OCR
Oxford Cambridge and RSA

GCE

Mathematics (MEI)

Unit **4751**: Introduction to Advanced Mathematics (C1)

Advanced Subsidiary GCE

Mark Scheme for June 2016

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

1. These are the annotations, (including abbreviations), including those used in scoris, which are used when marking

Annotation in scoris	**Meaning**
BP	Blank Page – this annotation **must** be used on all blank pages within an answer booklet (structured or unstructured) and on each page of an additional object where there is no candidate response.
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

2. Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		1	1 **[1]**		
1	(ii)		27	2 **[2]**	condone ± 27; **B1** for $[\pm]3^3$ or $\sqrt{729}$ or for $\left[9^{\frac{1}{2}} = \right]3$ or ± 3 soi	
1	(iii)		$\frac{25}{16}$ or $1\frac{9}{16}$ isw	2 **[2]**	**B1** for $\frac{5}{4}$ or $\frac{1}{\frac{16}{25}}$ or $\frac{16}{25}$ oe	B0 for 1.5625 without fractions seen; if this is found, check for possible use of calculator throughout the paper
2			substitution to eliminate one variable simplification to $ax = b$ or $ax - b = 0$ form, or equivalent for y (9/7, 22/7) oe or $x = 9/7$ $y = 22/7$ oe isw	M1 M1 A2 **[4]**	or multiplication or division to make one pair of coefficients the same; condone one error in either method or appropriate subtraction / addition; condone one further error in either method **A1** each	 independent of first M1 A0 for just rounded decimals or for $-9/-7$ oe
3	(i)		$x < -11/2$ oe www as final answer	2 **[2]**	**M1** for $-2x > 11$ oe or $x < 11/-2$	if working with equals throughout, give 2 for correct final answer, 0 otherwise

Question			Answer	Marks	Guidance	
3	(ii)		$250c^{10}d^{-2}$ or $\frac{250c^{10}}{d^2}$ as final answer	2 [2]	**B1** for two correct elements; must be multiplied if B0, allow **SC1** for $125c^6d^3$ obtained from numerator or for all elements correct but added	
4			$a(2c - 5) = 3c + 2a$ or $2ac - 5a = 3c + 2a$	M1	for multiplying up correctly (may also expand brackets)	annotate this question if partially correct
			$a(2c - 5) - 2a = 3c$ or $2ac - 7a = 3c$ or ft	M1	for collecting a terms on one side, remaining term[s] on other [need not be simplified]	ft only if two or more a terms
			$a(2c - 7) = 3c$ or ft	M1	for factorising a terms, need not be simplified; may be implied by final answer	ft only if two or more a terms, needing factorising may be earned before 2nd M1
			$[a =]\frac{3c}{2c-7}$ or simplified equivalent or ft as final answer	M1 [4]	for division by their two-term factor (accept a 3 term factor that would simplify to 2 terms); for all 4 marks to be earned, work must be fully correct and simplified and not have a triple-or quadruple-decker answer	candidates whose final answer expresses c in terms of a: treat as MR after the first common M and mark equivalently, applying MR−1 if they gain further Ms. So that a final answer, correctly obtained, of $[c =]\frac{7a}{2a-3}$ or simplified equivalent earns 3 marks in total
5	(i)		$11\sqrt{2}$	2 [2]	**M1** for $\left[\sqrt{50} =\right]5\sqrt{2}$ or $\left[3\sqrt{8} =\right]6\sqrt{2}$	

Question			Answer	Marks	Guidance	
5	(ii)		attempting to multiply numerator and denominator of fraction by $4+\sqrt{3}$	M1		
			$2+\sqrt{3}$ or $2+1\sqrt{3}$ or $c = 2$ and $d = 1$	A2	or **B1** for denominator = 13 soi or numerator = $26+13\sqrt{3}$ soi	
			or			
			cross-multiplying by $4-\sqrt{3}$ and forming a pair of simultaneous equations in c and d, with at most one error	M1		
			$c = 2$ and $d = 1$	A2	**A1** for one correct	
				[3]		

Question			Answer	Marks	Guidance	
6			$1 - 20x + 150x^2 - 500x^3 + 625x^4$ as final answer	4 [4]	part marks can be awarded for earlier stages if final answer incorrect or not fully simplified: **M3** for 4 terms correct or for all coefficients correct except for sign errors or for correct answer seen then further 'simplified' or for all terms correct eg seen in table but not combined (condone eg $+(-20x)$ or $+-20x$ instead of $-20x$) **M2** for 3 terms correct or for correct expansion seen without correct evaluation of coefficients [if brackets missing in elements such as $(-5x)^2$ there must be evidence from calculation that $25x^2$ has been used] binomial coefficients such as ${}^4C_2$ are not sufficient – must show understanding of these symbols by at least partial evaluation; or **M1** for 1 4 6 4 1 soi, eg in Pascal's triangle or in expansion where powers of 5 have been ignored	for binomial coefficients, ${}^4C_2$ or factorial notation is not sufficient but accept $\frac{4\times3\times2\times1}{2\times1\times2\times1}$ oe etc any who multiply out instead of using binomial coeffts: look at their final answer and mark as per main scheme if 3 or more terms are correct, otherwise M0

Question			Answer	Marks	Guidance	
7	(i)		[x =] 5, [x =] –1 www	2 **[2]**	**M1** for $x - 2 = \pm 3$ or for $(x - 5)(x + 1)$ [=0]	0 for just $x = 5$ or for $x - 2 = 3$
7	(ii)		parabola shape curve the correct way up	1	must extend beyond x-axis;	condone 'U' shape or very slight curving back in/out; condone some doubling / feathering – deleted work sometimes still shows up in rm assessor; must not be ruled; condone fairly straight with clear attempt at curve at minimum; be reasonably generous on attempt at symmetry e.g. condone minimum on y-axis for this mark
			intersecting x-axis at 5 and –1 or ft from (i) and y-axis at –5	1		
			turning point (2, –9)	1 **[3]**	seen on graph or identified as tp elsewhere in this part	may be implied by 2 and –9 marked on axes 'opposite' turning point
8			$8 + 2a + c = 11$	B1	accept 2^3 instead of 8	
			$-1 - a + c = 8$	B1	or $c - (a + 1) = 8$ oe (often from division) accept $(-1)^3$ instead of –1	
			Correct method for eliminating one variable, condoning one further error	M1	dep on two equations in a and c and at least B1 earned	
			$a = -2$, $c = 7$	A2 **[5]**	**A1** for one correct	
9	(i)		–5.7 to –5.8, –2.2 to –2.3, –1 isw	2 **[2]**	**B1** for 2 correct or for all 3 only stated in coordinate form, ignoring y coordinates	

Question			Answer	Marks	Guidance	
9	(ii)		$1=(x + 2)(x^2 + 7x + 7)$	M1	condone missing brackets if expanded correctly; or **M1** for correct expansion of $(x + 2)(x^2 + 7x + 7)$	
			correct completion with at least one interim stage of working to given answer: $x^3 + 9x^2 + 21x + 13 = 0$	A1		
			[$x = -1$ is root so] $(x + 1)$ is factor soi	M1	implied by division of cubic by $x + 1$	condone some confusion of root/factor for this mark if division of cubic by $x + 1$ seen
			correctly finding other factor as $x^2 + 8x + 13$	M2	**M1** for correct division of cubic by $(x + 1)$ as far as obtaining $x^2 + 8x$ (may be in grid) or for two correct terms of $x^2 + 8x + 13$ obtained by inspection	allow seen in grid without + signs
			$\frac{-8 \pm \sqrt{8^2 - 4 \times 13}}{2}$ oe	M1	for use of formula, condoning one error, for $x^2 + 8x + 13= 0$	or **M1** for $(x+4)^2 = 4^2 - 13$ oe or further stage, condoning one error
			$\frac{-8 \pm \sqrt{12}}{2}$ isw or $-4 \pm \sqrt{3}$ isw and $x = -1$	A1	$x = -1$ may be stated earlier	isw wrong simplification or giving as coordinates
				[7]		

Question			Answer	Marks	Guidance	
9	(iii)	A	drawing the translated quadratic or showing that the horizontal gap between the relevant parts of the curve is always less than 3 estimated coordinates of the point of intersection (1.8 to 2, 0.2 to 0.3)	B1 B1 **[2]**	must be a reasonable translation of given quadratic, only intersecting given curve once; intersections with x axis -3 to -2.5 and 1.5 to 2; ignore above $y = 1$	
9	(iii)	B	$y = x^2 + x - 5$ or $y = \left(x + \frac{1}{2}\right)^2 - \frac{21}{4}$	2 **[2]**	**M1** for [y =] $(x-3)^2 + 7(x-3) + 7$ oe or for simplified equation with 'y =' omitted or for $y = (x-a)(x-b)$ where a and b are the values $3 + \frac{-7 \pm \sqrt{21}}{2}$ oe (may have been wrongly simplified)	M0 for use of estimated roots in (*A*)

Question			Answer	Marks	Guidance	
10	(i)		[Grad AB =] $\frac{7-3}{2-0}$ or $\frac{4}{2}$	M1		
			[Grad BC =] $\frac{-1-3}{8-0}$ or $\frac{-4}{8}$	M1		allow just a simplified version of 2 or –½ for one method mark, but for both to be gained, there must be evidence that the gradients have been obtained independently
			product of gradients = –1 [when lines are at right angles]	A1	or 'negative reciprocal [so perpendicular] oe; may be implied by correct calculation	may be seen earlier, but correct working must support the statement
			or	**or**		
			$AB^2 = 2^2 + 4^2$ [=20] , $BC^2 = 8^2 + 4^2$ [=80] and $AC^2 = 6^2 + 8^2$ [= 100]	M2	or equiv for AB etc; allow at unsimplified stage; or **M1** for just one correct expression for one of the sides	allow just a simplified version of eg $AB^2 = 20$ for one method mark, but for both to be gained, there must be evidence that the lengths or their squares have been obtained independently
			$AB^2 + BC^2 = AC^2$ [so by Pythagoras, angle ABC = 90°] oe	A1	may be implied by correct calculation	may be seen earlier, but correct working must support the statement another possible method: M1 for finding midpt ofAC as (5, 3), M1 for showing dist from midpt to A, B and C is 5 and M1 for using angle in a semicircle to show that ABC = 90°
				[3]		

Question			Answer	Marks	Guidance	
10	(ii)		centre D = $\left(\frac{2+8}{2}, \frac{7+-1}{2}\right)$ or (5, 3) soi	B1	may be implied by circle eqn	if already found in (i), must be used in (ii) to get the mark here
			radius = 5 or $r^2 = 25$ or for finding dist between A, B or C and their centre D oe	B1	may be implied by circle eqn	if already found in (i), must be used in (ii) to get the mark here
			$(x-a)^2 + (y-b)^2 = r^2$ soi	M1	general formula may be quoted or implied by eqn using their values, but it must be clear that they are using their r^2 rather than their r or their d or d^2	for this method mark, allow use of their values, even if obtained from AB or BC as diameter
			$(x-5)^2 + (y-3)^2 = 25$ or 5^2 isw	A1	alternative method: allow B4 for $(y-7)(y+1)+(x-2)(x-8)=0$	
				[4]		
10	(iii)		[grad AD =] $\frac{7-3}{2-5}$ isw or $-\frac{4}{3}$ oe	B1	or may use CD $\left(\frac{-1-3}{8-5}\right)$, AC $\left(\frac{7-(-1)}{2-8}\right)$ or ft their D from (ii) or **B1** for correct differentiation: $2x + 2y\frac{dy}{dx} - 10 - 6\frac{dy}{dx} = 0$ oe	if D wrong, check back to (ii) for any ft NB: A(2, 7) B(0, 3) and C(8, −1)
			grad tgt = ¾ oe www or −1/ their grad AD oe	M1		
			$y - 7 =$ their ¾ $(x - 2)$ or $7 =$ their $\frac{3}{4} \times 2 + c$	M1	**M0** if grad AD used; **M0** for a spurious gradient used	perp gradient to AB or BC used: may earn 2nd M1 only
			$4y = 3x + 22$ oe where a, b, c are integers, isw	A1	allow correct answer to imply 3rd M1, provided first two Ms have been earned	
				[4]		

Question			Answer	Marks	Guidance	
11	(i)		(0, –3)	B1	condone $y = –3$, isw	if not coordinates, must be clear which is x and which is y
			(– ½ , 0) and (3, 0) www	B2 **[3]**	condone $x = – ½$ and 3; **B1** for one correct www or **M1** for $(2x + 1)(x – 3)$ or correct use of formula or reversed coordinates	
11	(ii)		$2x^2 – 6x – 6[= 0]$ isw or $x^2 – 3x – 3$ [= 0] or $2y^2 – 18y + 30$ [=0]	M1	for equating curve and line, and rearrangement to zero, condoning one error	allow rearranging to constant if they go on to attempt completing the square
			use of formula or completing the square, with at most one error	M1	no ft from $2x^2 – 6x = 0$ or other factorisable equations	if completing the square must get to the stage of complete square only on lhs as in 9(ii)
			$\left(\frac{6 \pm \sqrt{84}}{4}, \frac{18 \pm \sqrt{84}}{4}\right)$ or $\left(\frac{3 \pm \sqrt{21}}{2}, \frac{9 \pm \sqrt{21}}{2}\right)$ oe isw	A2 **[4]**	**A1** for one set of coords or for x values correct (or ys from quadratic in y); need not be written as coordinates	A0 for unsimplified y coords eg $\frac{3+\sqrt{21}}{2}+3$

Question			Answer	Marks	Guidance	
11	(iii)		$2x^2 - 5x - 3 = x + k$	M1	for equating curve and line	
			$2x^2 - 6x - 3 - k$ [= 0]	M1	for rearrangement to zero, condoning one error, but must include k ;this second M1 implies the first, eg it may be obtained by subtracting the given equations	
			$b^2 - 4ac < 0$ oe for non-intersecting lines	M1	eg allow for just quoting this condition; may be earned near end with correct inequality sign used there allow 'discriminant is negative' if further work implies $b^2 - 4ac$	some may use condition for intersecting lines or for a tangent and then swap condition at the end; only award this M1 and the final A mark if the work is completely clear
			$36 - 8 \times -(3 + k)$ [< 0] oe	A1	for correct substitution into $b^2 - 4ac$; no ft from wrong equation; if brackets missing or misplaced, must be followed by a correct simplified version	can be earned with equality or wrong inequality, or in formula – this mark is not dependent on the 3rd M mark;
			$k < -\frac{15}{2}$ oe	A1	isw if 3rd M1 not earned, allow **B1** for $-\frac{15}{2}$ obtained for k with any symbol	

Question			Answer	Marks	Guidance	
11	(iii) cont		or, for those using a tangent condition with trials to find the boundary value			mark one mark scheme or another, to the advantage of the candidate, but not a mixture of schemes
			rearrangement with correct boundary value of k eg $2x^2 - 6x + 4.5$ [= 0] or $2x^2 - 6x - (3 - 7.5)$ [= 0]	M2	**M1** for $2x^2 - 5x - 3 = x - 7.5$	M0 for trials with wrong values without further progress, though may still earn an M1 for $b^2 - 4ac < 0$
			showing $36 - 8 \times - (3 - 7.5) = 0$ or $36 - 8 \times 4.5 = 0$ oe	M1	may be in formula implies previous M2	
			$k < -\frac{15}{2}$ oe	A2	**B1** for $-\frac{15}{2}$ obtained for k as final answer with any symbol	
			or, for using tangent with differentiation			
			$y' = 4x - 5$	M1		
			[when $y = x + k$ is tgt] $4x - 5 = 1$	M1		
			$x = 1.5$, $y = -6$	A1		
			$-6 = 1.5 + k$ or $k = -7.5$ oe	A1		
			$k < -7.5$ oe	A1		
				[5]		

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