



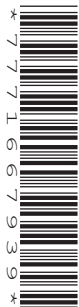
Oxford Cambridge and RSA

**Monday 3 June 2019 – Morning**

**A Level Further Mathematics A**

**Y540/01 Pure Core 1**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

**You may use:**

- a scientific or graphical calculator

**INSTRUCTIONS**

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g\text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

**INFORMATION**

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

**1 In this question you must show detailed reasoning.**

The quadratic equation  $x^2 - 2x + 5 = 0$  has roots  $\alpha$  and  $\beta$ .

(a) Write down the values of  $\alpha + \beta$  and  $\alpha\beta$ . [1]

(b) Hence find a quadratic equation with roots  $\alpha + \frac{1}{\beta}$  and  $\beta + \frac{1}{\alpha}$ . [3]

**2 Indicate by shading on an Argand diagram the region**

$\{z : |z| \leq |z - 4|\} \cap \{z : |z - 3 - 2i| \leq 2\}$ . [3]

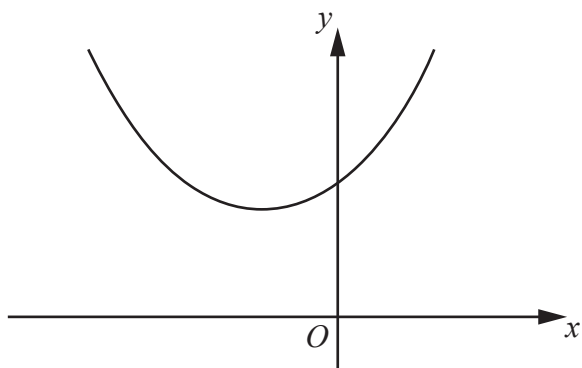
**3 In this question you must show detailed reasoning.**

You are given that  $x = 2 + 5i$  is a root of the equation  $x^3 - 2x^2 + 21x + 58 = 0$ .

Solve the equation. [4]

**4** Using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , show that  $\sum_{r=1}^{10} r(3r - 2) = 1045$ . [3]

- 5 The diagram shows part of the curve  $y = 5 \cosh x + 3 \sinh x$ .



- (a) Solve the equation  $5 \cosh x + 3 \sinh x = 4$  giving your solution in exact form. [4]
- (b) **In this question you must show detailed reasoning.**

Find  $\int_{-1}^1 (5 \cosh x + 3 \sinh x) dx$  giving your answer in the form  $ae + \frac{b}{e}$  where  $a$  and  $b$  are integers to be determined. [3]

- 6 You are given that  $y = \tan^{-1} \sqrt{2x}$ .

(a) Find  $\frac{dy}{dx}$ . [2]

(b) Show that  $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{\sqrt{x}}{(x+2x^2)} dx = k\pi$  where  $k$  is a number to be determined in exact form. [4]

- 7 The function  $\operatorname{sech} x$  is defined by  $\operatorname{sech} x = \frac{1}{\cosh x}$ .

(a) Show that  $\operatorname{sech} x = \frac{2e^x}{e^{2x} + 1}$ . [2]

(b) Using a suitable substitution, find  $\int \operatorname{sech} x dx$ . [4]

- 8 The equation of a plane is  $4x + 2y + z = 7$ .  
The point  $A$  has coordinates  $(9, 6, 1)$  and the point  $B$  is the reflection of  $A$  in the plane.

Find the coordinates of the point  $B$ .

[6]

9 **In this question you must show detailed reasoning.**

You are given the complex number  $\omega = \cos\frac{2}{5}\pi + i\sin\frac{2}{5}\pi$  and the equation  $z^5 = 1$ .

(a) Show that  $\omega$  is a root of the equation. [2]

(b) Write down the other four roots of the equation. [1]

(c) Show that  $\omega + \omega^2 + \omega^3 + \omega^4 = -1$ . [2]

(d) Hence show that  $\left(\omega + \frac{1}{\omega}\right)^2 + \left(\omega + \frac{1}{\omega}\right) - 1 = 0$ . [3]

(e) Hence determine the value of  $\cos\frac{2}{5}\pi$  in the form  $a + b\sqrt{c}$  where  $a$ ,  $b$  and  $c$  are rational numbers to be found. [4]

10 You are given the matrix  $\mathbf{A}$  where  $\mathbf{A} = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$ .

(a) Find, in terms of  $a$ , the determinant of  $\mathbf{A}$ , simplifying your answer. [2]

(b) Hence find the values of  $a$  for which  $\mathbf{A}$  is singular. [2]

You are given the following equations which are to be solved simultaneously.

$$ax + 2y = 6$$

$$ay + 2z = 8$$

$$4x + 5y + z = 16$$

(c) For each of the values of  $a$  found in part (b) determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution.

[7]

- 11 A particle is suspended in a resistive medium from one end of a light spring. The other end of the spring is attached to a point which is made to oscillate in a vertical line.

The displacement of the particle may be modelled by the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 10 \sin t$$

where  $x$  is the displacement of the particle below the equilibrium position at time  $t$ .

When  $t = 0$  the particle is stationary and its displacement is 2.

- (a) Find the particular solution of the differential equation. [11]
- (b) Write down an approximate equation for the displacement when  $t$  is large. [2]

**END OF QUESTION PAPER**

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