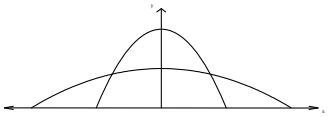
4733 Probability & Statistics 2

General: Conclusions to hypothesis tests must acknowledge uncertainty. Thus "time is unchanged" is A0. Similarly, "Significant evidence that time is unchanged" is also A0.

1	**(i)**	Biased in favour of those with strong political interest	B2	**2**	"Biased", "unrepresentative", "not indept" or equiv [but *not* "not random"] stated, with sensible reason. [SR: partial answer, B1]
	(ii)	Obtain list of all pupils Allocate numbers sequentially Choose using random numbers	B1 B1 B1	**3**	List, can be implied; number serially or randomly, not just "number pupils" Select consistently with method of numbering, not just "select randomly" [SR: systematic: List B1, every n^{th} B1, random start B1] [SR: names in a hat: B2]
2	**(i)**	$\Phi\left(\frac{24-30}{12}\right) - \Phi\left(\frac{20-30}{12}\right)$ $= \Phi(-0.5) - \Phi(-0.833)$ $= (1 - 0.6915) - (1 - 0.7976) =$ **0.1061**	M1 A1 M1 A1	**4**	Standardise one, allow $\sqrt{12}$, 12^2, $\sqrt{n}$ Both standardisations correct, allow cc here Correct handling of tails [0.3085 – 0.2024] Answer, a.r.t. 0.106, c.a.o.
	(ii)	Not symmetrical (skewed) Therefore inappropriate	M1 A1	**2**	Any comment implying not symmetric Conclude "not good model" [Partial answer: B1]
3		$H_0 : \mu = 28$ $H_1 : \mu \neq 28$ $\sigma^2 = 37.05 \times 40/39$ [= 38]	B2 M1		Both hypotheses correctly stated; one error, allow wrong or no letter, but not x or t or $\bar{x}$, B1 Multiply 37.05 or $\sqrt{37.05}$ by $n/(n-1)$ or $\sqrt{[n/(n-1)]}$
	α	$z = \frac{26.44 - 28}{\sqrt{38/40}} = -1.601$ Compare –1.645, or 0.0547 with 0.05	M1 A1 B1		Standardise with $\sqrt{n}$, allow $\sqrt{}$ errors, cc, + Correct z, a.r.t –1.60, or $p \in$ [0.0547, 0.0548] Explicit comparison of z with –1.645 or p with 0.05
	β	Critical value $28 - z\sigma/\sqrt{n}$ [= 26.397] $z = 1.645$ Compare 26.44 with 26.40	M1 B1 A1√		Allow "±", √ errors, cc, ignore other tail $z = 1.645$ in CV expression, and compare 26.44 CV, √ on their z, rounding to 3 SF correct
		Do not reject H_0 [can be implied] Insufficient evidence that time taken has changed.	M1 A1√	**8**	Needs $\sqrt{n}$, correct method & comparison, *not* $\mu = 26.44$ Conclusion interpreted in context, √ on z,
4	**(i)**	$\frac{53-50}{\sigma/\sqrt{10}} < 2.326$ $\sigma >$ **4.08** **AG** [Allow ≥]	M1 A1 B1 A1	**4**	Standardise with 10 or $\sqrt{10}$ and Φ^{-1} Both sides same sign, $\sqrt{10}$, don't worry about < 2.326 or 2.33 seen Convincingly obtain $\sigma > 4.08$ to 3 SF, one other step [SR: Substitution: standardise & substitute 4.08 M1; 0.0101 A1; 4.07 or 4.075 tried, M1; full justification A1]
	(ii)	P(Type I) = 0.01 used, e.g. Geo(0.01) $0.99^4 \times 0.01$ = **0.0096**	M1 M1 A1	**3**	Not enough merely to state $p = 0.01$ $p^4 \times q$ Answer, a.r.t. 0.0096
5	**(i)**	$\int_{-1}^{1} \frac{3}{4}(x^2 - x^4)dx = \frac{3}{4}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1}$ [= 1/5] $1/5 - 0^2$ = **1/5**	M1 A1 B1 A1	**4**	Attempt $\int_{-1}^{1} x^2 f(x)dx$ Correct indefinite integral Mean 0 clearly indicated Answer 1/5 or a.r.t. 0.200, don't need $\mu = 0$
	(ii)	(a)  (b) Areas equal, more spread out, so g_{max} lower (c) W greater as more spread out	B1 M1 A1 B1dep depB1	**5**	Correct graph, don't need f(x) as well. Don't allow if graph goes further below axis than "pips". Don't worry too much about exact shape Mention areas or total probability Convincing argument, not just "flatter" W greater… …with convincing reason

6	(a)	Po(2.375) $e^{-2.375}\left(\frac{2.375^3}{3!}+\frac{2.375^4}{4!}\right)$ [= 0.2079 + 0.1233] = **0.3310**	M1 M1 A1 A1	**4**	Po(19/8) stated or implied One correct Poisson formula, *not* tables Complete correct expression, including addition Answer, a.r.t. 0.331 [SR: Po(2) or Po(2.4) and tables, M1]
	(b)	(i) *n* large OR $n > 50$ *p* small OR $np < 5$	B1 B1	**2**	Or equivalent [Allow ≤ and ≥ throughout] Or equivalent, e.g. $np \approx npq$, or $p < 0.1$ [Treat "$np < 5$, $npq < 5$" as single wrong statement]
		(ii) B(108, $\frac{1}{36}$) ≈ Po(3) 1 – P(≤ 3) = 1 – 0.6472 = **0.3528**	M1 M1 A1 M1 A1	**5**	Correct binomial distribution stated or implied Po(*np*), √ on their *n*, *p* Po(3) Use Po tables, "1 –", *or* correct formula, ± 1 term, e.g. 0.1847; a.r.t. 0.353, allow from exact Binomial
7	(i)	Dropped catches must occur independently of one another and at constant average rate	B1 B1	**2**	"independently", in context, allow "random" "Constant average rate", in context ["Singly" doesn't gain B1]
Marks can be awarded in either part	(ii)	Use: "Reject H_0 when correct" Po(10) P(≥ 16) = 1 – P(≤ 15) = 1 – 0.9513 Probability 0.0487	M1 M1 M1 A1		Find P(≥ *r*) where $r > \lambda$, e.g. P(≥ 6) from Po(2) Po(10) stated or implied [can be recovered in (iii)] Seek biggest prob < 0.05, e.g. 0.0835 or 0.0166, allow 0.0293 but no other LH tail Answer in range [0.0487, 0.0488], cwd, cwo
	(iii)	$H_0 : \lambda = 10$ or 2 [or μ] $H_1 : \lambda > 10$ or 2 [or μ] α: P(≥ 14) = 1 – 0.8645 = 0.1355 > 0.05	B2 A1 B1		Hypotheses fully correct, allow λ or μ [SR: one error, B1, but *r* or *R* or *x* or $\bar{x}$: B0] $p \in$ [0.135, 0.136] from Po(10) Compare explicitly with 0.05 or 0.0487
		β: Critical region $r \geq 16$, $p = 0.0487$ Compare $r = 14$	A1√ B1√		√ on answer from (ii)
		Do not reject H_0 [can be implied] Insufficient evidence of an increase in the number of dropped catches	M1 A1√	**10**	Method correct, √ on *p*, must be upper tail and "≥" Conclusion interpreted in context [SR: P(≤ 14) = 0.9165 < 0.95: (B2 M1) A0 B1 M0A0; same for P(> 14) or P(= 14)] [SR: N(10,10): (ii) 0.05 M0. (iii) (B2) M1 A0 B1 M0A0]
8	(i)	$H_0 : p = 0.4$ or $\mu = 4.8$ $H_1 : p > 0.4$ or $\mu > 4.8$ B(12, 0.4) P(≥ 9) = 1 – 0.9847 = 0.0153 < 0.05 Reject H_0 [can be implied] Significant evidence of increase in proportion of audience members who know sponsor's name	B2 M1 A1 B1√ M1 A1√	**7**	Both fully correct, B2. [SR: one error, B1, but *x* or *R* or *r* or $\bar{x}$: B0] B(12, 0.4) stated or implied, e.g. 0.9972 or 0.9847 Or: CR is ≥ 9 and $p \in$ [0.015, 0.0153] Explicitly compare with 0.05, or 9 with ≥ 9, √ on < Reject H_0, √ on probability, must be "≥" Conclusion interpreted in context [SR: P(≤ 9) or P(= 9) or P(> 9): (B2 M1) A0 B1 M0A0] [SR: N(4.8, 2.88): (B2) M1 A0 B0 M0A0]
	(ii)	N(160, 96) $\frac{(x-0.5)-160}{\sqrt{96}} = 1.645$ Solve to find *x* [= 176.6] Minimum value is **177**	B1 B1 M1 A1 B1 M1 A1	**7**	Normal, mean 160 Variance (or SD) 96 [96/400: B2M0] Standardise unknown with *np* and √*npq* or *npq*, & equate to Φ^{-1}; √96 and signs correct, ignore cc RHS = 1.645 Solve [implied by 177 or 176.6 or 176.1] 177 only, from 176.6, CWO [cc error: 6 ex 7]