AQA

A-LEVEL
MATHEMATICS

Pure Core 4 – MPC4
Mark scheme

6360
June 2014

Version/Stage: Final V1.0

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a)	$\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)=t \qquad \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)=-\frac{4}{t^2}$	B1		ACF - **Both** correct.
	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-\frac{4}{t^2}}{t}$	M1		Attempt at their $\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}$
	At $t=2 \quad \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{1}{2}$	A1	3	CSO
(b)	$t=\frac{4}{y+1}$ **and** $x=\mathrm{f}(y)$	M1		Attempt to isolate t **and** attempt to substitute
	$x=\frac{1}{2}\left(\frac{4}{y+1}\right)^2+1$	A1	2	ACF
	Total		5	
	Alternatives			
(b)	$x-1=\frac{t^2}{2} \qquad (y+1)^2=\left(\frac{4}{t}\right)^2$	M1		Solve for $\frac{t^2}{2}$ and $\left(\frac{4}{t}\right)^2$ and multiply
	$(x-1)(y+1)^2=8$	A1	2	ACF
(b)	$t^2=2x-2 \quad \& \quad y=f(x)$	M1		Attempt to find t^2 in terms of x **and** attempt to substitute.
	$y=\frac{4}{\pm\sqrt{2x-2}}-1$	A1	2	or $(y+1)^2=\frac{16}{2x-2}$ ACF

Q	Solution	Mark	Total	Comment
2(a)	$4x^3 - 2x^2 + 16x - 3 =$ $Ax(2x^2 - x + 2) + B(4x - 1)$	M1		Attempt to multiply by $2x^2 - x + 2$ or long division with $2x$ seen or substitute two values of x
	$A = 2$	A1		A stated or written in expression
	$B = 3$	A1	3	B stated or written in expression
(b)	$\int 2x + \frac{3(4x-1)}{2x^2 - x + 2}\,\mathrm{d}x =$			
	$x^2 +$	B1ft		ACF **ft** on their A
	$3\ln(2x^2 - x + 2) \quad (+C)$	B1ft		**ft** on their B
	$2 = (-1)^2 + 3\ln(2(-1)^2 - (-1) + 2) + C$	M1		Substitute $(-1, 2)$ into an expression of form $y = ax^2 + b\ln(2x^2 - x + 2) + C$ **and** attempt to find the constant
	$y = x^2 + 3\ln(2x^2 - x + 2) + 1 - 3\ln 5$	A1	4	CAO
	Total		**7**	

(a) If **M1** is not awarded then award **SC1** for **either** $A = 2$ (*or* $2x$) **or** $B = 3$.

NMS $A = 2$ **and** $B = 3$ scores **SC3**; as the values of A and B can be found by inspection.

Q	Solution	Mark	Total	Comment
3(a)	$(1-4x)^{\frac{1}{4}} = 1+\frac{1}{4}(-4x)+kx^2$	**M1**		k is any non-zero numerical expression
	$= 1-x-\frac{3}{2}x^2$	**A1**	**2**	Simplified to this form , but allow -1.5
(b)	$(2+3x)^{-3} = 2^{-3}\left(1+\frac{3}{2}x\right)^{-3}$	**B1**		OE e.g. $\frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$
	$\left(1+\frac{3}{2}x\right)^{-3} = 1-3\times\frac{3}{2}x+\frac{-3\times-4}{2}\left(\frac{3}{2}x\right)^2$	**M1**		Condone missing brackets and one sign error
	$(2+3x)^{-3} = \frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2$	**A1**	**3**	or $\frac{1}{8}\left(1-\frac{9}{2}x+\frac{27}{2}x^2\right)$
	Alternative $(2+3x)^{-3} =$ $2^{-3}+(-3)2^{-4}(3x)+\frac{1}{2}(-3)(-4)2^{-5}(3x)^2$	**(M1)**		Condone missing brackets and one sign error.
	$=\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2$	**(A2)**	**(3)**	**A1 not** available
(c)	$\left(1-x-\frac{3}{2}x^2\right)\left(\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2\right)$	**M1**		Product of their expansions
	$= \frac{1}{8}-\frac{11}{16}x+\frac{33}{16}x^2$	**A1**	**2**	
	Total		**7**	

Q	Solution	Mark	Total	Comment
4 (a)	$A = 5000$	B1	1	
(b)(i)	$25000 = 5000p^{10} \Rightarrow p^{10} = 5$	B1	1	First equation seen and correct. **AG**
(ii)	$\ln p^t = t \ln p$	B1		PI
	$\ln\left(\frac{75000}{A}\right) = \ln p^t$	M1		**Correctly** taking logs of both sides. OE eg $\ln 75000 = \ln A + \ln p^t$
	$t = \frac{10 \ln 15}{\ln 5}$ or $t = 16.8....$	A1		OE e.g. $t = \frac{\ln 15}{\ln 1.175}$ or $16.79\ldots$ $t = \frac{\ln 15}{\ln 5^{\frac{1}{10}}}$ etc.
	2018	B1	4	
(c)(i)	$5000p^{T-10} = 2500q^T$	B1		Correct opening expression
	$\ln 2 + (T - 10)\ln p = T \ln q$	M1		Use laws of logs correctly to obtain a linear equation in T. Powers must involve T and $T \pm 10$.
	$T = \frac{10 \ln p - \ln 2}{\ln p - \ln q}$	m1		Make T the subject of their expression correctly.
	$p^{10} = 5 \Rightarrow 10 \ln p = \ln 5 \Rightarrow T = \frac{\ln\left(\frac{5}{2}\right)}{\ln\left(\frac{p}{q}\right)}$	A1	4	$p^{10} = 5 \Rightarrow 10 \ln p = \ln 5$ used to get **AG**
(ii)	2023	B1	1	
	Total		**11**	

Q	Solution	Mark	Total	Comment
5 (a)(i)	$R=5$	B1		
	$\tan\alpha=\frac{4}{3}$	M1		$R\sin\alpha=4$ or $R\cos\alpha=3$ using their R $\sin\alpha=4$ $\cos\alpha=3$ is **M0**
	$\alpha=53.1°$	A1	3	53.1° **only**
(ii)	$5\sin(2\theta+53.1)°=5$	M1		Candidate's R and α but must use 2θ - PI.
	$[(2\theta+53.1)°=90°$ and $450°]$			
	$\theta=18.4°$	A1		Accept $\theta=18.5°$
	$\theta=198.4°$	A1ft	3	$180°+'their'\,18.4°$
(b)(i)	$\frac{2\tan\theta}{1-\tan^2\theta}\times\tan\theta=2$	M1		**Use** of correct form of $\tan 2\theta$
	$2\tan^2\theta=2(1-\tan^2\theta)$ $4\tan^2\theta=2$ $2\tan^2\theta=1$	A1	2	Correct derivation of **AG**.
(ii)	$\theta=35.3°$	B1		
	$\theta=144.7°$	B1	2	
(c)(i)	$8\times\frac{1}{8}-4\times\frac{1}{2}+1=0\Rightarrow 2x-1$ is a factor	B1	1	Accept $1-2+1=0$ but need the conclusion
(ii)	$4(2\cos^2\theta-1)\cos\theta+1=8x^3-4x+1$	B1	1	$\cos 2\theta=2\cos^2\theta-1$ used correctly in deriving **AG**
(iii)	$8x^3-4x+1=(2x-1)(4x^2+2x-1)$	B1		Award for quadratic factor
	$x=\frac{-2\pm\sqrt{20}}{8}$ or $\frac{-2\pm2\sqrt{5}}{8}$	M1		Correct solution of their quadratic – ACF.
	$(\cos 72°>0)\Rightarrow\cos 72°=\frac{\sqrt{5}-1}{4}$	A1	3	**CSO**
	Total		**15**	

(a)(ii) Either $\theta=18.4°$ or $\theta=198.4°$ earns **A1** and any extras in the interval together with the two correct values earns **A1 A0ft**
Award **SC1** for both answers to greater degree of accuracy 18.43494 … and 198.43494561…

(b)(ii) Either $\theta=35.3°$ or $\theta=144.7°$ earns **B**1 and any extras in the interval together with the two correct values earns **B**1 **B0**
Award **SC1** for both answers to greater degree of accuracy 35.26413… and 144.735561…

Q	Solution	Mark	Total	Comment
6(a)	$\left(\overrightarrow{OP}\right)=\begin{bmatrix}5\\-8\\2\end{bmatrix}$ $\left(\overrightarrow{OQ}\right)=\begin{bmatrix}11\\-14\\8\end{bmatrix}$	**B1**		PI by correct $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ below
	$\left(\overrightarrow{PQ}\right)=\begin{bmatrix}11\\-14\\8\end{bmatrix}-\begin{bmatrix}5\\-8\\2\end{bmatrix}$ or $\begin{bmatrix}6\\-6\\6\end{bmatrix}$	**M1**		$\overrightarrow{PQ}=\pm$ their$\left(\overrightarrow{OQ}-\overrightarrow{OP}\right)$
	$\overrightarrow{PQ}=6\begin{bmatrix}1\\-1\\1\end{bmatrix}$	**A1**	**3**	or $\begin{bmatrix}6\\-6\\6\end{bmatrix}$ stated to be parallel to $\begin{bmatrix}1\\-1\\1\end{bmatrix}$
(b)(i)	$\lambda=1$ **or** $\mu=-2$	**B1**		
	$b=-5+3$ or $b=-8+6$, (their λ or μ) **or** $c=3+1$ or $c=6-2$, (their λ or μ)	**M1**		Attempt to find the value of b **or** c
	$b=-2$ **and** $c=4$	**A1**	**3**	$b=-2$ **shown** and $c=4$
(ii)	$\overrightarrow{RS}=\begin{bmatrix}5+2t\\-8-3t\\2+t\end{bmatrix}-\begin{bmatrix}3\\-2\\4\end{bmatrix}$	**M1**		**Clear** attempt to find $\pm\overrightarrow{RS}$
	$2+2t+6+3t-2+t=0$	**m1**		$\overrightarrow{RS}\bullet\begin{bmatrix}1\\-1\\1\end{bmatrix}=0$ or $\overrightarrow{RS}\bullet\begin{bmatrix}6\\-6\\6\end{bmatrix}=0$ $=0$ PI; correct direction vector
	$t=-1$	**A1**		
	S is at $(3,-5,1)$	**A1**	**4**	Accept as a column vector.
	Total		**10**	

Q	Solution	Mark	Total	Comment
7(a)(i)	$-2\sin 2y\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**B1**		
	$+3y\,\mathrm{e}^{3x}+\mathrm{e}^{3x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$	**M1**		$py\,\mathrm{e}^{3x}+q\mathrm{e}^{3x}\dfrac{\mathrm{d}y}{\mathrm{d}x}$
		A1		Product rule correct
	$= 0$	**B1**		PI
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}(\mathrm{e}^{3x}-2\sin 2y)+3y\mathrm{e}^{3x}=0$	**m1**		Attempt to factorise.
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{-3y\mathrm{e}^{3x}}{\mathrm{e}^{3x}-2\sin 2y}$	**A1**	**6**	OE
(ii)	At A $\quad\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\pi$	**B1**	**1**	Must have scored **all** 6 marks in (a)(i)
(b)	$\left(y-\dfrac{\pi}{4}\right)=\dfrac{1}{\pi}(x-\ln 2)$	**M1**		Finding the equation of **normal** with gradient $\frac{-1}{\text{their}(a)(\text{ii})}$.
	At B $\quad y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$	**A1**	**2**	
	Total		**9**	
(b)	Alternative using $y=mx+c$			
	$\dfrac{\pi}{4}=\dfrac{1}{\pi}\ln 2+c \quad \left(y=\dfrac{1}{\pi}x+c\right)$	**M1**		Use $y=mx+c$ and find c using their gradient.
	At B $\quad y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$	**A1**	**2**	Must see $y=\dfrac{\pi}{4}-\dfrac{\ln 2}{\pi}$ or a statement that c is the required y-coordinate

Q	Solution	Mark	Total	Comment
8 (a)	$16x = A(1+x)^2 + B(1-3x)(1+x) + C(1-3x)$	B1		OE
	$x = -1 \quad -16 = 4C$ $x = \frac{1}{3} \quad \frac{16}{3} = A\left(\frac{4}{3}\right)^2$	M1		Use $x = \frac{1}{3}$ or $x = -1$ to find a value for A or C.
	$A = 3 \quad B = 1 \quad C = -4$	A1 A1	4	Any two correct All three correct
(b)	$\int \frac{1}{e^{2y}}\,dy = \int \frac{16x}{(1-3x)(1+x)^2}\,dx$	B1		
	or $\int \frac{dy}{e^{2y}} = \int \frac{3}{1-3x} + \frac{1}{1+x} - \frac{4}{(1+x)^2}\,dx$			or correct **ft** separation on non-zero A B C
	$\frac{-e^{-2y}}{2}$	B1		**OE**
	$= -\ln(1-3x)$	B1ft		OE ft on $\frac{A}{-3}\ln(1-3x)$
	$+\ln(1+x)$	B1ft		OE ft on $B\ln(1+x)$
	$+\frac{4}{1+x}$	B1ft		OE ft on $\frac{C}{-1}(1+x)^{-1}$
	$-\frac{1}{2} = (-\ln 1 + \ln 1) + 4 + \text{constant}$	M1		Use $(0,0)$ **and** attempt to find a value for the constant.
	$-\frac{1}{2}e^{-2y} = -\ln(1-3x) + \ln(1+x) + \frac{4}{1+x} - \frac{9}{2}$	A1	7	ACF
	Total		**11**	
	TOTAL		**75**	

(b) For **M1** candidates must have a term of the form $ke^{\pm 2y}$ on one side and at least one ln term on the other, substitute $(0,0)$ **and** find a value for the constant.