AQA

A-LEVEL
Mathematics

MS2B - Statistics 2B

Mark scheme

6360

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Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS2B

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question.

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks.

GN3 In general, a correct answer (to accuracy required) without units scores full marks.

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks.

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown.

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34 .

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3}) .

GN8 **Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.**

Q1	Solution	Marks	Total	Comments
(a)(i)	$\int_8^{16} \frac{1}{160}x\,dx = \left[\frac{1}{320}x^2\right]_8^{16}$	M1		Integration completed, ignore limits Not awarded for work seen in part (b)
	$= \frac{16}{20} - \frac{4}{20} = \frac{6}{10}$	A1		PI Any form
	So c is 4 more than 16, c = 20	A1		Completed by any method CAO
	Alternative, by use of area			
	0.5 × (0.05 + 0.1) × 8 + [(c – 16) × 0.1 = 1] 0.6 + [(0.1c – 1.6) = 1] Leading to c = 20	(M1) (A1) (A1)		For first area For first area = 0.6 CAO
			3	
(ii)	19	B1		CAO
			1	
(b)	$\int_8^m \frac{1}{160}x\,dx = 0.5$	M1		Complete including limits and = 0.5 Or use of F(x) derived in part (a)
	$\left[\frac{1}{320}x^2\right]_8^m = \frac{1}{320}[m^2 - 8^2] = 0.5$	A1		Equation correct in any form
	$m^2 – 8^2 = 160$, $m^2 = 224$			
	Median = $\sqrt{224} = 4\sqrt{14} = 14.96…$	A1		Surd or AWFW 14.9 to 15.0 NB not 15 exactly by using f(x) =0.1 backwards from x = 16
			3	
		Total	**7**	

Q2	Solution	Marks	Total	Comments
(a)	$P(X = 10) = 0.001$ **shown by one of the following** by using $(0.5)^{10} = 0.000976\ldots = 0.001$ to 3 dp **Or** Using B(10, 0.5) by $P(X=10) = P(X \leq 10) – P(X \leq 9)$ = 1.000 – 0.999 **Or** Using $P(X = 9) = 10 \times (0.5)^{10}$ or 0.00976 or (0.9990 – 0.9893) and then subtraction from 1 $P(X = 9) = 0.010$	 B1 B1		**AG** $(0.5)^{10}$ or 0.000976… seen Not simply stating $P(X=10) = 0.001$ CAO seen anywhere
			2	
(b)(i)	Mean prize = (50 × 0.044 + 200 × '0.01' + 800 × 0.001) = 5p (4.93p from exact values) **In pence**: $E(X^2) = 50^2 \times 0.044 + 200^2 \times$ '0.01' $+ 800^2 \times 0.001$ (= 1150) Var(X) = "1150" – "5"^{2} = 1125 so SD = $\sqrt{1125}$ = 33.5 **In pounds:** $E(X^2) = 0.5^2 \times 0.044 + 2^2 \times$ '0.01' $+ 8^2 \times 0.001$ (= 0.115) Var(X) = "0.115" – "0.05"^{2} = 0.1125 so SD = $\sqrt{0.1125}$ = £0.335 [= 33.5p] [SC Use of $E(X - \mu)^2$ must include $(-5)^2 \times 0.945 + 45^2 \times 0.044 + 195^2 \times$ '0.01' $+ 795^2 \times 0.001$ OE for M1m1]	 M1 A1 M1 m1 A1 (M1) (m1) (A1)		Or equivalent in £. Their '0.01' but must use (0 and) 50/0.5, 200/2 & 800/8 AWFW 4.9 to 5.0. Allow without working for B2 Their '0.01' but must use 50, 200 & 800 Their $E(X^2)$ and their mean AWFW 33 to 34 AG Their '0.01' but must use 0.5, 2 & 8 Their $E(X^2)$ and mean AWFW £0.33 to £0.34 do not condone omission of £ sign if left as 0.335 AG
			5	
(ii)	Doubling the prizes would make the expected prize 10p ('= charge for the game' or 'doubled') **Or** the standard deviation 67p (or doubled) Because cost = expected prize … … this would be a fair game, **Or …** no point in Rodney running the game.	 B1 E1		AWFW 9.8 to 10 Either AWFW 66 to 68 Do not award E1 if $E(2X) \neq 9.8$ to 10 OE – concept of fairness Either OE – concept of zero profit
			2	
		Total	**9**	

Q3	Solution	Marks	Total	Comments
(a)	H_0: There is no association between main course and dessert chosen.	B1		Allow 'independent'
			1	
(b)	<table><tr><th></th><th>B</th><th>L</th><th>C</th><th>V</th></tr><tr><td>**F S**</td><td></td><td></td><td></td><td></td></tr><tr><td>**I C**</td><td></td><td></td><td>**25.2**</td><td>**23.1**</td></tr><tr><td>**G**</td><td></td><td></td><td>**15.6**</td><td>**14.3**</td></tr></table>	B2, 1		B1 for any one correct B2 for all four correct
			2	
(c)	X^2 value = $\frac{8.8^2}{27.2} + \frac{0^2}{16} + \frac{(-3.2)^2}{19.2} + \frac{(-5.6)^2}{17.6} + \frac{(-2.7)^2}{35.7} + \frac{3^2}{21} +$ $\frac{2.8^2}{25.2} + \frac{(-3.1)^2}{23.1} + \frac{(-6.1)^2}{22.1} + \frac{(-3)^2}{13} + \frac{0.4^2}{15.6} + \frac{8.7^2}{14.3}$	M1		Any one fraction or decimal correct Possibly implied by correct X^2 value
	= 2.84(7)..+ 0 + 0.53..+ 1.78..+ 0.20..+ 0.42(9)..+ 0.31..+0.41(6)..+1.68..+ 0.69..+ 0.01..+ 5.29..	A1		At least 11 (or 10, excluding 0) correct to at least 2 dp. (ie 1 slip allowed) Possibly implied by correct X^2 value.
	= 14.2	A1		AWRT
	3 × 2 = 6 degrees of freedom Critical value = 12.592	B1		AWRT 12.6
	14.2 > 12.6 so reject H_0	A1dep		Dep on TS and CV both correct. Comparison stated or diagram Accept X^2 > cv
	There is (significant evidence of) an association between main course and dessert chosen.	A1dep		Dep on previous A1. Conclusion in context, must refer to association or dependence.
			6	
(d)	More than expected beef eaters chose fruit salad More than expected vegetarians chose gateau Fewer than expected beef eaters chose gateau Fewer than expected vegetarians chose fruit	E1		For any one of these
			1	
		Total	**10**	

Q4	Solution	Marks	Total	Comments
(a)(i)	Using Po(2.7) $e^{-2.7} \times \frac{2.7^2}{2} = 0.245$	M1 A1		Stated or implied AWRT
			2	
(a)(ii)	Use of Po(9) 0.7060 used as P($X \leq 10$) Subtraction of 0.0550 from a top value (0.5874, 0.7060 or 0.8030) 0.7060 – 0.0550 = 0.651	M1 A1 m1 A1		Must see use of 0.0550, 0.1157, 0.2068,0.5874, 0.7060 or 0.8030 to at least 3 sf AWRT 0.706. Stated or implied by final answer AWRT 0.055. Stated or implied by final answer (0.532, 0.651 or 0.748) AWRT
			4	
(b)(i)	0.785	B1		AWRT
			1	
(b)(ii)	0.2019 × (1 – 0.5697) (= 0.0869) (+) 0.0907 × (1 – 0.7834) (= 0.0196) = 0.1065	M1 M1 A1		Allow 3 dp rounding Allow 3 dp rounding AWFW 0.106 to 0.107
			3	
		Total	**10**	

Q5	Solution	Marks	Total	Comments
(a)	(E(Y) = 2E(X) – 5 so) E(X) = 8	B1		CAO
	(Var(Y) = 4Var(X) so) Var(X) = $\frac{3}{4}$	B1		CAO any equivalent form
			2	
(b)(i)	$\frac{(b-a)^2}{12} = \frac{3}{4}$	M1		Application of formula (their ¾)
	$(b-a)^2 = 9$			
	so $(b-a) = (\pm)3$	A1		Ignore –3 at this stage. No FT here.
	$\frac{(b+a)}{2} = 8$ (so $(b+a) = 16$)	M1		Application of formula (their 8) PI by final answer
	$a = 6.5$, $b = 9.5$	A1		CAO For both in any form. If there are extra solutions then A0.
			4	
(ii)	Use of $\frac{9-6.5}{3}$	M1		Or by subtraction 1 – $\frac{0.5}{3}$
	$\frac{5}{6}$ = (0.833)	A1		CAO Any equivalent form
			2	
		Total	**8**	

Q6	Solution	Marks	Total	Comments
(a)(i)	Mid interval value = mean = 1.64	B1		CAO
			1	
(a)(ii)	Use of t_9 value 2.262	B1		AWRT 2.26
	$2.262 \times \frac{s}{\sqrt{10}} = 0.23$	M1		Or 2 × and 0.46
	$s = \sqrt{10} \times 0.23 \div 2.262 = 0.3215\ldots$ (= 0.322)	A1		Arithmetic indicated or 4 sf answer AG Use of 2.26 gives 0.3218…
			3	
(b)	Use of t_9 value 1.833 $1.64 \pm 1.833 \times \frac{0.322}{\sqrt{10}}$ = (1.45, 1.83)	B1 M1 A1		0.322 or 0.3215… or 0.32 AWRT 1.83
			3	
(c)	**1.85** is outside the 90% interval but it/1.85 is inside the 95% interval.	B1F		**Follow through as long as the c.i. in part (b) excludes 1.85** OE Must compare 1.85 with each interval. Any 'it' must be unambiguous. Numerical comparison alone is not sufficient. (Eg. 1.85>1.83, and 1.81 <1.85 <1.87)
	Eg. Cannot decide whether suitable or not Or More samples needed Or Low probability of being suitable for mining	E1dep		Dep on B1F. Must make an **overall** statement which is inconclusive about suitability, even if two individual comments about uncertainty have already been given. Do not accept "suitable" or "not suitable" as a definite conclusion. E0 if there is any suggestion that 95% c.i. is 'more accurate'.
			2	
		Total	**9**	

Q7	Solution	Marks	Total	Comments
(a)(i)	H_0: $\mu = 334$, H_1: $\mu < 334$ Sample mean = 320.8 Use of z value (±) 1.6449 test stat = $\frac{320.8 - 334}{(17 \div \sqrt{5})}$ = –1.736… Reject H_0 (or accept H_1), because –1.736 < –1.6449 **or** 1.736 > 1.6449 **or** diagram **or** ts < cv **or** |ts| > |cv| Significant evidence that the mean collection time has reduced.	B1 B1 B1 M1 A1 A1dep E1dep		Both (here or in **(ii)**) CAO (here or in **(ii)**) ignore notation AWFW 1.64 to 1.65 Ignore sign here AWRT -1.74 Condone inconsistency between diagram and numerical statement Dep on A1 and z value B1 Dep on A1 and z value B1 (but not on A1dep) In context, must refer to 'mean' and 'time' or 334 minutes. Must not be too definite.
			7	
(a)(ii)	(H_0: $\mu = 334$, H_1: $\mu < 334$) (sample mean = 320.8 and) s = 15.89… t_4 = –2.132 test stat = $\frac{320.8 - 334}{(15.90 \div \sqrt{5})}$ = –1.857 No significant evidence that the mean collection time has reduced.	B1 B1 M1 A1 E1dep		AWRT 15.9 AWRT –2.13 Do not ignore sign here. Allow their s. AWFW –1.85 to –1.86 Dep on A1 and t value B1. In context must refer to 'mean' and 'time' or 334 minutes. Accept in terms of mean being unchanged. Must not be too definite.
			5	
(b)	**(a)(ii)** & Type II error.	E1		CAO
			1	
		Total	**13**	

Alternatives for those using critical values of time

a(i)	cv = 334 – 1.6449 × 17 ÷ √5 = 321.5 Comparison 320.8 < 321.5, so reject H_0	M1 A1 A1dep		Condone 1.28 or 1.96 for 1.65 for M1 AWRT Dep on A1 and z value B1
a(ii)	cv = 334 – 2.132 × 15.90 ÷ √5 = 318.8 Comparison 318.8 < 320.8, so accept H_0 etc.	M1 A1 A1dep		Condone 1.53 or 2.77 for 2.13 for M1 AWRT Dep on A1 and t value B1.

Alternatives for those using confidence interval approach

a(i)	ci = 320.8 ± 1.6449 × 17 ÷ √5 = (308.3) to 333.3 Comparison 333.3 < 334, so reject H_0	M1 A1 A1dep		Condone 1.28 or 1.96 for 1.65 for M1 AWRT Dep on A1 and *z* value B1
a(ii)	ci = 320.8 ± 2.132 × 15.90 ÷ √5 = (304.6) to 337.0 Comparison 334< 337.0, so accept H_0 etc.	M1 A1 A1dep		Condone 1.53 or 2.77 for 2.13 for M1 AWRT Dep on A1 and *t* value B1.

Alternatives for those using *p* value approach

Q7	Solution	Marks	Total	Comments
(a)(i)	H_0: $\mu = 334$, H_1: $\mu < 334$ Sample mean = 320.8 test stat = $\frac{320.8 - 334}{(17 \div \sqrt{5})}$ = –1.736… Giving a *p* value of 0.0413 (or 0.0826) **Correct** comparison 0.0413 < 0.05 (or 0.0826 < 0.1) , so reject H_0 Significant evidence that the mean collection time has reduced.	B1 B1 M1 A1 A1 A1dep E1dep		Both (here or in **(ii)**) CAO (here or in **(ii)**) Ignore sign here PI by correct *p* value AWRT -1.74 PI by correct *p* value AWRT Or diagram Dep on A1 and *z* value B1 Dep on A1dep. In context, must refer to 'mean' and 'time' or 334 minutes. Must not be too definite.
			7	
(a)(ii)	(H_0: $\mu = 334$, H_1: $\mu < 334$) (sample mean = 320.8 and) s = 15.89 … test stat = $\frac{320.8 - 334}{(15.90 \div \sqrt{5})}$ = –1.857 Giving a *p* value of 0.0684 (or 0.137) No significant evidence that the (mean) collection time has reduced.	B1 M1 A1 A1 E1dep		AWRT 15.9 AWFW –1.85 to –1.86 AWRT Dep on *p* value A1. In context must refer to 'mean' and 'time' or 334 minutes. Accept in terms of mean being unchanged. Must not be too definite.
			5	
(b)	**(a)(ii)** & Type II error.	E1		CAO
			1	
		Total	**13**	

Q8	Solution	Marks	Total	Comments
(a)	F(2) = $\frac{15}{80}$	B1		This fraction OE seen or implied by correct answer
	P(X > 2) = 1 – F(2) = $\frac{13}{16}$ (= 0.8125)	B1		Any equivalent fraction or 0.812, 0.8125, 0.813.
			2	
(b)	Differentiate to get f(x) = $\frac{1}{20}x^3$	B1		
	E(Y) = $\int_1^3 \left(\frac{1}{x}\right)\left(\frac{1}{20}x^3\right)dx = \int_1^3 \frac{1}{20}x^2 dx$	M1		Allow omission of limits for M1
	= $\left[\frac{x^3}{60}\right]_1^3 = \frac{26}{60}$	A1		Any equivalent form or at least 3sf
	E(Y^2) = $\int_1^3 \left(\frac{1}{x^2}\right)\left(\frac{1}{20}x^3\right)dx = \int_1^3 \frac{1}{20}x\,dx$	M1		Allow omission of limits for M1
	=$\left[\frac{x^2}{40}\right]_1^3 = \frac{1}{5}$	A1		Any equivalent form
	Var(Y) = $\frac{1}{5} - \left(\frac{26}{60}\right)^2 = \frac{180}{900} - \frac{169}{900}$	M1		Their E(Y^2) and E(Y)^2 but at least one must be correct
	= $\frac{11}{900}$	A1		Or exact equivalent or AWRT 0.0122
			7	
		Total	**9**	