

ADVANCED GCE

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

4753/01

Candidates answer on the Answer Booklet

OCR Supplied Materials:

- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:

None

Wednesday 20 January 2010
Afternoon

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 Solve the equation $e^{2x} - 5e^x = 0$. [4]

2 The temperature T in degrees Celsius of water in a glass t minutes after boiling is modelled by the equation $T = 20 + be^{-kt}$, where b and k are constants. Initially the temperature is 100°C , and after 5 minutes the temperature is 60°C .

(i) Find b and k . [4]

(ii) Find at what time the temperature reaches 50°C . [2]

3 (i) Given that $y = \sqrt[3]{1 + 3x^2}$, use the chain rule to find $\frac{dy}{dx}$ in terms of x . [3]

(ii) Given that $y^3 = 1 + 3x^2$, use implicit differentiation to find $\frac{dy}{dx}$ in terms of x and y . Show that this result is equivalent to the result in part (i). [4]

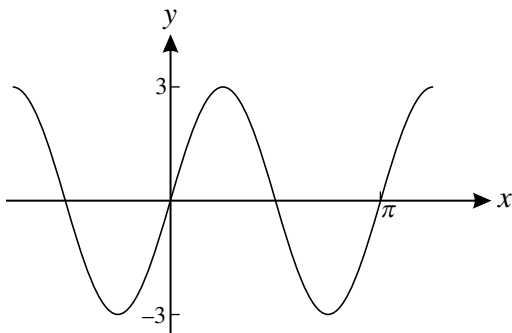
4 Evaluate the following integrals, giving your answers in exact form.

(i) $\int_0^1 \frac{2x}{x^2 + 1} dx$. [3]

(ii) $\int_0^1 \frac{2x}{x + 1} dx$. [5]

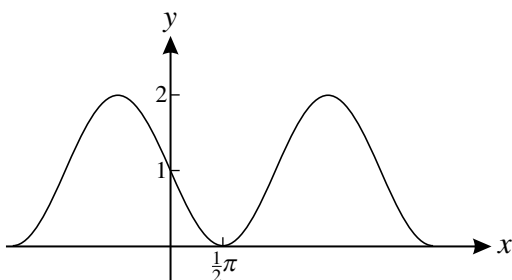
5 The curves in parts (i) and (ii) have equations of the form $y = a + b \sin cx$, where a , b and c are constants. For each curve, find the values of a , b and c .

(i)



[2]

(ii)



[2]

- 6 Write down the conditions for $f(x)$ to be an odd function and for $g(x)$ to be an even function.
Hence prove that, if $f(x)$ is odd and $g(x)$ is even, then the composite function $gf(x)$ is even. [4]
- 7 Given that $\arcsin x = \arccos y$, prove that $x^2 + y^2 = 1$. [Hint: let $\arcsin x = \theta$.] [3]

Section B (36 marks)

- 8 Fig. 8 shows part of the curve $y = x \cos 3x$.
The curve crosses the x -axis at O, P and Q.

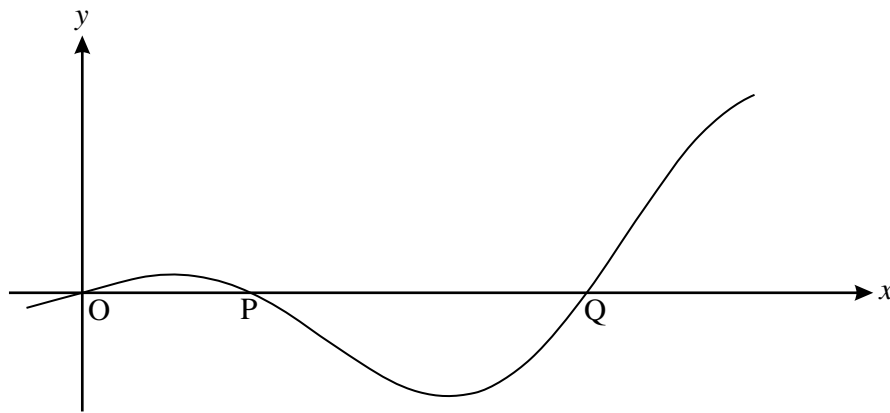


Fig. 8

- (i) Find the exact coordinates of P and Q. [4]
- (ii) Find the exact gradient of the curve at the point P.
Show also that the turning points of the curve occur when $x \tan 3x = \frac{1}{3}$. [7]
- (iii) Find the area of the region enclosed by the curve and the x -axis between O and P, giving your answer in exact form. [6]

[Question 9 is printed overleaf.]

- 9 Fig. 9 shows the curve $y = f(x)$, where $f(x) = \frac{2x^2 - 1}{x^2 + 1}$ for the domain $0 \leq x \leq 2$.

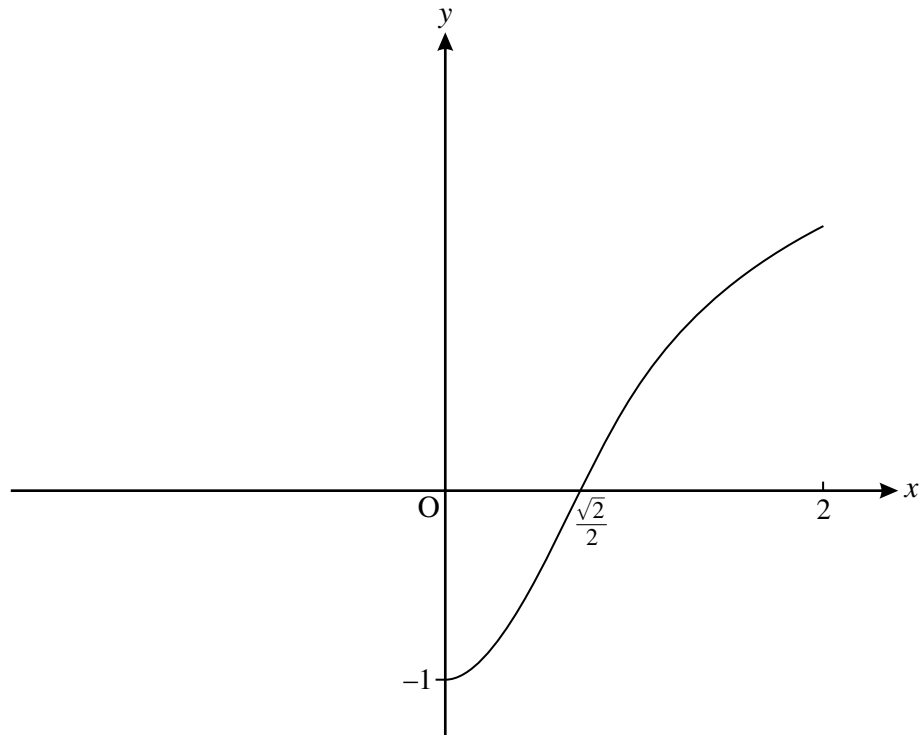


Fig. 9

- (i) Show that $f'(x) = \frac{6x}{(x^2 + 1)^2}$, and hence that $f(x)$ is an increasing function for $x > 0$. [5]
- (ii) Find the range of $f(x)$. [2]
- (iii) Given that $f''(x) = \frac{6 - 18x^2}{(x^2 + 1)^3}$, find the maximum value of $f'(x)$. [4]

The function $g(x)$ is the inverse function of $f(x)$.

- (iv) Write down the domain and range of $g(x)$. Add a sketch of the curve $y = g(x)$ to a copy of Fig. 9. [4]
- (v) Show that $g(x) = \sqrt{\frac{x+1}{2-x}}$. [4]

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