

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

**Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education**

MATHEMATICS

4724

Core Mathematics 4

Monday **12 JUNE 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

- 1 Find the gradient of the curve $4x^2 + 2xy + y^2 = 12$ at the point $(1, 2)$. [4]
- 2 (i) Expand $(1 - 3x)^{-2}$ in ascending powers of x , up to and including the term in x^2 . [3]
- (ii) Find the coefficient of x^2 in the expansion of $\frac{(1 + 2x)^2}{(1 - 3x)^2}$ in ascending powers of x . [4]
- 3 (i) Express $\frac{3 - 2x}{x(3 - x)}$ in partial fractions. [3]
- (ii) Show that $\int_1^2 \frac{3 - 2x}{x(3 - x)} dx = 0$. [4]
- (iii) What does the result of part (ii) indicate about the graph of $y = \frac{3 - 2x}{x(3 - x)}$ between $x = 1$ and $x = 2$? [1]
- 4 The position vectors of three points A , B and C relative to an origin O are given respectively by
- $$\begin{aligned}\overrightarrow{OA} &= 7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}, \\ \overrightarrow{OB} &= 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \\ \text{and } \overrightarrow{OC} &= 5\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}.\end{aligned}$$
- (i) Find the angle between AB and AC . [6]
- (ii) Find the area of triangle ABC . [2]
- 5 A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2 .
- (i) Write down a differential equation which models this situation. [2]
- (ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]
- 6 (i) Show that the substitution $u = e^x + 1$ transforms $\int \frac{e^{2x}}{e^x + 1} dx$ to $\int \frac{u - 1}{u} du$. [3]
- (ii) Hence show that $\int_0^1 \frac{e^{2x}}{e^x + 1} dx = e - 1 - \ln\left(\frac{e + 1}{2}\right)$. [5]

7 Two lines have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + \lambda(3\mathbf{i} + \mathbf{j} + a\mathbf{k}) \quad \text{and} \quad \mathbf{r} = -8\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - \mathbf{k}),$$

where a is a constant.

(i) Given that the lines are skew, find the value that a cannot take. [6]

(ii) Given instead that the lines intersect, find the point of intersection. [2]

8 (i) Show that $\int \cos^2 6x \, dx = \frac{1}{2}x + \frac{1}{24} \sin 12x + c$. [3]

(ii) Hence find the exact value of $\int_0^{\frac{1}{12}\pi} x \cos^2 6x \, dx$. [6]

9 A curve is given parametrically by the equations

$$x = 4 \cos t, \quad y = 3 \sin t,$$

where $0 \leq t \leq \frac{1}{2}\pi$.

(i) Find $\frac{dy}{dx}$ in terms of t . [3]

(ii) Show that the equation of the tangent at the point P , where $t = p$, is

$$3x \cos p + 4y \sin p = 12. \quad [3]$$

(iii) The tangent at P meets the x -axis at R and the y -axis at S . O is the origin. Show that the area of triangle ORS is $\frac{12}{\sin 2p}$. [3]

(iv) Write down the least possible value of the area of triangle ORS , and give the corresponding value of p . [3]

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