A-level
Mathematics
MPC3-Pure Core 3
Mark scheme

June 2018

Version/Stage: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

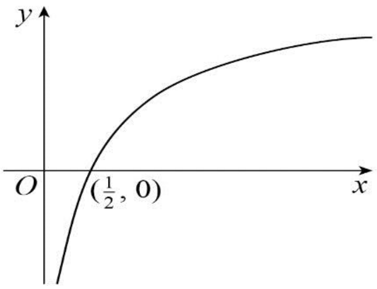
Otherwise we require evidence of a correct method for any marks to be awarded.

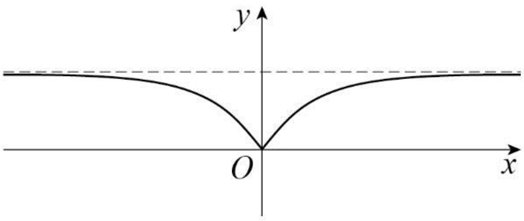
Q1	Solution	Mark	Total	Comment
(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) Ax\left(5+3x^2\right)^{-\frac{1}{2}}$	M1		
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) \quad 3x\left(5+3x^2\right)^{-\frac{1}{2}}$ **OE**	A1	2	correct possibly unsimplified
(b)	$x+B\cos 4x$	M1		
	$x-\frac{1}{4}\cos 4x$	A1		
	$x-\frac{1}{4}\cos 4x+c(onstant)$	A1	3	CAO with $+c$
	Total		5	

(a) Allow $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right) \quad \frac{1}{2}\times 2\times 3x\left(5+3x^2\right)^{-\frac{1}{2}}$ etc for full marks with **ISW**

Also allow use of implicit differentiation $2y\frac{\mathrm{d}y}{\mathrm{d}x}=6x$ for **M1** and $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6x}{2y}$ **OE** for **A1**

Q2	Solution	Mark	Total	Comment
(a)	$\dfrac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$	M1		Condone 'poor' use of brackets
	Use of $\sin^2 x+\cos^2 x=1$ **OE** $\left(\dfrac{\mathrm{d}(\cot x)}{\mathrm{d}x}=\right)\ \dfrac{-1}{\sin^2 x}=-\operatorname{cosec}^2 x$	A1	2	**AG** be convinced
(b)(i)	$\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)-1-3\operatorname{cosec}^2 3y$ **OE**	M1 A1	2	$-1+k\operatorname{cosec}^2(3p)\quad k\neq 0$, $p=x$ or y correct
(ii)	$y=\dfrac{\pi}{12}\Rightarrow\left(\dfrac{\mathrm{d}x}{\mathrm{d}y}=\right)-1-3\operatorname{cosec}^2\left(\dfrac{3\pi}{12}\right)$	M1		Correct substitution of $\pi/4$ into expression of form $-1+k\operatorname{cosec}^2 3y\quad k\neq 0$ **PI**
	Grad of tangent = 1/ (dx/dy)	M1		**FT** reciprocal of "their" dx/dy
	Grad of tangent = $-\dfrac{1}{7}$	A1		Must have scored **M1M1**
	$y=-\dfrac{1}{7}x+\dfrac{\pi}{12}+\dfrac{1}{7}$	A1cso	4	
	Total		8	
(a)	$\dfrac{\sin x-\sin x-\cos x\cos x}{\sin x^2}$ scores **M1A0** Numerator must have 'two negatives' Product rule scores **M0**			
(b)(ii)	The two **M1** marks could be seen in either order 2nd **M** mark can be earned by $x=(their(-7))y+c$ but **A** mark only earned when $-\dfrac{1}{7}$ seen			

Q3	Solution	Mark	Total	Comment
(a)(i)		**M1**		"log" graph correct shape in 1st and 4th quadrants Graph not touching y-axis
		A1	2	cuts x-axis at $\left(\frac{1}{2},0\right)$ stated or 0.5 marked on x-axis and no other intersections with coordinate axes
(ii)	Stretch + either I or II Parallel to x-axis I	**M1**		**Alt 1:** Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ **M1**
	SF $\frac{1}{2}$ II	**A1**		$k = -\dfrac{3}{2}$ **A1**
	Followed by Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$	**M1**		Followed by Stretch in x-direction **M1** SF $\frac{1}{2}$ **A1**
	$k = -\dfrac{3}{4}$	**A1**	4	
(b)(i)	$x_1 = 1;\ x_2 = 1.5;\ x_3 = 2;\ x_4 = 2.5;\ x_5 = 3$	**B1**		all 5 correct x-values (& no extras) used
	$\dfrac{0.5}{3}\left\{\mathrm{f}(1)+\mathrm{f}(3)+2\mathrm{f}(2)+4\mathrm{f}(1.5)+4\mathrm{f}(2.5)\right\}$	**M1**		Simpson's rule used correctly
	$\mathrm{f}(1) = \ln 7 \approx 1.94591...\ \mathrm{f}(1.5) = \ln 9 \approx 2.19722...$ $\mathrm{f}(2) = \ln 11 \approx 2.397895...\ \mathrm{f}(2.5) = \ln 13 \approx 2.564949...$ $\mathrm{f}(3) = \ln 15 \approx 2.708050...$	**A1**		at least 4 correct y-values in exact form or decimals, rounded or truncated to 4dp or better (in table or formula) **PI**
	Answer = 4.7497	**A1**	4	**CAO** must see this value exactly **NB NOT** for 4.749845… to 6 dp (obtained by exact integration)
(ii)	$I = \int \ln \mathrm{e}\ \mathrm{d}x + \int \ln(3+4x)\ \mathrm{d}x$	**M1**		or single integral split as sum of two correct terms (including brackets), but condone omission of dx
	$= \left[x \ln \mathrm{e}\right]_1^3 + A = 2\ln \mathrm{e} + A = A + 2$	**A1**	2	**AG** be convinced
	Total		**12**	
(a)(ii)	Condone, stretch **(**in) x(-axis), scores **M1**, then **A1** available with correct SF			
	Alt 2: Translation $\begin{bmatrix} k \\ 0 \end{bmatrix}$ $k = -\dfrac{3}{4}$ **M1A1** Translation $\begin{bmatrix} 0 \\ l \end{bmatrix}$ $l = \ln 2$ **M1A1**			
(b)(i)	May use $\dfrac{0.5}{3}\left\{\ln 7+\ln 15+4(\ln 9+\ln 13)+2\ln 11\right\} = \dfrac{1}{6}\left\{\ln 105+4(\ln 117)+2\ln 11\right\}$ etc to earn **B1 M1 A1** (implied)			
(ii)	Use of Simpson's rule scores **M0**			

Q4	Solution	Mark	Total	Comment
(a)(i)	$k = \frac{\pi}{2}$	B1	1	Not 90º alone, but $\frac{\pi}{2}$ or 90 scores **B1**
(ii)		B1	1	modulus graph as shown with cusp at O roughly symmetrical and curve approaching but not crossing asymptote
(iii)	Line $y = 3 + 7x$ drawn	M1		gradient >0 which would cut y-axis above upper asymptote
	Line cuts curve exactly once so there is exactly one real root	A1	2	
(b)	$f(x) = \lvert\tan^{-1} x\rvert - 7x - 3$;			
	$f(-0.4) = 0.1805...$ $f(-0.3) = -0.6085...$	M1		Both values correct
	change of sign indicates α lies between – 0.4 and – 0.3	A1	2	(maybe rounded or truncated to 1dp)
(c)	$x_2 = -0.374$ $x_3 = -0.377$	B1,B1	2	exactly these values
	Total		8	
(a)(i) (b)	Answer may be in body of script $g(x) = \lvert\tan^{-1} x\rvert$ and $h(x) = 7x + 3$; **Alternative:** $g(-0.4) = 0.38...$ $h(-0.4) = 0.2$ $g(-0.3) = 0.29...$ $h(-0.3) = 0.9$ values correct when rounded or truncated to 1dp for **M1** then $g(-0.4) > h(-0.4)$, $g(-0.3) < h(-0.3)$ hence α lies between –0.4 and –0.3 **A1** Condone "less than or equal to" for < etc; allow "x", "root", "solution" for "α" but not "it"			

Q5	Solution	Mark	Total	Comment
				M1 M1 may be in either order
(a)(i)	$x=\sqrt{2y+3}$	**M1**		Interchange *x* and *y* ;
	$x^2=2y+3 \Rightarrow y=\frac{1}{2}(x^2-3)$	**M1**		Isolate *y/x* ;
	$\left[f^{-1}(x)\right]=\frac{1}{2}(x^2-3)$	**A1**	**3**	**OE**
(ii)	$x \ldots 1$	**B1**	**1**	
(b)	$x^2+4x=(x+2)^2-4$	**M1**		Showing $x=-2$ when $\frac{dy}{dx}=0$
	Minimum value of g(x) is – 4 **PI**	**A1**		
	Range of g is $g(x) \ldots -4$	**A1**	**3**	
(c)(i)	$gf(x)=\left(\sqrt{2x+3}\right)^2+4\sqrt{2x+3}$ **oe**	**B1**	**1**	
(ii)	$4\sqrt{2x+3}=18-2x$	**M1**		Square root term isolated
	$2\sqrt{2x+3}=9-x$			
	$4(2x+3)=(9-x)^2$ **oe**	**A1**		Squaring both sides
	[$8x+12=81-18x+x^2$ **]**			
	$x^2-26x+69$ [= 0]	**A1**		
	$x=3,\quad x=23$	**A1**		
	[Reject $x=23$ since does not satisfy equation]			
	[Only valid solution is] $x=3$	**A1cso**	**5**	
	Total		**13**	

(a)(i) Condone $y=\frac{1}{2}(x^2-3)$ or $\frac{1}{2}(x^2-3)$ for final **A1**

(b) **Note:**
SC3 for $g(x) \ldots -4$;
SC2 for $g(x) \geqslant -4$ or $y \ldots -4$ or $g \ldots -4$ or Range $\ldots -4$ or $\ldots -4$
SC1 for $y>-4$ etc
BUT $x \ldots -4$, $f(x) \ldots -4$ scores **SC0**

(c)(ii) **NMS** or if **M1** is not earned "$x=3$" scores **SC1,** **BUT** must have scored **B1** in **(c)(i)**

Alternative: $g(y)=21$; $y^2+4y-21=0$ **M1** $\Rightarrow y=-7, y=3$ **A1 A1**; $x=3$, 23 **A1**
$x=3$ and no other solution **A1cso**

Q6	Solution	Mark	Total	Comment
	$u = 2 + \ln x \Rightarrow \mathrm{d}u = \frac{1}{x}\mathrm{d}x$ **OE**	**B1**		may have $x = \mathrm{e}^{u-2}$ etc
		M1		clear attempt to get integral all in terms of *u* including d*x* in terms of d*u*
	$\int \frac{u-2}{u^2}\,\mathrm{d}u$	**A1**		correct – condone omission of d*u* if seen on later line
	$\int \left(\frac{A}{u} + \frac{B}{u^2}\right)\mathrm{d}u = A\ln u + \frac{C}{u}$	**dM1**		
	$\ln u + \frac{2}{u}$	**A1**		Integration by parts gives $\int = \ln u + \frac{2}{u} - 1$
	$\left[\ln 3 + \frac{2}{3}\right] - [\ln 2 + 1]$	**dM1**		correct substitution of correct limits but must have earned previous **dM1**
	$= -\frac{1}{3} + \ln\left(\frac{3}{2}\right)$	**A1**	7	
	Total		**7**	
	Accept $\int \frac{Au}{u^2}\,\mathrm{d}u = 0.5A\ln u^2$ May see integration by parts For second **dM1** candidate may correctly change back to '*x*' then use original limits '+c' on final line loses final **A1**			

Q7	Solution	Mark	Total	Comment
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{-3x}-3(x-1)\mathrm{e}^{-3x}$	M1 A1		$\mathrm{e}^{-3x}+A(x-1)\mathrm{e}^{-3x}$ **OE** $A=-3$
	Equate gradient to 0, hence $x=\frac{4}{3}$	A1cso	3	**AG** be convinced, must see a middle line
(b) (i)	$u=(x-1)^2 \quad \frac{\mathrm{d}v}{\mathrm{d}x}=\mathrm{e}^{-6x}$	M1		u and $\frac{\mathrm{d}v}{\mathrm{d}x}$ correct and 4 terms in this form
	$\frac{\mathrm{d}u}{\mathrm{d}x}=2(x-1) \quad v=-\frac{1}{6}\mathrm{e}^{-6x}$	A1		all correct
	$I=-\frac{1}{6}(x-1)^2\mathrm{e}^{-6x}+\frac{1}{3}\int(x-1)\mathrm{e}^{-6x}\,\mathrm{d}x$	dM1		correct sub of their terms into parts formula
	$\int(x-1)\mathrm{e}^{-6x}\mathrm{d}x=-\frac{(x-1)}{6}\mathrm{e}^{-6x}+\int\frac{1}{6}\mathrm{e}^{-6x}\mathrm{d}x$	M1		must be correct but may be seen anywhere in solution
	$=-\frac{(x-1)}{6}\mathrm{e}^{-6x}-\frac{1}{36}\mathrm{e}^{-6x}$	A1		
	$\int=-\frac{1}{6}(x-1)^2\mathrm{e}^{-6x}-\frac{(x-1)}{18}\mathrm{e}^{-6x}-\frac{1}{108}\mathrm{e}^{-6x}$	A1	6	**OE** condone omission of +c **SEE BELOW FOR ALT SOLN**
(ii)	$\pi\int_1^2(x-1)^2\mathrm{e}^{-6x}\mathrm{d}x$	B1		must have $\pi, \mathrm{d}x$ and correct limits
	$(\pi)\left(-\frac{1}{6}\mathrm{e}^{-12}-\frac{1}{18}\mathrm{e}^{-12}-\frac{1}{108}\mathrm{e}^{-12}+\frac{1}{108}\mathrm{e}^{-6}\right)$	M1		correct sub of 1 and 2 into their part **(b)(i)** answer, but must be terms of the form $(Ax^2+Bx+C)\mathrm{e}^{-6x}$ **OE**
	(Volume =) $\frac{\pi}{108}\left(\mathrm{e}^{-6}-25\mathrm{e}^{-12}\right)$ **OE**	A1	3	like terms collected
	Total		**12**	

(a) Withhold final **A1** for verification

(b)(i) Condone omission of dx throughout

Alt: If a candidate expands $(x-1)^2=x^2-2x+1$, then

$u=x^2 \quad \frac{\mathrm{d}v}{\mathrm{d}x}=\mathrm{e}^{-6x} \quad \frac{\mathrm{d}u}{\mathrm{d}x}=2x \quad v=-\frac{1}{6}\mathrm{e}^{-6x}$ **M1A1** u and $\frac{\mathrm{d}v}{\mathrm{d}x}$ correct and 4 terms in this form

$\int x^2\mathrm{e}^{-6x}\mathrm{d}x=-\frac{1}{6}x^2\mathrm{e}^{-6x}+\frac{1}{6}\int 2x\mathrm{e}^{-6x}\,\mathrm{d}x$ **dM1** correct sub of their terms into parts formula

$\int x\mathrm{e}^{-6x}\mathrm{d}x=-\frac{x}{6}\mathrm{e}^{-6x}+\int\frac{1}{6}\mathrm{e}^{-6x}\mathrm{d}x$ **M1** must be correct but may be seen anywhere in solution

$=-\frac{x}{6}\mathrm{e}^{-6x}-\frac{1}{36}\mathrm{e}^{-6x}$ **A1**

$\int=\mathrm{e}^{-6x}(-\frac{1}{6}x^2+\frac{5}{18}x-\frac{13}{108})$ **OE** **A1**

Q8	Solution	Mark	Total	Comment
(a)	$\dfrac{\sec\theta(\sec\theta+1)+\sec\theta(\sec\theta-1)}{(\sec\theta-1)(\sec\theta+1)}$ **OE**	**B1**		Common denominator (could be 2 separate fractions), all in terms of secθ
	Use of $\sec^2\theta = 1+\tan^2\theta$	**M1**		used
	$\dfrac{2\sec^2\theta}{\tan^2\theta}$			**OR** $\dfrac{2+2\tan^2\theta}{\tan^2\theta}$ $=2+2\cot^2\theta = 2\text{cosec}^2\theta$
	$=\dfrac{2}{\sin^2\theta} = 2\text{cosec}^2\theta$	**A1**	**3**	**AG be convinced**
				Penalise poor notation for final **A1** **(consult team leader)**
(b)	$2\text{cosec}^2\theta = 8-\cot\theta$			
	$2(1+\cot^2\theta) = 8-\cot\theta$	**M1**		Use of $\text{cosec}^2\theta = 1+\cot^2\theta$
	$2\cot^2\theta+\cot\theta-6=0$	**A1**		Must see = 0, this line or next line
	$(2\cot\theta-3)(\cot\theta+2)$ $[=0]$	**dM1**		Correctly factorising/solving their quadratic in cot (**PI** by next **A1** earned)
	$\tan\theta = \dfrac{2}{3}, -\dfrac{1}{2}$ or $\cot\theta = \dfrac{3}{2}, -2$	**A1**		**OE**
	0.58800.., 3.72959.., 2.6779.., 5.819537..	**B1**		sight of **any** of these values correct to at least 3sf rounded or truncated **PI**
	(x =) 0.0940, 1.66, 1.14, 2.71	**B1**		3 correct at least 3sf
		B1	**7**	all 4 correct to 3sf and no others in interval (ignore answers outside interval)
	Total		**10**	

(a) **Alternative 1:** $\dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta} = \dfrac{1+\cos\theta+1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$ **B1**, $1-\cos^2\theta = \sin^2\theta$ used **M1 etc**

Alternative 2: $2\text{cosec}^2\theta = \dfrac{2}{\sin^2\theta} = \dfrac{2}{1-\cos^2\theta}$ **B1** $\dfrac{2}{(1-\cos\theta)(1+\cos\theta)} = \dfrac{1}{1-\cos\theta}+\dfrac{1}{1+\cos\theta}$ **M1 etc**

(b) **Alternative:** $2 = 8\sin^2\theta - \sin\theta\cos\theta$ hence $2(1+\tan^2\theta) = 8\tan^2\theta - \tan\theta$ **M1**

with **A1** for a quadratic in $\tan\theta$: $6\tan^2\theta - \tan\theta - 2 = 0$ **;** $(3\tan\theta-2)(2\tan\theta+1)=0$ **dM1** factors **etc**