

General Certificate of Education June 2010

Mathematics

MS2B

Statistics 2B

Mark Scheme

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Key to mark scheme and abbreviations used in marking

M	mark is for method				
m or dM	mark is dependent on one or more M marks and is for method				
A	mark is dependent on M or m marks and is for accuracy				
В	mark is independent of M or m marks and is for method and accuracy				
Е	mark is for explanation				
√or ft or F	follow through from previous				
	incorrect result	MC	mis-copy		
CAO	correct answer only	MR	mis-read		
CSO	correct solution only	RA	required accuracy		
AWFW	anything which falls within	FW	further work		
AWRT	anything which rounds to	ISW	ignore subsequent work		
ACF	any correct form	FIW	from incorrect work		
AG	answer given	BOD	given benefit of doubt		
SC	special case	WR	work replaced by candidate		
OE	or equivalent	FB	formulae book		
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme		
–x EE	deduct x marks for each error	G	graph		
NMS	no method shown	c	candidate		
PI	possibly implied	sf	significant figure(s)		
SCA	substantially correct approach	dp	decimal place(s)		

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

MS2B		1		
Q	Solution	Marks	Total	Comments
1	$\overline{x} = 82 \; ; \; s^2 = 31.1 \; (s = 5.58)$	B1B1		
	Assumption: The number of customers served daily at the post office counter forms a Normal distribution.	B1		
	$H_0: \mu = 79$ $H_1: \mu > 79$	B1		
	$t = \frac{82 - 79}{5.58 / \sqrt{12}}$	M1		$\frac{\text{their } \overline{x} - 79}{\text{their } s / \sqrt{12}}$
	t = 1.86	A1		(AWRT)
	$v = 11 \implies t_{\text{crit}} = 1.796$	B1		
	Reject H ₀	A1		
	Sufficient evidence at 5% level of significance to support Judith's belief.	E1	9	Iff $t_{\rm calc} > t_{\rm crit}$
2	Total		9	
	$ \begin{array}{ c c c c c }\hline O_i & E_i & (o_i-E_i -0.5) & \alpha^2/E_i\\\hline 24 & 28 & 3.5 & 0.4375\\\hline 56 & 52 & 3.5 & 0.2356\\\hline 11 & 7 & 3.5 & 1.7500\\\hline 9 & 13 & 3.5 & 0.9423\\\hline & & & & & & & \\\hline 3.3654\\\hline \end{array} $ H ₀ : No association between drug and prevention of sickness $ H_1: \text{ Association between drug and prevention of sickness} $ $ \mathcal{X}_{5\%}^2 = 3.841 $ Accept H_0 No evidence at the 5% level of	M1 M1 M1 A1 B1 B1 A1ft		E attempted Yates' correction attempted χ^2 attempted AWFW 3.36 to 3.37 (at least H ₀ stated correctly)
	significance to support the claim that the drug is effective against sickness. Total	E1ft	8	
<u> </u>	Total		U	

MS2B

MS2B Q	Solution	Marks	Total	Comments
3(a)(i)				
	-3k	B2,1	2	Horizontal line $f(x) = k$ From $-3k$ to k If $\frac{1}{2}$ then max. B1
(ii)	Area = $4k \times k = 1$ $k^2 = \frac{1}{4}$	M1		SC If use $k = \frac{1}{2}$ to show that the Area = 1 then \Rightarrow B1
(b)	$k = \frac{1}{2} (k > 0)$	A1	2	AG
(6)	$\mathrm{E}(X) = \frac{1}{2}(-3k+k)$			
	$=-k$ $=-\frac{1}{2}$	B1		CAO
	$\operatorname{Var}(X) = \frac{1}{12}(k - 3k)^{2} = \frac{16k^{2}}{12} = \frac{4k^{2}}{3}$ $= \frac{1}{3}$	M1		CAO
(c)(i)	st. dev $(X) = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ or $\sqrt{\frac{1}{3}}$	A1	3	OE (exact)
	$P\left(X \ge -\frac{1}{4}\right) = \frac{1}{2} \times \frac{3}{4}$	M1		
(ii)	$=\frac{3}{8}$ (0.375)	A 1	2	
(ii)	$P\left(X \neq -\frac{1}{4}\right) = 1$	B1	1	
	Total		10	

MSZB (cont)				
Q	Solution	Marks	Total	Comments
4	$\overline{x} = \frac{0.35}{10} = 0.035$ $s^{2} = \frac{0.12705}{9} = 0.0141 \text{ or } (s = 0.1188)$ $t_{\text{crit}} = 3.250$ $99\% \text{ CI for } \mu:$	B1		both
	$ 0.035 \pm 3.25 \times \frac{\sqrt{0.0141}}{\sqrt{10}} \\ 0.035 \pm 0.1221 $ $(-0.087, 0.157)$	M1 A1	5	$0.035 \pm 3.25 \times \frac{\sqrt{0.012705}}{\sqrt{9}}$ Iff \overline{x} , s and $t_9 = 3.25$ all correct in expression CAO (3dp only)
	Total		5	

MS2B (cont)	Solution	Marks	Total	Comments
5(a)(i)		Maiks	1 Otal	Comments
3(a)(1)	$X \sim Po(7)$ $P(X \le 5) = 0.301$	B1	1	AWFW 0.300 and 0.301
(ii)	$P(X=7) = \frac{e^{-7} \times 7^7}{7!}$	M1		$P(X \le 7) - P(X \le 6)$
	=0.149	A1	2	= 0.5987 – 0.4497 (M1) = 0.149 (A1)
(iii)	$0.65 \le p \le 0.66$	В3		$P(X \le 9) - P(X \le 4)$
	$0.72 \le p \le 0.73$ or $0.52 \le p \le 0.53$	(B2)		$ \begin{cases} P(X \le 10) - P(X \le 4) \\ P(X \le 9) - P(X \le 5) \end{cases} $
	0.60	(B1)	3	$P(X \le 10) - P(X \le 5)$
(b)	No. telephone calls received per hour = $Y \sim P_0 (0.875)$	B1	1	
(c)(i)	Maximum number = 4	B1	1	
	$P(Y < 4) = P(Y = 0,1,2,3)$ $= e^{-0.875} \left(1 + \frac{7}{8} + \frac{49}{128} + \frac{343}{3072} \right)$ $= 0.4169 (1 + 0.875 + 0.3828 + 0.1117)$ $= 0.987740443$	B2		Any correct expression (B2) or AWFW 0.987 to 0.988
	$P(Y \ge 4) = 1 - 0.9877$ = 0.0123	M1 A1	4	1-(their $P(Y < 4)$) AWFW 0.0122 and 0.0123
				SC $P(Y \le 4) = 0.997 \text{ to } 0.998$ or any correct expression B2 P(Y > 4) = 0.002 to 0.003 M1A0
(d)	λ probably not constant The number of calls in any time interval of 1 hour is likely to vary throughout the day.	E1	1	'System Down' ⇒ not independent
	Total		13	

Q Q	Solution	Marks	Total	Comments
6(a)(i)	$P(R \ge 5) = 0.3 + 0.25 + 0.1 + 0.05$			
	= 0.70	B1	1	CAO
(ii)	$E(R) = \sum rp$			
	$=3\times0.1+4\times0.2+5\times0.3+$	M1		
	$6 \times 0.25 + 7 \times 0.1 + 8 \times 0.05$			
	=5.2	A1	2	
(iii)	$E(R^2) = 9 \times 0.1 + 16 \times 0.2$			
	$+25\times0.3+36\times0.25$	M1A1		(Correct expression or 28.7)
	+ 49×0.1+64×0.05	WITAI		(Correct expression of 28.7)
	1 15 × 0.1 + 01 × 0.05			
	(=28.7)			
	$Var(R) = 28.7 - 5.2^2$	M1		
	=1.66	A1	4	AG
(b)(i)	$P(R+S=6) = 0.1 \times 0.15 = 0.015$	B1		$P(R=3 \text{ and } S \le 5) = 0.1 \times 0.85 = 0.085$
	$P(R+S=7) = 0.1 \times 0.4 + 0.2 \times 0.15$			
	=0.04+0.03	B1		$P(R = 4 \text{ and } S \le 4) = 0.2 \times 0.55 = 0.110$
	= 0.07			,
	$P(R+S=8) = 0.2 \times 0.4 + 0.1 \times 0.3$			
	+0.3×0.15			
	= 0.08 + 0.03 + 0.045	B1		$P(R=5 \text{ and } S \le 3) = 0.3 \times 0.15 = 0.045$
	= 0.155			
	- 0.133			
	$P(R+S \le 8) = 0.015 + 0.07 + 0.155$	M1		$P(R+S \le 8) = 0.085 + 0.110 + 0.045$
	,			(K+S 20) = 0.003 + 0.110 + 0.013
	= 0.24	A1	5	=0.24 (AG)
(ii)	$p = {}^{5}C_{4}(0.24)^{4}(0.76)$	M1		First term correct
	$+(0.24)^{5}$	M1 M1		+ correct second term
	=0.0126+0.000796			or correct numerical values
	J			(must use $p = 0.24$)
	= 0.0134	A1	3	(0.013 to 0.0135)
(iii)				Alternative: (using (b)(i))
	$P(R=4 R+S\leq 8)$			$= \frac{P(R = 4 \text{ and } S \le 4)}{P(R + S \le 8)}$
	$= \frac{P(R = 4 \text{ and } R + S \le 8)}{P(R + S \le 8)}$, ,
	$P(R+S\leq 8)$			$=\frac{0.11}{0.24}=\frac{11}{24}$
				0.24 24
	-0.03+0.08	B1		(numerator) or 0.11 seen
	=	M1		(÷ 24 iff numerator < 0.24)
	$=\frac{11}{24} (0.458)$	A1	3	CAO
		AI		
	Total		18	

MS2B (cont) Q	Solution	Marks	Total	Comments
7(a)	Median = 1	B1		
	Lower quartile = $\frac{1}{2}$	B1	2	
(b)	<u> </u>	DI	2	
	$F(1) = \frac{1}{2}$			
	For $1 \le x \le 4$			
	$\int \frac{1}{18} (x-4)^2 \mathrm{d}x$	M1		ignore limits
	$= \left[\frac{1}{54} (x-4)^3 \right]_1^x$ $= \left[\frac{1}{54} (x-4)^3 + \frac{1}{2} \right]$	A1		Correct integration + correct limits seen or used
	$F(x) = \left[\frac{1}{54}(x-4)^3 + \frac{1}{2}\right] + \frac{1}{2}$	m1		adding $\frac{1}{2}$ or $F(1)$
	$=1+\frac{1}{54}(x-4)^3$	A1	4	CAO (AG)
	Alternative			Alternative
	$\int \frac{1}{18} (x-4)^2 dx = \frac{1}{54} (x-4)^3 + c$	(M1)		$\int \frac{1}{18} (x-4)^2 dx $ (M1)
	$\int_{18}^{18} (x^{-4})^{-4} dx = \frac{1}{54} (x^{-4})^{-1} c$	(1111)		$\int \frac{1}{18} (x-4) dx (WII)$
	$F(1) = \frac{1}{2} \implies c = 1$	(m1) (A1)		$= \int_{1}^{x} \frac{1}{18} \left(x^{2} - 8x + 16 \right) dx$
	$F(x) = 1 + \frac{1}{54}(x-4)^3$	(A1)		$= \frac{1}{18} \left[\frac{x^3}{3} - 4x^2 + 16x \right]_1^x $ (A1)
	J 4	, ,		$F(x) = \frac{1}{2} + \frac{1}{54} [x^3 - 12x^2 + 48x]_1^x (m1)$
				$= \frac{1}{2} + \frac{1}{54} \left(x^3 - 12x^2 + 48x - 37 \right)$
				$=1+\frac{1}{54}\left(x^3-12x^2+48x-64\right)$
	50 46			$=1+\frac{1}{54}(x-4)^3 \qquad (A1)$
(c)	$P(2 \le X \le 3) = \frac{53}{54} - \frac{46}{54}$	M1		F(3)-F(2)
	$P(2 \le X \le 3) = \frac{53}{54} - \frac{46}{54}$ $= \frac{7}{54} (0.130)$	A1		0.1296
(d)(i)	$F(q) = \frac{3}{4}$	M1		use of $F(q) = \frac{3}{4}$
	$F(q) = \frac{3}{4}$ $1 + \frac{1}{54}(q - 4)^{3} = \frac{3}{4}$ $\frac{1}{54}(q - 4)^{3} = -\frac{1}{4}$ $(\times 54) \implies (q - 4)^{3} = -13.5$	M1		(either)
	$\frac{1}{54}(q-4)^3 = -\frac{1}{4}$			
	$(\times 54) \implies (q-4)^3 = -13.5$	A1	3	AG
(ii)	$q-4=\sqrt[3]{-13.5}=-2.3811 \ q=1.619 \ (3dp)$	B1	1	CAO
	Total		<u>12</u>	
	TOTAL		75	