# GCE

## Further Mathematics B (MEI)

**Y421/01:** Mechanics major

Advanced GCE

## Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

## Text Instructions

### Annotations and abbreviations

| Annotation in scoris | Meaning |
|---|---|
| ✓and ✗ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |
| | |
| **Other abbreviations in mark scheme** | **Meaning** |
| E1 | Mark for explaining a result or establishing a given result |
| dep* | Mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |
| AG | Answer given |
| awrt | Anything which rounds to |
| BC | By Calculator |
| DR | This indicates that the instruction **In this question you must show detailed reasoning** appears in the question. |

## Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

**M**
A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**
Mark for a correct result or statement independent of Method marks.

**E**
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 3 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 1 | (a) | | $-4+1+k=0$ | M1 | 1.1a | Consideration of **i**-component (allow sign errors) | |
| | | | $k=3$ | A1 | 1.1 | Sight of 3 scores both marks | $k=-3$ with no working scores no marks |
| | | | | **[2]** | | | |
| 1 | (b) | | e.g. taking moments about O: 4(k) – 3(2) | M1 | 1.1a | Taking moments correctly about any point with correct number of terms | |
| | | | 6 | A1 | 1.1 | | |
| | | | clockwise | B1 | 2.5 | oe (could be seen on a diagram) | |
| | | | | **[3]** | | | |
| 2 | | | $[\rho]=\mathrm{ML}^{-3}$ | B1 | 1.2 | | |
| | | | $[\mathrm{u}]=\mathrm{LT}^{-1}$ | B1 | 1.2 | | |
| | | | $[\mu]=[\rho \mathrm{ul}]=(\mathrm{ML}^{-3})(\mathrm{LT}^{-1})\mathrm{L}$ | M1 | 1.1 | Rearrange correctly to make $\mu$ the subject and use the fact that R is dimensionless and substitute their $[\rho]$ and $[\mathrm{u}]$ | |
| | | | $[\mu]=\mathrm{ML}^{-1}\mathrm{T}^{-1}$ | A1 | 1.1 | | Condone lower case |
| | | | | **[4]** | | | |
| | | | | | | **SC**: B2 for $\mathrm{kg\,m^{-1}s^{-1}}$ | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 3 | (a) | | $-8\mathbf{i}+10\mathbf{j}=2(\mathbf{v}-(3\mathbf{i}-2\mathbf{j}))$ | M1* | 3.3 | Use of impulse = change of momentum – must be using m = 2 | Must be using subtraction |
| | | | $\mathbf{v}=-\mathbf{i}+3\mathbf{j}$ | A1 | 1.1 | oe (e.g. as a column vector) | |
| | | | $(\lvert\mathbf{v}\rvert=)\sqrt{(-1)^2+3^2}$ | M1dep* | 1.1 | Correct method for either $\lvert\mathbf{v}\rvert$ or $\lvert\mathbf{v}\rvert^2$ from their $\lvert\mathbf{v}\rvert$ | |
| | | | $\lvert\mathbf{v}\rvert=\sqrt{10}$ | A1 | 1.1 | oe (awrt 3 sf) | 3.162277… |
| | | | | [4] | | | |
| 3 | (b) | | The ball is modelled as a particle | B1 | 3.5b | The impact of the ball and bat is instantaneous | |
| | | | | [1] | | | |
| 4 | (a) | | | M1 | 2.1 | Table of values idea – correct number of terms, dimensionally consistent | Allow one a slip – be on the look out for moments about F and C |
| | | | $(2.5\mathrm{a})(20\mathrm{a}^2)+\left(5\mathrm{a}+\frac{1}{3}\mathrm{a}\right)\left(\frac{1}{2}(4\mathrm{a})(\mathrm{a})\right)=\ldots$ | A1 | 1.1 | | |
| | | | $\bar{x}\left(20\mathrm{a}^2+\frac{1}{2}(4\mathrm{a})(\mathrm{a})\right)$ | A1 | 1.1 | | |
| | | | $\bar{x}=\frac{91}{33}\mathrm{a}$ | A1 | 2.2a | Must be exact and must come from exact working | For reference only: 2.757575… |
| | | | | [4] | | | |
| 4 | (b) | | $\tan\theta=\frac{2\mathrm{a}}{5\mathrm{a}-\bar{x}}$ | M1 | 1.1 | Tan of a relevant angle; allow reciprocal | M0 for $\tan\theta=\frac{2\mathrm{a}}{\bar{x}}$ |
| | | | $\theta=41.7^\circ$ | A1 | 1.1 | oe (radians: 0.728317…) – answer correct to at least 3 sf | 41.729512… |
| | | | | [2] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 5 | (a) | | $\dot{\mathbf{r}}(t) = 3\mathbf{i} - 6e^{-3t}\mathbf{j}$ | M1* | 3.1b | Attempt at differentiation – must be of the form $3\mathbf{i} + ke^{-3t}\mathbf{j}$ where $k \neq 0$ | |
| | | | $\dot{\mathbf{r}}(0) = 3\mathbf{i} - 6\mathbf{j}$ | M1dep* | 3.4 | Substitute t = 0 into their $\dot{\mathbf{r}}(t)$ | |
| | | | $\text{KE} = \frac{1}{2}(4)(3^2 + 6^2)$ | M1dep* | 3.3 | Correct method for finding KE with their $\dot{\mathbf{r}}(0)$ | Dependent on first two M marks |
| | | | 90 (J) | A1<br>[4] | 1.1 | | |
| 5 | (b) | | $18e^{-3t} = 2$ | M1* | 3.4 | Differentiating their $\dot{\mathbf{r}}(t)$ and equating to 2 | Their acceleration must be of the form $k_1e^{-3t}\mathbf{j}$ |
| | | | $e^{-3t} = \frac{1}{9}$<br>$-3t = \ln\left(\frac{1}{9}\right) \Rightarrow t = \ldots$ | M1dep* | 1.1 | Correct method (i.e. taking logs correctly) to find t | |
| | | | $t = 0.732$ | A1<br>[3] | 1.1 | oe e.g. $\frac{1}{3}\ln 9$ - if not given exact then must be given to at least 2 sf | 0.7324081… |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 6 | | | $mga = \frac{1}{2}mv^2$ | M1* | 3.3 | Equating KE gained for P with the PE lost | v is the speed of P before impact |
| | | | $v^2 = 2ga$ or $v = \sqrt{2ga}$ | A1 | 1.1 | $v$ or $v^2$ of P before impact | |
| | | | $\frac{1}{2}mv_Q + mv_P = m\sqrt{2ga}$ | M1* | 3.3 | Attempt at conservation of momentum – correct number of terms – must be using their speed of P before impact in terms of g and a | Correct masses for P and Q – note that $v_Q$ may already have been substituted |
| | | | $v_Q - v_P = -e\left(0 - \sqrt{2ga}\right)$ | M1* | 3.3 | Attempt at Newton's experimental law – correct number of terms and consistent with CLM – must be using their speed of P before impact in terms of g and a | This mark may be earned later after $v_Q$ and $v_P$ have been found |
| | | | $v_Q = \sqrt{2ga}$ or $\frac{1}{2}\left(\frac{1}{2}m\right){v_Q}^2 = \frac{1}{2}mga$ | B1 | 3.3 | www where $v_Q$ is the speed of Q after impact – for reference $v_Q = \frac{2}{3}\sqrt{2ga}(1+e)$ | Condone $\frac{1}{2}m{v_Q}^2 = mga$ |
| | | | $\left(v_P = \frac{1}{2}\sqrt{2ga}\text{ or } v_P = \frac{\sqrt{2ga}}{3}(2-e)\right)$ | | | Where first $v_P$ expression comes from $\frac{1}{2}mv_Q + mv_P = m\sqrt{2ga}$ and $v_Q = \sqrt{2ga}$ | No marks for these |
| | | | $\sqrt{2ga} - \frac{1}{2}\sqrt{2ga} = e\left(\sqrt{2ga}\right)$ or $\frac{1}{2}\left(\frac{1}{2}m\right)\left(\frac{2}{3}\sqrt{2ga}(1+e)\right)^2 = \frac{1}{2}mga$ | M1dep* | 2.1 | Setting up an equation involving g, a and e – dependent on all previous M marks only and must have found $v_P$ and/or $v_Q$ from a correct method | Either substituting into NEL or considering energy of Q after collision |
| | | | $e = \frac{1}{2}$ | A1 | 2.2a | | |
| | | | | [7] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| | | | **NOTE THAT THERE ARE TWO PAGES FOR THIS QUESTION – PLEASE ENSURE BOTH ARE CHECKED** | | | | |
| **7** | | | **DR** | | | | |
| | | | $V=\pi\int_0^{\ln 2}\left(e^{\frac{3}{2}y}\right)^2 dy$ | **M1*** | **1.1** | For $\int\left(e^{\frac{3}{2}y}\right)^2 dy$ or $\int\left(e^{\frac{2}{3}y}\right)^2 dy$ | |
| | | | $=\pi\left[\frac{1}{3}e^{3y}\right]_0^{\ln 2}$ | **A1** | **1.1** | For $\frac{1}{3}e^{3y}$ | Limits not required for M and first A mark |
| | | | $=\frac{1}{3}\pi\left(e^{3\ln 2}-e^0\right)=\frac{7}{3}\pi$ | **A1** | **1.1** | For correct substitution of limits and removing of logs | Must see one line of working from integrated expression to answer |
| | | | $V\bar{y}=\pi\int_0^{\ln 2} y\left(e^{\frac{3y}{2}}\right)^2 dy$ | **M1*** | **2.1** | For $\int yx^2 dy$ leading to $=\pm\alpha ye^{3y}\pm\beta\int e^{3y}dy$ or consistent with their V if using $\int\left(e^{\frac{2}{3}y}\right)^2 dy$ | Clear indication of integrating exponential term and differentiating *y* term |
| | | | $=\pi\left\{\left[\frac{1}{3}ye^{3y}\right]_0^{\ln 2}-\frac{1}{3}\int_0^{\ln 2}e^{3y}dy\right\}$ | | | | |
| | | | $=\pi\left[\frac{1}{3}ye^{3y}-\frac{1}{9}e^{3y}\right]_0^{\ln 2}$ | **A2** | **1.1**<br>**1.1** | Both terms integrated correctly (A1 for one error) | Limits not required for M mark and both A marks |
| | | | $\bar{y}=\frac{\frac{8}{3}\ln 2-\frac{7}{9}}{\frac{7}{3}}$ | **M1dep*** | **1.1** | M1 for $\bar{y}=\frac{V\bar{y}}{V}$ - dependent on both previous M marks | Must have used correct limits correctly |
| | | | $\bar{y}=\frac{8}{7}\ln 2-\frac{1}{3}$ | **A1** | **2.2a** | oe | Allow absence of $\pi$ throughout |
| | | | | **[8]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 8 | (a) | | Driving force = $\frac{25000}{v}$ or $\frac{25000}{7}$ | B1 | 1.2 | | |
| | | | $800g\sin 5$ | B1 | 1.1 | Not for $mg\sin 5$ - award when value for m substituted | Or implied by their answer |
| | | | $\frac{25000}{v} - 750 - 800g\sin 5 = 800a$ | M1 | 3.3 | N2L with either 3 or 4 terms – allow for the M mark $D - 750 - 800g\sin 5 = 800a$ or $D - 750 - 800g\sin 5 = 0$ | Condone sign errors and sin/cos confusion – lhs must be a weight component and the rhs must be mass only |
| | | | $v = 7 \Rightarrow a = 2.67 \text{ m s}^{-2}$ | A1 | 1.1 | Answer to least 3 sf | 2.67265943… |
| | | | $a = 0 \Rightarrow v = 17.4 \text{ m s}^{-1}$ | A1 | 1.1 | Answer to least 3 sf | 17.442253… |
| | | | | [5] | | | |
| 8 | (b) | | WD by car = 25000(10.4) | B1 | 1.1 | 260000 | |
| | | | WD against reistance = 750(131) | B1 | 1.1 | 98250 | |
| | | | Change in PE = $800g(131\sin 5)$ | B1 | 1.1 | 89512.4340… | Not for $mg(131\sin 5)$ - however, can be awarded if m implied or substituted later |
| | | | Change in KE = $\pm\frac{1}{2}(800)(v^2 - 7^2)$ | B1 | 1.1 | Use of correct formula for KE | As above - must use value for m at some pt. |
| | | | $\frac{1}{2}(800)(v^2 - 7^2) + 800g(131\sin 5)$ $= (25000)(10.4) - 750(131)$ | M1 | 3.3 | Use of work-energy principle, all terms present – all values (condone g) substituted or implied by later working (so not in terms of m but award when m substituted or implied) | Allow sign errors but must be a change in KE (so must imply subtraction of the two KE terms) |
| | | | $v = 15.2 \text{ m s}^{-1}$ | A1 | 2.2a | Answer to least 3 sf | 15.1523567… |
| | | | | [6] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 8 | (b) | | **ALT** | | | Using diff. equations | |
| | | | $x=\int\frac{800v}{\frac{25000}{v}-750-800g\sin 5}dv$ or<br><br>$t=\int\frac{800}{\frac{25000}{v}-750-800g\sin 5}dv$ | | | Award B1 for either of these two integrals (allow numerical equivalents)<br><br>Then the remaining 5 marks are for the correct answer of 15.2 (no interim marks after the first B mark) | So may have attempted to simplified the constant terms |
| 9 | (a) | | Initial PE = $-mgl\cos\alpha$, KE = 0 | **B1** | **1.1** | oe – other reference levels for GPE = 0 (e.g. $mgl(1-\cos\alpha)$ using a reference level of l below 0) | KE = 0 can be implied |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| | | | At angle $\theta$, PE = $-mgl\cos\theta$, KE = $\frac{1}{2}mv^2$ | B1 | 1.1 | Or B1 for KE = $\frac{1}{2}mv^2$ and B1 for PE = $\pm mgl(\cos\theta - \cos\alpha)$<br>Note that $\frac{1}{2}mv^2 = \pm mgl(\cos\theta - \cos\alpha)$ implies both B marks | |
| | | | $\frac{1}{2}ml^2\left(\frac{d\theta}{dt}\right)^2 = mgl(\cos\theta - \cos\alpha)$ | M1 | 3.3 | Conservation of energy and use of $v = l\frac{d\theta}{dt}$ | Must be three terms but allow sign errors and sin/cos confusion |
| | | | $\left(\frac{d\theta}{dt}\right)^2 = \frac{2g}{l}\cos\theta - \frac{2g}{l}\cos\alpha$ | A1<br>[4] | 1.1 | $k_1 = -\frac{2g}{l}\cos\alpha$ | |
| 9 | (b) | | $T - mg\cos\theta = ml\omega^2$ | M1* | 3.3 | Applying N2L radially – correct number of terms – allow r for l | For acceleration allow $\frac{v^2}{l}$ or a |
| | | | $T - mg\cos\theta = ml\left[\frac{2g}{l}(\cos\theta - \cos\alpha)\right]$ | M1dep* | 3.4 | M1 for substituting their $v^2$ or $\omega^2$ consistently – must be using l not r | But M0 if in terms of $k_1$ |
| | | | $T = 3mg\cos\theta - 2mg\cos\alpha$ | A1<br>[3] | 1.1 | $k_2 = -2mg\cos\alpha$ | |
| 9 | (c) | | $\cos\alpha \approx \cos\theta \approx 1$ | M1 | 3.1a | Setting $\cos\alpha$ and $\cos\theta$ equal to 1 in their expression for T | |
| | | | $T \approx mg$ | A1<br>[2] | 2.2b | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 9 | (d) | | $2\left(\frac{d\theta}{dt}\right)\left(\frac{d^2\theta}{dt^2}\right)=\frac{2g}{l}(-\sin\theta)\left(\frac{d\theta}{dt}\right)$ | M1* | 3.4 | Differentiate $\omega^2$ with respect to t $\frac{d\theta}{dt}$ or $\dot{\omega}$ must appear on both sides before cancelling | Or use N2L tangentially (correct number of terms) e.g. $-mg\sin\theta = ml\frac{d^2\theta}{dt^2}$ - must be using $\theta$ |
| | | | $2\frac{d^2\theta}{dt^2}\approx -2\frac{g}{l}\theta$ | M1dep* | 2.3 | Cancel $\dot{\theta}$ terms and use small angle approximation for sin | |
| | | | $\frac{d^2\theta}{dt^2}\approx -\omega^2\theta$ where $\omega^2 = gl^{-1}$ so motion is approximately simple harmonic | A1 | 2.4 | Must state that this is simple harmonic | |
| | | | | [3] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 10 | (a) | | $m\ddot{x} = -\frac{kmg}{x^2} - F$ | M1* | 3.3 | N2L with correct number of terms – accept any form for a – M0 if $\frac{1}{2}m\ddot{x} = -\frac{kmg}{x^2} - F$ seen | Allow $a = \frac{1}{2}\frac{d}{dx}(v^2)$ or for acceleration allow any of $a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$ but not if circular motion implied |
| | | | $m\ddot{x} = -\frac{kmg}{x^2} - \mu mg$ | M1dep* | 3.4 | Use of $F = \mu R$ and substitute in their application of N2L | Allow any correct form for a |
| | | | $v\frac{dv}{dx} + \frac{kg}{x^2} + \mu g = 0 \Rightarrow \frac{1}{2}\frac{d}{dx}(v^2) + \frac{kg}{x^2} + \mu g = 0$<br>$\Rightarrow \frac{d}{dx}(v^2) + \frac{2kg}{x^2} + 2\mu g = 0$ | A1 | 2.2a | Use of $a = v\frac{dv}{dx}$ to get to **AG**<br>Note that $\frac{d}{dx}\left(\frac{1}{2}mv^2\right) = -\frac{kmg}{x^2} - F$ is equivalent to the first M mark (work-energy principle for a variable force) | Must see the $\frac{1}{2}\frac{d}{dx}(v^2)$ term before **AG** |
| | | | | **[3]** | | | |
| 10 | (b) | | When $x = a, v^2 = 2gk\left(\frac{1}{a} - \frac{1}{a}\right) + 2\mu g(a - a) = 0$ therefore v = 0 at x = a | B1 | 3.5a | Must explicitly show that when x = a, v = 0 | Allow $v^2 = 0$ |
| | | | $\frac{d}{dx}\left[2gk\left(\frac{1}{x} - \frac{1}{a}\right) + 2\mu g(a - x)\right] = -\frac{2gk}{x^2} - 2\mu g$ | M1 | 2.1 | Differentiate $v^2$ to get two terms of the form $\pm\frac{\alpha}{x^2} \pm \beta$ | Allow for differentiating v |
| | | | $\frac{d}{dx}(v^2) + \frac{2kg}{x^2} + 2\mu g$<br>$= -\frac{2gk}{x^2} - 2\mu g + \frac{2kg}{x^2} + 2\mu g = 0$ | A1 | 2.2a | Correctly shown – could re-arrange their derivative for $v^2$ to the AG e.g.<br>$\frac{d}{dx}(v^2) = -\frac{2gk}{x^2} - 2\mu g$ therefore<br>$\frac{d}{dx}(v^2) + \frac{2gk}{x^2} + 2\mu g = 0$ | Must be = 0 |
| | | | | **[3]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 10 | (b) | | **ALT** | | | Solving differential equation | |
| | | | $v^2 = -\int \frac{2kg}{x^2} + 2\mu g \, dx \Rightarrow v^2 = \frac{2kg}{x} - 2\mu g x + c$ | M1* | | Separating and integrating to the form $v^2 = \pm\frac{\alpha kg}{x} \pm \beta\mu g x$ | +c not required |
| | | | v = 0 at x = a $\Rightarrow c = 2\mu g a - \frac{2kg}{a}$ | M1dep* | | Using given conditions to find c | |
| | | | $v^2 = \frac{2kg}{x} - 2\mu g x + 2\mu g a - \frac{2kg}{a}$ <br> $\Rightarrow v^2 = 2gk\left(\frac{1}{x} - \frac{1}{a}\right) + 2\mu g(a - x)$ | A1 | | AG – so sufficient working must be shown | |
| 10 | (c) | | Remain at A if $\frac{kmg}{a^2} \le \mu mg$ <br> $\mu \ge \frac{k}{a^2}$ | **M1** <br> **A1** <br> **[2]** | **3.1b** <br> **1.1** | Considering relationship between attractive force and friction (accept any inequality or equals) <br> cao | Allow x for a for the M mark |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 11 | (a) | | | M1* | 3.3 | Attempt at conservation of linear momentum – correct number of terms | Let $\mathbf{u}_A = u_A\mathbf{i} + v_A\mathbf{j}$ and $\mathbf{u}_B = u_B\mathbf{i} + v_B\mathbf{j}$ |
| | | | $0.2u_A + 0.6u_B = 0.2(-4) + 0.6(2)$ | A1 | 1.1 | | |
| | | | | M1* | 3.3 | Attmpt at NLR – correct number of terms and speed of approach must be multiplied by coefficient of restitution | |
| | | | $-4-2=-0.5(u_A - u_B)$ | A1 | 1.1 | oe $4+2=0.5(u_A - u_B)$. Must be consistent with CLM e.g. $0.2u_A - 0.6u_B = 0.2(-4)+0.6(2)$ and $-4-2=-0.5(u_A + u_B)$ would score the first 4 marks | |
| | | | | M1dep* | 2.1 | Solving simultaneous equations and correct **j** components ($\mathbf{u}_A$ and $\mathbf{u}_B$ must be of the form $\alpha\mathbf{i} + \beta\mathbf{j}$) | No components of $u_A$ and $u_B$ for this mark (unless done correctly) |
| | | | $\mathbf{u}_A = 9.5\mathbf{i} + 2\mathbf{j}$ | A1 | 2.2a | | |
| | | | $\mathbf{u}_B = -2.5\mathbf{i} + 3\mathbf{j}$ | A1 | 1.1 | ISW if speeds found after correct velocities | |
| | | | | [7] | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 11 | (b) | | Velocity of B after impact is $-2e\mathbf{i}+3\mathbf{j}$ | B1 | 3.1b | Condone $2e\mathbf{i}+3\mathbf{j}$ (or for dividing their answer by 2 if using v for 2e) | May be seen in an equation for e |
| | | | $\frac{1}{2}(0.6)\left[\left(2^2+3^2\right)-\left((\pm 2e)^2+3^2\right)\right]=1.152$ | M1 | 3.4 | Or for $\frac{1}{2}(0.6)\left[2^2-(\pm 2e)^2\right]=1.152$ or for $\frac{1}{2}(0.6)\left((\pm 2e)^2+3^2\right)=2.748$ or for $\frac{1}{2}(0.6)(\pm 2e)^2=0.048$ - allow slip in one power only – must be using 2e (either seen or implied by dividing by 2) | Must be using speeds after collision (so must be using 2**i** + 3**j** or just 2**i**) and correct mass – if using say v for 2e then this mark can only be earned when dividing by 2 |
| | | | $e=0.2$ | A1<br>[3] | 1.1 | | |
| 11 | (c) | | $\tan\alpha=\frac{2}{3}$ or $\tan\chi=\frac{3}{2}$ | B1 | 1.1 | In degrees: $\alpha=33.6900675...$ | Radians:<br>$\alpha=0.5880026...$<br>$\beta=0.1325515...$ |
| | | | $\tan\beta=\frac{2e}{3}$ or $\tan\delta=\frac{3}{2e}$ | B1ft | 1.1 | With their e - if correct for reference $\beta=7.59464336...$ | |
| | | | Deflected angle = $\tan^{-1}\left(\frac{2}{3}\right)+\tan^{-1}\left(\frac{2e}{3}\right)$ | M1 | 3.1b | or $180-\left(\tan^{-1}\left(\frac{3}{2}\right)+\tan^{-1}\left(\frac{3}{2e}\right)\right)$ | |
| | | | $41.3^\circ$ | A1<br>[4] | 1.1 | 41.284710… (correct to 1 d.p.) | 0.72055413… |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 12 | (a) | | | M1* | 3.3 | Resolving vertically – correct number of terms but allow sin/cos confusion and sign errors | Must be mg for the weight – condone any clear notation (even AP, BP) for the tension in the two strings – no marks if T is used for both strings |
| | | | $T_{AP}\cos\theta = T_{BP}\sin\theta + mg$ | A1 | 1.1 | | |
| | | | | M1* | 3.3 | N2L horizontally – correct number of terms – accept a for acceleration – allow sin/cos confusion and sign errors | |
| | | | $T_{AP}\sin\theta + T_{BP}\cos\theta = mr\omega^2$ | A1 | 1.1 | | |
| | | | $T_{BP}\sin^2\theta + mg\sin\theta + T_{BP}\cos^2\theta = mr\omega^2\sin\theta$ | M1dep* | 1.1 | Correctly eliminating $T_{AP}$ from their two equations | Or correct method for solving simultaneous equations for $T_{BP}$ |
| | | | $T_{BP} = m\left(r\omega^2\cos\theta - g\sin\theta\right)$ | A1<br>[6] | 2.2a | **AG** | Must show sufficient working as **AG** |
| 12 | (b) | | $T_{AP}\cos^2\theta = mr\omega^2\sin\theta - T_{AP}\sin^2\theta + mg\cos\theta$ | M1 | 1.1 | Either substitutes given result into one of their two equation from **(a)** correctly to find $T_{AP}$ | Or re-starts and eliminates $T_{BP}$ using given answer in **(a)** |
| | | | $T_{AP} = m\left(r\omega^2\sin\theta + g\cos\theta\right)$ | A1<br>[2] | 1.1 | Correct answer with no working scores both marks | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 12 | (c) | | $\sin\theta = \frac{3}{5}, \cos\theta = \frac{4}{5}, r = \frac{12}{5}a$ | B1 | 1.1 | Correct r and one of sin or cos correct | |
| | | | $T_{BP} > 0 \Rightarrow \left(\frac{12}{5}a\right)\left(\frac{4}{5}\right)\omega^2 - \frac{3}{5}g > 0$ | M1 | 3.1b | Setting $T_{BP} > 0$ and substitute values | Allow = 0 |
| | | | $48a\omega^2 > 15g \Rightarrow 16a\omega^2 > 5g$ | A1 | 2.2a | **AG** | Must show sufficient working as **AG** |
| | | | | **[3]** | | | |
| 12 | (d) | | $m\left(r\omega^2\cos\theta - g\sin\theta\right) = m\left(r\omega^2\sin\theta + g\cos\theta\right)$ | M1* | 3.1b | Setting $T_{AP} = T_{BP}$ | |
| | | | $\frac{12}{5}a\Omega^2\left(\frac{4}{5}\right) - \frac{3}{5}g = \frac{12}{5}a\Omega^2\left(\frac{3}{5}\right) + \frac{4}{5}g$ | M1dep* | 3.4 | Substituting values and obtain an equation in terms of a, g and $\Omega$ | Condone use of $\omega$ for the angular speed |
| | | | $\Omega^2 = \frac{35g}{12a}$ or $\Omega = \sqrt{\frac{35g}{12a}}$ | A1 | 1.1 | Could be implied in their working | |
| | | | Period = $\frac{2\pi}{\Omega}$ | M1dep* | 1.2 | Dependent on both previous M marks | |
| | | | $= 2\pi\sqrt{\frac{12a}{35g}}$ | A1 | 1.1 | oe | |
| | | | | **[5]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 13 | (a) | | | **M1*** | **3.3** | Moments about A for AC – correct number of terms – allow sin/cos confusion and sign errors | |
| | | | $4aR_C \sin\theta = aT\cos\theta$ | **A1** | **1.1** | | $R_C$ is the normal contact force at C |
| | | | | **M1*** | **3.3** | Moments about A for AB – correct number of terms – allow sin/cos confusion and sign errors | |
| | | | $aT\cos\theta + 2aW\sin\theta + (4a - x)(4W\sin\theta)$ $= 4aR_B \sin\theta$ | **A1** | **1.1** | | $R_B$ is the normal contact force at B |
| | | | $R_B + R_C = 4W + W$ | **B1** | **3.3** | Resolving vertically | |
| | | | $\dfrac{aT\cos\theta + 18aW\sin\theta - 4Wx\sin\theta}{4a\sin\theta} + \dfrac{T\cos\theta}{4\sin\theta} = 5W$ | **M1dep*** | **2.1** | Correct method to eliminate $R_B$ and $R_C$ to obtain a linear equation in T | Dependent on all previous M marks and B mark |
| | | | $2aT\cos\theta = 2aW\sin\theta + 4Wx\sin\theta$ | | | | |
| | | | $\Rightarrow T = W\left(1 + \dfrac{2x}{a}\right)\tan\theta$ | **A1** | **2.2a** | **AG** | Sufficient working must be shown as **AG** |
| | | | | **[7]** | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| 13 | (b) | | $DE^2 = a^2 + a^2 - 2(a)(a)\cos 2\theta$ | B1 | 3.1a | Correct application of cosine rule for triangle DAE | B1 for $DE = 2a\sin\theta$ or $\frac{1}{2}DE = a\sin\theta$ |
| | | | $T = \frac{W}{0.25a}(DE - 0.25a)$ or $T = \frac{W}{0.125a}(0.5DE - 0.125a)$ | M1 | 3.3 | Correct application of Hooke's law for their extension | Condone x or e for extension |
| | | | $W\left(1+\frac{2x}{a}\right)\tan\theta = \frac{4W}{a}\left(\sqrt{2a^2 - 2a^2\cos 2\theta} - \frac{1}{4}a\right)$ | A1 | 1.1 | Or A2 (rather than the next M) for $W\left(1+\frac{2x}{a}\right)\tan\theta = \frac{4W}{a}\left(2a\sin\theta - \frac{1}{4}a\right)$ or $W\left(1+\frac{2x}{a}\right)\tan\theta = \frac{8W}{a}\left(a\sin\theta - \frac{1}{8}a\right)$ | |
| | | | $\left(1+\frac{2x}{a}\right)\tan\theta = \frac{4}{a}\left(2a\sin\theta - \frac{1}{4}a\right)$ | M1 | 3.1a | Correct use of double-angle formula to give an equation in terms of x, a and $\theta$ | |
| | | | $x = \frac{1}{2}a(8\cos\theta - \cot\theta - 1)$ | A1<br>**[5]** | 2.2a | **AG** | **AG** so sufficient working must be shown |
| 13 | (c) | | **DR** | | | | |

| Question | | | Answer | Marks | AOs | Guidance | |
|---|---|---|---|---|---|---|---|
| | | | $\frac{dx}{d\theta}=\frac{a}{2}\left(-8\sin\theta+\operatorname{cosec}^2\theta\right)=0$ | M1* | 1.1 | Attempt differentiation and set equal to zero – must be of the form $\pm 8\sin\theta \pm \operatorname{cosec}^2\theta$ with no extra terms | |
| | | | $\sin^3\theta=\frac{1}{8}\Rightarrow\theta=\frac{\pi}{6}$ | A1 | 1.1 | www | Allow in degrees throughout (30) |
| | | | $x=\frac{1}{2}a\left(8\cos\left(\frac{\pi}{6}\right)-\cot\left(\frac{\pi}{6}\right)-1\right)$ | M1dep* | 3.4 | Substitute their value of $\theta$ and obtain a value for x | |
| | | | $x=\frac{3\sqrt{3}-1}{2}a$ | A1 | 1.1 | 2.10a (allow 2.1a) | For reference: 2.098076…a |
| | | | $\left.\frac{d^2x}{d\theta^2}\right\|_{\theta=\frac{\pi}{6}}=\frac{a}{2}\left(-8\cos\left(\frac{\pi}{6}\right)-2\left(\operatorname{cosec}^2\left(\frac{\pi}{6}\right)\cot\left(\frac{\pi}{6}\right)\right)\right)$ <br> $=-6a\sqrt{3}<0$ therefore maximum | A1 <br> **[5]** | 3.2a | Correctly showing that value of x is a maximum (condone in terms of degrees) <br> $=-10.39...a$ | Or equivalent justification that this value is a maximum |

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