RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for January 2013

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*/DM1	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
ft or √	Follow through

Subject-specific Marking Instructions for GCE Mathematics Pure strand

a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c. The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.
Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f. Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

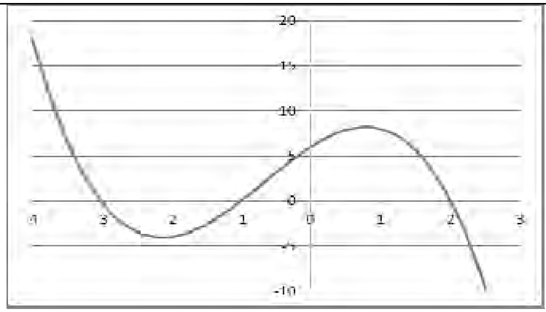
If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$\frac{6 \pm \sqrt{(-6)^2 - 4\times1\times-2}}{2\times1}$	M1	Valid attempt to use quadratic formula	**No marks for attempting to factorise**
			$= \frac{6 \pm \sqrt{44}}{2}$	A1		
			$= 3 \pm \sqrt{11}$	A1	Both roots correct and simplified	
			OR: $(x-3)^2 - 9 - 2 = 0$ $x - 3 = \pm\sqrt{11}$	M1 A1	Correct method to complete square	Must get to $(x - 3)$ and ± stage for the M mark, constants combined correctly gets A1
			$x = 3 \pm \sqrt{11}$	A1 **[3]**	Rearranged to correct form **cao**	
1	(ii)		$\frac{dy}{dx} = 2x - 6$	B1		
			$= -16$	B1 **[2]**	**www**	
2	(i)		$n = 0$	**B1** **[1]**	Allow 3^0	
2	(ii)		$\frac{1}{t^3} = 64$ (or 4^3)	M1	or $t^3 = \frac{1}{64}$ or $64t^3 = 1$ or $\left(\frac{1}{t}\right)^3 = 64$	Allow embedded
			$t = \frac{1}{4}$	A1 **[2]**	4^{-1} is **A0** $t = \pm\frac{1}{4}$ is **A0**	4^{-1} **www** alone implies **M1 A0**
2	(iii)		$2p^2 = 8$	M1	or $8p^6 = 8^3$. Allow $2p^{\frac{6}{3}} = 8$ for **M1**	If not 512, evidence of 8 × 8 × 8 needed.
			$p = 2$	A1	**www**	**SC** Spotted **B1** for 2, **B1** for -2, **B1** for justifying exactly 2 solutions
			or $p = -2$	A1 **[3]**	**www**	**SC** $8p^2 = 8$, $p = \pm 1$ **B1**

Question			Answer	Marks	Guidance	
3	(i)			B1 B1 B1 **[3]**	-ve cubic with 3 distinct roots (0, 6) labelled or indicated on y-axis – seen elsewhere not enough (-3, 0), (-1, 0) and (2, 0) labelled or indicated on x-axis and no other x-intercepts.	Must not stop at x-axis. Condone errors in curvature at the extremes unless extra turning point(s)/root(s) clearly implied. **Must have a curve for 2nd and 3rd marks** Do not allow final B1 if shown as repeated root(s)
3	(ii)		Reflection in the y axis	B1 B1 **[2]**	Not mirrored/flipped etc. or $x = 0$. No/through/along etc. Must be "in". Cannot get 2nd B1 without some indication of a reflection e.g. flip etc. Do not **ISW** if contradictory statement seen	**Alt** Stretch (scale) factor –1 **B1** parallel to the x axis for **B1** **Must be a single transformation for any marks**
4	(i)		$2x^2 - 3x - 5 = \dfrac{-10x-11}{2}$ $4x^2 + 4x + 1 = 0$ $(2x + 1)(2x + 1) = 0$ $x = -\dfrac{1}{2}$ $y = -3$	*M1 A1 DM1 A1 A1 **[5]**	Substitute for x/y or attempt to get an equation in 1 variable only Obtain correct 3 term quadratic – could be a multiple e.g. $2x^2 + 2x + 0.5 = 0$ Correct method to solve resulting 3 term quadratic	or $10x + 2(2x^2 - 3x - 5) + 11 = 0$ If x is eliminated, expect $k(8y^2 + 48y + 72) = 0$ **SC** If DM0 and $x = -\dfrac{1}{2}$ spotted **B1** for x value, **B1** for y value **B1** justifying only one root
4	(ii)		Line is a tangent to the curve	B1√ **[1]**	**Must be consistent with their answers to their quadratic in (i).** **1 repeated root –** indicates one point. Accept tangent, meet at, intersect, touch etc. but do not accept cross **2 roots –** indicates meet at two points **0 roots –** indicates do not meet. Do not accept "do not cross"	**Follow through from their solution to (i)**

Question			Answer	Marks	Guidance	
5	(i)		$5x^2 + 17x - 12 - 3(x^2 - 4x + 4)$	M1	Attempt to expand both pairs of brackets	
				A1	$5x^2 + 17x - 12$ **and** $x^2 - 4x + 4$ soi ; may be unsimplified, no more than one incorrect term, no "extra" terms at all. No "invisible brackets"	
			$= 2x^2 + 29x - 24$	A1	$2x^2 + 29x - 24$	**ISW** if they then put expression equal to zero and go on to "solve"
				[3]		
5	(ii)		$-5x^2 + 2kx^2 + 6x^2$	M1	Correct method to multiply out 3 brackets or correctly identify all x^2 terms	No more than 8 terms, but ignore sign errors/accuracy of non x^2 terms
				A1	All x^2 terms correct, no extras	
			$k = -2$	A1		
				[3]		

Question			Answer	Marks	Guidance	
6	(i)		$\frac{p-7}{-4-{}^-2}$ or $\frac{7-p}{-2-{}^-4}$	M1	uses $\frac{y_2-y_1}{x_2-x_1}$ (at least 3out of 4 correct)	**Alternative method:** Equation of line through one of the given points with gradient 4 **M1** Substitutes **other point** into their equation **M1**
			$\frac{p-7}{-4-{}^-2}=4$ or $\frac{7-p}{-2-{}^-4}=4$	A1	Correct, unsimplified **equation**	Obtains $p = -1$ (Accept $y = -1$)**A1**
			$p = -1$	A1 **[3]**		**Note:** Other "informal" methods can score full marks provided **www**
6	(ii)		$\frac{{}^-2+6}{2}=m,\quad \frac{7+q}{2}=5$	M1	Correct method (may be implied by one correct coordinate)	Use the same marking principle for candidates who add/subtract half the difference to an end point or use similar triangles or other valid "informal" methods.
			$m = 2$	A1		
			$q = 3$	A1 **[3]**		
6	(iii)		$\sqrt{(-2-d)^2+(7-3)^2}$	*M1	Correct method to find line length/square of line length using Pythagoras' theorem (at least 3out of 4 correct)	
			$d^2 + 4d + 20 = 52$	B1	$\left(2\sqrt{13}\right)^2 = 52$ or $2\sqrt{13}=\sqrt{52}$	
			$d^2 + 4d - 32 = 0$ $(d + 8)(d - 4) = 0$	DM1	Correct method to solve 3 term quadratic, must involve their "52"	**SC: B1** for each value of d found or "spotted" from correct working
			$d = -8$ or 4	A1 **[4]**		**Note:** Other "informal" methods can score full marks provided **www**

Question			Answer	Marks	Guidance	
7	(i)		$y = 9x^5$	M1	Obtain kx^5	If individual terms are differentiated then **M0A0B0**
				A1	Correct expression for y ($9x^5$)	
			$\frac{dy}{dx} = 45x^4$	B1 ft **[3]**	Follow through from their single kx^n, $n \neq 0$. Must be simplified.	$\frac{3x^2 + x^4}{x}$ is **not** a misread **M0A0B0**
7	(ii)		$y = x^{\frac{1}{3}}$	B1	$\sqrt[3]{x} = x^{\frac{1}{3}}$	
				B1	$kx^{-\frac{2}{3}}$	**SC** $\sqrt[3]{x} = x^{-\frac{1}{3}}$ differentiated to
			$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$	B1 **[3]**	$\frac{1}{3}x^{-\frac{2}{3}}$. Allow $0.\dot{3}$ (not finite)	$-\frac{1}{3}x^{-\frac{4}{3}}$ **B1**
7	(iii)		$y = \frac{1}{2}x^{-3}$	M1	kx^{-4} seen	
			$\frac{dy}{dx} = -\frac{3}{2}x^{-4}$	A1 **[2]**		
8			$(3k-1)^2 - 4 \times k \times -4$	*M1	Attempts $b^2 - 4ac$ or an equation or inequality involving b^2 and $4ac$. Must involve k^2 in first term (but no x anywhere). If $b^2 - 4ac$ not stated, must be clear attempt.	Must be working with the discriminant explicitly and not only as part of the quadratic formula. Allow $\sqrt{b^2 - 4ac}$ for first **M1 A1**
			$= 9k^2 + 10k + 1$	A1	Correct discriminant, simplified to 3 terms	
			$9k^2 + 10k + 1 < 0$	M1	States discriminant < 0 or $b^2 < 4ac$.	Can be awarded at any stage. Doesn't need first M1. No square root here.
			$(9k + 1)(k + 1) < 0$	DM1	Correct method to find roots of a three term quadratic	
			$-1, -\frac{1}{9}$	A1	Both values of k correct	
			$-1 < k < -\frac{1}{9}$	M1	Chooses "inside region" of inequality	Allow correct region for their inequality
				A1 **[7]**	Allow "$k < -\frac{1}{9}$ **and** $k > -1$" etc. must be strict inequalities for A mark	Do not allow "$k < -\frac{1}{9}$ **or** $k > -1$";

Question			Answer	Marks	Guidance	
9	(i)		Centre (1, −5) $(x-1)^2+(y+5)^2-19-1-25=0$ $(x-1)^2+(y+5)^2=45$ Radius $=\sqrt{45}$	B1 M1 A1 **[3]**	Correct centre Correct method to find r^2 Correct radius. **Do not allow if wrong centre used in calculation of radius.**	 $r^2=(\pm5)^2+(\pm1)^2+19$ for the M mark **A0** if $\pm\sqrt{45}$
9	(ii)		$7^2+(-2)^2-14-20-19$ $=0$	B1 **[1]**	Substitution of coordinates into equation of circle in any form or use of Pythagoras' theorem to calculate the distance of (7, -2) from *C*	**No follow through** for this part as **AG. Must be consistent–** do not allow finding the distance as $\sqrt{45}$ if no/wrong radius found in 9(i).
9	(iii)		gradient of radius $=\frac{-5-(-2)}{1-7}$ or $\frac{-2-(-5)}{7-1}$ $=\frac{1}{2}$ gradient of tangent $=-2$ $y+2=-2(x-7)$ $2x+y-12=0$	M1 A1√ B1√ M1 A1 **[5]**	uses $\frac{y_2-y_1}{x_2-x_1}$ with **their C** (3/4 correct) Follow through from their C, allow unsimplified single fraction e.g. $\frac{-3}{-6}$ Follow through from their gradient, even if M0 scored. Allow $\frac{-1}{\text{their fraction}}$ **B1** correct equation of straight line through (7, −2), any non-zero numerical gradient oe 3 term equation in correct form i.e. $k(2x+y-12)=0$ where k is an integer **cao**	**Follow through from 9(i) until final mark.** If (−1,5) is used for C, then expect Gradient of radius $=\frac{5-(-2)}{-1-7}=-\frac{7}{8}$ Gradient of tangent $=\frac{8}{7}$ <u>Alternative markscheme for implicit differentiation</u>: **M1** Attempt at implicit diff as evidenced by $2y\frac{dy}{dx}$ term **A1** $2x+2y\frac{dy}{dx}-2+10\frac{dy}{dx}=0$ **A1** Substitution of (7, −2) to obtain gradient of tangent = −2 Then **M1 A1** as main scheme

Question			Answer	Marks	Guidance	
10			$\frac{dy}{dx} = x^2 - 9x^{-2}$	B1	x^2 from differentiating first term	
				M1	kx^{-2}	
				A1	$-9x^{-2}$ (no + c)	
			Gradient of line = 8	B1		
			$x^2 - 9x^{-2} = 8$	M1	Equate their $\frac{dy}{dx}$ to 8 (or their gradient of line, if clear)	**Note:** If equated to +/-1/8 then M0 but the next M1 and its dependencies are available
			$x^4 - 8x^2 - 9 = 0$ $k^2 - 8k - 9 = 0$	*M1	Use a correct substitution to obtain a 3 term quadratic or factorise into 2 brackets each containing x^2	If no substitution stated and treated as a quadratic (e.g. quadratic formula), no more marks until square rooting seen
			(k – 9)(k + 1) = 0	DM1	Correct method to solve 3 term quadratic – **dependent on previous M1**	**SC:** If spotted after first five marks- (3, 12) **B1** (-3, -12) **B1** Justifies exactly two solutions **B3**
			k = 9 (don't need k = −1)	A1	No extras	
			x = 3, −3	DM1	Attempt to find x by square rooting – accept one value	
			y = 12, −12	A1 **[10]**	No extras	

More Additional Guidance for Q10

If curve equated to line and before differentiating:

First four marks **B1 M1 A1 B1** available as main scheme
Then **M0** for equating as this not been explicitly done
Allow the **M1** for the substitution
DM1 for quadratic as main scheme (dependent on a correct substitution)
A0 for the 9 (as follows wrong working)
DM1 for square rooting (dependent on a correct substitution)
A0 for the co-ordinates (as follows wrong working). Max mark **7/10**

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0**.

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times 2}$	earns **M1** (minus sign incorrect at start of formula)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	earns **M1** (18 for *c* instead of −18)
$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$	**M0** (2 sign errors: initial sign and *c* incorrect)
$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times -5}$	**M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

← This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

OCR (Oxford Cambridge and RSA Examinations)
1 Hills Road
Cambridge
CB1 2EU

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