the magnitude of the force exerted on the plank by the support at A. *(2 marks)*

(d) Explain how you have used the fact that the plank is uniform in your solution. *(1 mark)*

4 A particle moves so that at time t seconds its velocity $\mathbf{v}\,\text{m}\,\text{s}^{-1}$ is given by

$$\mathbf{v} = (4t^3 - 12t + 3)\mathbf{i} + 5\mathbf{j} + 8t\mathbf{k}$$

(a) When $t = 0$, the position vector of the particle is $(-5\mathbf{i} + 6\mathbf{k})$ metres.

Find the position vector of the particle at time t. *(4 marks)*

(b) Find the acceleration of the particle at time t. *(2 marks)*

(c) Find the magnitude of the acceleration of the particle at time t. Do not simplify your answer. *(2 marks)*

(d) Hence find the time at which the magnitude of the acceleration is a minimum. *(2 marks)*

(e) The particle is moving under the action of a single variable force $\mathbf{F}$ newtons. The mass of the particle is 7 kg.

Find the minimum magnitude of $\mathbf{F}$. *(2 marks)*

5 A golf ball, of mass m kg, is moving in a straight line across smooth horizontal ground. At time t seconds, the golf ball has speed $v\,\mathrm{m\,s^{-1}}$. As the golf ball moves, it experiences a resistance force of magnitude $0.2mv^{\frac{1}{2}}$ newtons until it comes to rest. No other horizontal force acts on the golf ball.

Model the golf ball as a particle.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.2v^{\frac{1}{2}} \qquad \textit{(1 mark)}$$

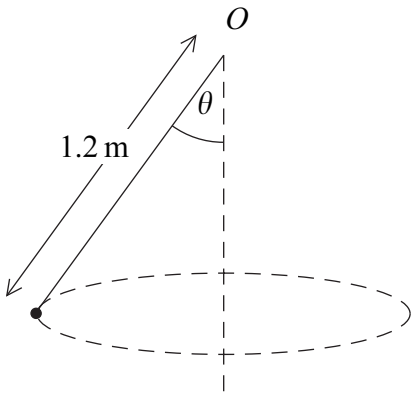
(b) When $t = 0$, the speed of the golf ball is $16\,\mathrm{m\,s^{-1}}$.

Show that $v = (4 - 0.1t)^2$. *(5 marks)*

(c) Find the value of t when $v = 1$. *(3 marks)*

(d) Find the distance travelled by the golf ball as its speed decreases from $16\,\mathrm{m\,s^{-1}}$ to $1\,\mathrm{m\,s^{-1}}$. *(4 marks)*

6 A particle, of mass 4 kg, is attached to one end of a light inextensible string of length 1.2 metres. The other end of the string is attached to a fixed point O. The particle moves in a horizontal circle at a constant speed. The angle between the string and the vertical is θ.



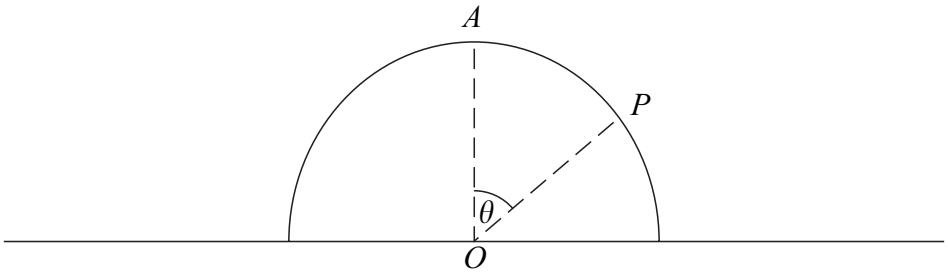
(a) Find the radius of the horizontal circle in terms of θ. *(1 mark)*

(b) The angular speed of the particle is 5 radians per second. Find θ. *(6 marks)*

7 A smooth hemisphere, of radius a and centre O, is fixed with its plane face on a horizontal surface. A particle, of mass m, can move freely on the surface of the hemisphere.

The particle is placed at the point A, the highest point of the hemisphere, and is set in motion along the surface with speed u.

(a) While the particle is in contact with the hemisphere at a point P, OP makes an angle θ with the upward vertical.



Show that the speed of the particle at P is

$$(u^2 + 2ga[1 - \cos\theta])^{\frac{1}{2}}$$ *(5 marks)*

(b) The particle leaves the surface of the hemisphere when $\theta = \alpha$.

Find $\cos\alpha$ in terms of a, u and g. *(5 marks)*

Turn over for the next question

8 A bungee jumper, of mass 49 kg, is attached to one end of a light elastic cord of natural length 22 metres and modulus of elasticity 1078 newtons. The other end of the cord is attached to a horizontal platform, which is at a height of 60 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached, and falls vertically. The bungee jumper can be modelled as a particle. Assume that Hooke's Law applies whilst the cord is taut and that air resistance is negligible throughout the motion.

When the bungee jumper has fallen x metres, his speed is $v\,\mathrm{m\,s^{-1}}$.

(a) By considering energy, show that, when x is greater than 22,

$$5v^2 = 318x - 5x^2 - 2420$$ *(6 marks)*

(b) Explain why x must be greater than 22 for the equation in part (a) to be valid. *(1 mark)*

(c) Find the maximum value of x. *(4 marks)*

(d) (i) Show that the speed of the bungee jumper is a maximum when $x = 31.8$. *(3 marks)*

(ii) Hence find the maximum speed of the bungee jumper. *(2 marks)*

END OF QUESTIONS

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