4724 Core Mathematics 4

1		Attempt to factorise numerator and denominator	M1	$\frac{A}{\mathrm{f}(x)}+\frac{B}{\mathrm{g}(x)}$;fg= $6x^2-24x$
		Any (part) factorisation of both num and denom	A1	Corres identity/cover-up
		Final answer = $-\frac{5}{6x}, \frac{-5}{6x}, \frac{5}{-6x}, -\frac{5}{6}x^{-1}$ Not $-\frac{\frac{5}{6}}{x}$	A1	
			3	

2		Use parts with $u = x, \mathrm{d}v = \sec^2 x$	M1	result $\mathrm{f}(x) +/- \int \mathrm{g}(x)\,\mathrm{d}x$
		Obtain correct result $x \tan x - \int \tan x\,\mathrm{d}x$	A1	
		$\int \tan x\,\mathrm{d}x = k \ln \sec x$ or $k \ln \cos x$, where $k = 1$ or -1	B1	or $k \ln\lvert\sec x\rvert$ or $k \ln\lvert\cos x\rvert$
		Final answer = $x \tan x - \ln\lvert\sec x\rvert + c$ or $x \tan x + \ln\lvert\cos x\rvert + c$	A1	
			4	

3	**(i)**	$1 + \frac{1}{2}.2x + \frac{\frac{1}{2}.-\frac{1}{2}}{2}\left(4x^2 \text{ or } 2x^2\right) + \frac{\frac{1}{2}.-\frac{1}{2}.-\frac{3}{2}}{6}\left(8x^3 \text{ or } 2x^3\right)$	M1	
		$= 1 + x$	B1	
		… $-\frac{1}{2}x^2 + \frac{1}{2}x^3$ (AE fract coeffs)	A1 (**3**)	For both terms
	(ii)	$(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3$	B1	or $(1+x)^3 = 1 + 3x + 3x^2 + x^3$
		Either attempt at their **(i)** multiplied by $(1+x)^{-3}$	M1	or **(i)** long div by $(1+x)^3$
		$1 - 2x$ …. $\sqrt{}\,1 + (a-3)x$	A1	f.t. **(i)** = $1 + ax + bx^2 + cx^3$
		… $+ \frac{5}{2}x^2$.... $\sqrt{}\,(-3a + b + 6)x^2$	A1	
		… $-2x^3$ $\sqrt{}\,(6a - 3b + c - 10)x^3$	A1 (**5**)	(AE fract.coeffs)
	(iii)	$-\frac{1}{2} < x < \frac{1}{2}$, or $\lvert x\rvert < \frac{1}{2}$	B1 (**1**)	
			9	

4		Attempt to expand $(1+\sin x)^2$ and integrate it	*M1	Minimum of $1+\sin^2 x$
		Attempt to change $\sin^2 x$ into $f(\cos 2x)$	M1	
		Use $\sin^2 x = \frac{1}{2}(1-\cos 2x)$	A1	dep M1 + M1
		Use $\int \cos 2x \, dx = \frac{1}{2}\sin 2x$	A1	dep M1 + M1
		Use limits correctly on an attempt at integration	dep* M1	Tolerate g $(\frac{1}{4}\pi)$ − 0
		$\frac{3}{8}\pi - \sqrt{2} + \frac{7}{4}$ AE(3-term)F	A1	WW 1.51… $\rightarrow$ M1 A0
			6	
5	**(i)**	Attempt to connect du and dx, find $\frac{du}{dx}$ or $\frac{dx}{du}$	M1	But not e.g. du = dx
		Any correct relationship, however used, such as dx = 2u du	A1	or $\frac{du}{dx} = \frac{1}{2}x^{-½}$
		Subst with clear reduction (≥ 1 inter step) to **AG**	A1 (**3**)	WWW
	(ii)	Attempt partial fractions	M1	
		$\frac{2}{u} - \frac{2}{1+u}$	A1	
		√ $A \ln u + B \ln(1+u)$	√A1	Based on $\frac{A}{u} + \frac{B}{1+u}$
		Attempt integ, change limits & use on f(u)	M1	or re-subst & use 1 & 9
		$\ln \frac{9}{4}$ AEexactF (e.g. 2 ln 3 –2 ln 4 + 2 ln 2)	A1 (**5**)	Not involving ln 1
			8	

6	**(i)**	Solve $0 = t-3$ & subst into $x = t^2 - 6t + 4$	M1	
		Obtain $x = -5$	A1 (**2**)	(–5,0) need not be quoted
		N.B. If **(ii)** completed first, subst $y = 0$ into their cartesian eqn (M1) & find x (no f.t.) (A1)		
	(ii)	Attempt to eliminate t	M1	
		Simplify to $x = y^2 - 5$ ISW	A1 (**2**)	
	(iii)	Attempt to find $\frac{dy}{dx}$ or $\frac{dx}{dy}$ from cartes or para form	M1	Award anywhere in Que
		Obtain $\frac{dy}{dx} = \frac{1}{2t-6}$ or $\frac{1}{2y}$ or (-)$\frac{1}{2}(x+5)^{-\frac{1}{2}}$	A1	
		If t = 2, $x = -4$ and $y = -1$	B1	Awarded anywhere in **(iii)**
		Using their num (x, y) & their num $\frac{dy}{dx}$, find tgt eqn	M1	
		$x + 2y + 6 = 0$ AEF(without fractions) ISW	A1 (**5**)	
			9	
7	**(i)**	Attempt direction vector between the 2 given points	M1	
		State eqn of line using format (**r**) = (either end) + s(dir vec)	M1	's' can be 't'
		Produce 2/3 eqns containing t and s	M1	2 different parameters
		Solve giving $t = 3$, $s = -2$ or 2 or -1 or 1	A1	
		Show consistency	B1	
		Point of intersection = $(5,9,-1)$	A1 (**6**)	
	(ii)	Correct method for scalar product of 'any' 2 vectors	M1	Vectors from this question
		Correct method for magnitude of 'any' vector	M1	Vector from this question
		Use $\cos\theta = \frac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for the correct 2 vectors $\begin{pmatrix}1\\4\\-2\end{pmatrix}$ & $\begin{pmatrix}2\\-1\\3\end{pmatrix}$	M1	Vects may be mults of dvs
		62.2 (62.188157…) 1.09 (1.0853881)	A1 (**4**)	
			10	

8	**(i)**	$\frac{\mathrm{d}}{\mathrm{d}x}\left(y^3\right)=3y^2\frac{\mathrm{d}y}{\mathrm{d}x}$	B1	
		Consider $\frac{\mathrm{d}}{\mathrm{d}x}(xy)$ as a product	M1	
		$= \quad x\frac{\mathrm{d}y}{\mathrm{d}x}+y$	A1	Tolerate omission of '6'
		$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{6y-3x^2}{3y^2-6x}$ ISW AEF	A1 (**4**)	
	(ii)	$x^3=2^4$ or 16 and $y^3=2^5$ or 32	*B1	
		Satisfactory conclusion	dep* B1	**AG**
		Substitute $\left(2^{\frac{4}{3}},2^{\frac{5}{3}}\right)$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	or the numerator of $\frac{\mathrm{d}y}{\mathrm{d}x}$
		Show or use calc to demo that num = 0, ignore denom **AG**	A1 (**4**)	
	(iii)	Substitute (a,a) into eqn of curve	M1	& attempt to state 'a = …'
		$a=3$ only with clear ref to $a\neq 0$	A1	
		Substitute $(3,3)$ or (their a, their a) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$	M1	
		-1 only WWW	A1 (**4**)	from (their a,their a)
			12	
9	**(i)**	$\frac{\mathrm{d}\theta}{\mathrm{d}t}=\ldots$	B1	
		$k(160-\theta)$	B1 (**2**)	The 2 @ 'B1' are indep
	(ii)	Separate variables with $(160-\theta)$ in denom; or invert	*M1	$\int\frac{1}{160-\theta}\,\mathrm{d}\theta=\int k,\frac{1}{k},1\,\mathrm{d}t$
		Indication that LHS = $\ln \mathrm{f}(\theta)$	A1	If wrong ln, final 3@A = 0
		RHS = kt or $\frac{1}{k}t$ or t (+ c)	A1	
		Subst. $t=0,\theta=20$ into equation containing 'c'	dep* M1	
		Subst $t=5,\theta=65$ into equation containing 'c' & 'k'	dep* M1	
		$c=-\ln 140$ (-4.94) ISW	A1	
		$k=\frac{1}{5}\ln\frac{140}{95}$ (≈ 0.077 or 0.078) ISW	A1	
		Using their 'c' & 'k', subst t = 10 & evaluate θ	dep*M1	
		$\theta=96(95.535714)$ $\left(95\frac{15}{28}\right)$	A1 (**9**)	
			11	