

**Mark Scheme 4724
January 2006**

1	Attempt to factorise numerator and denominator num = $xx(x-3)$ <u>or</u> denom = $(x-3)(x+3)$ <u>Final</u> answer = $\frac{x^2}{x+3}$ [Not $\frac{xx}{x+3}$]	M1 A1 A1	Not num = $x(x^2 - 3x)$ 3 Do not ignore further cancellation.
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2	$\frac{d}{dx}(\sin y) = \cos y \cdot \frac{dy}{dx}$ $\frac{d}{dx}(xy) = x \frac{dy}{dx} + y$ s.o.i. $\cos y \cdot \frac{dy}{dx} = x \frac{dy}{dx} + y + 2x$ AEF [If written as $\frac{dy}{dx} = \cos y \frac{dy}{dx} = x \frac{dy}{dx} + y + 2x$, accept for prev B1 but not for following marks if the $\frac{dy}{dx}$ is used] $f(x, y) \frac{dy}{dx} = g(x, y)$ $\frac{y+2x}{\cos y - x}$ or $-\frac{y+2x}{x - \cos y}$ or $\frac{-2x-y}{x - \cos y}$	B1 B1 B1 M1 A1	[SR: If xy taken to LHS, accept $-x \frac{dy}{dx} + y$ as s.o.i.] Regrouping provided > one $\frac{dy}{dx}$ term 5 ISW Answer could imply M1
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3	(i) Quotient = $3x + \dots$ For evidence of correct division process $3x + 4$ $-6x - 13$	B1 M1 A1 A1	For correct leading term in quotient Or for cubic $\equiv (x^2 - 2x + 5)(gx + h) (+ \dots)$ For correct quotient 4 For correct remainder ISW
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	(ii) $a = 7$ $b = 20$ [SR: If B0+B0, award B1√ for $a = 1 + P$ AND $b = 7 + Q$; also SR B1 for $a = 20, b = 7$]	B1√ B1√	<u>Follow through</u> If rem in (i) is $Px + Q$, then B1√ for $a = 1 - P$ 2 and B1√ for $b = 7 - Q$
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4	(i) Parts using correct split of $u = x, \frac{dv}{dx} = \sec^2 x$ $x \tan x - \int \tan x dx$ $\int \tan x dx = -\ln \cos x$ or $\ln \sec x$ $x \tan x + \ln \cos x + c$ or $x \tan x - \ln \sec x + c$	M1 A1 B1 A1	1st stage result of form $f(x) + /- \int g(x) dx$ Correct 1 st stage 4
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	(ii) $\tan^2 x = + /- \sec^2 x + /- 1$ $\int x \sec^2 x dx - \int x dx$ s.o.i. $x \tan x + \ln \cos x - \frac{1}{2} x^2 + c$	M1 A1 A1√	or $\sec^2 x = + /- 1 + /- \tan^2 x$ Correct 1 st stage 3 f.t. their answer to part (i) $-\frac{1}{2} x^2$

5	(i)	$\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$	M1	Used, not just quoted
		$\frac{1}{t}$ or t^{-1}	A1	2 Not $\frac{2}{2t}$ as final answer
SR: M1 for Cart conv, finding $\frac{dy}{dx}$ & ans involv t + A1			M1	M1 is attempt only, accuracy not involved

	(ii)	Finding equation of tangent (using p or t)	M1	
		$py = x + p^2$ working	A1	2 AG; p essential; at least 1 line inter

	(iii)	$(25, -10) \Rightarrow p = -5$ or $-5y = x + 25$ seen	B1	$5y = x + 25$ seen \Rightarrow B0
		Substitution of their values of p into given tgt eqn	M1	Producing 2 equations
		Solving the 2 equations simultaneously	M1	
		$(-15, -2)$ $x = -15, y = -2$	A1	4 Common wrong ans $(15, 8) \Rightarrow$ B0, M2, A0

6	(i)	Attempt to connect $dx, d\theta$	M1	But not $dx = d\theta$
		$dx = 2 \sin \theta \cos \theta d\theta$	A1	AEF
		$\sqrt{\frac{x}{1-x}} = \frac{\sin \theta}{\cos \theta}$	B1	Ignore any references to \pm .
		Reduction to $\int 2 \sin^2 \theta d\theta$	A1	4 AG WWW

	(ii)	$\sin^2 \theta = k(+/-1 +/- \cos 2\theta)$	M1	Attempt to change $(2) \sin^2 \theta$ into $f(\cos 2\theta)$
		$2 \sin^2 \theta = 1 - \cos 2\theta$	A1	Correct attempt
		$\int \cos 2\theta d\theta = \frac{1}{2} \sin 2\theta$	B1	Seen anywhere in this part
		Attempting to change limits	M1	Or Attempting to resubstitute; Accept degrees
		$\frac{1}{2} \pi$	A1	5
		Alternatively Parts once & use $\cos^2 \theta = 1 - \sin^2 \theta$ $\frac{1}{2}(\theta - \sin \theta \cos \theta)$	(M2) (A1)	Instead of the M1 A1 B1 Then the final M1 A1 for use of limits

7	(i)	$A = 3$	B1	For correct value stated
		$C = 1$	B1	For correct value stated
		$11 + 8x \equiv A(1+x)^2 + B(2-x)(1+x) + C(2-x)$ e.g. $A - B = 0, 2A + B - C = 8, A + 2B + 2C = 11$	M1 A1	AEF; any suitable identity For any correct (f.t.) equation involving B
		$B = 3$	A1	5
	(ii)	$(1 - \frac{x}{2})^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$	B1	s.o.i.
		$(1+x)^{-1} = 1 - x + x^2 - \dots$	B1	s.o.i.
		$(1+x)^{-2} = 1 - 2x + 3x^2 - \dots$	B1, B1	s.o.i.
		Expansion = $\frac{11}{2} - \frac{17}{4}x + \frac{51}{8}x^2 + \dots$	B1	5 CAO. No f.t. for wrong A and/or B and/or C

SR(1) If partial fractions not used but product of **SR(2)** If partial fractions not used
 but $(11+8x)(2-x)^{-1}(1+x)^{-2}$ attempted, then denominator multiplied out, then
 B1 for $(1-\frac{x}{2})^{-1} = 1 + \frac{x}{2} + \frac{x^2}{4} + \dots$ B1 for denom = $2 + 3x(+0x^2) + \dots$
 B1,B1 for $(1+x)^{-2} = 1 - 2x + \dots + 3x^2 + \dots$ B1 for $(1+\frac{3x}{2})^{-1} = 1 - \frac{3x}{2} + \frac{9x^2}{4} + \dots$
 B1,B1 for $\frac{11}{2} - \frac{17}{4}x + \dots + \frac{51}{8}x^2 + \dots$ B1,B1,B1 for $\frac{11}{2} \dots - \frac{17}{4}x \dots + \frac{51}{8}x^2 + \dots$

N.B. In both SR, if final expansion given B0, -----allow SR B1 for $22 - 17x + 51/2 x^2$

8 (i) $\int (y-3)dy = \int (2-x)dx$ or equiv M1 For separation & integration of both sides
 $\frac{1}{2}y^2 - 3y = 2x - \frac{1}{2}x^2$ A1 or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2$
 For an arbitrary const on one/both sides *B1 } (or + M2 for equiv statement using limits)
 Substituting $(x, y) = (5, 4)$ or $(4, 5)$ & finding 'c' dep*M1 }
 $\frac{1}{2}y^2 - 3y = -\frac{1}{2}x^2 + 2x - \frac{3}{2}$ AEF ISW A1 **5** or $\frac{1}{2}(y-3)^2 = -\frac{1}{2}(x-2)^2 + 5$ AEF

(ii) Attempt to clear fracs (if nec) & compl square M1
 $a = 2, b = 3, k = 10$ A2 **3** For all 3; SR: A1 for 1 or 2 correct

(iii) Circle clearly indicated in a sketch B1
 Centre $(2, 3)$ or their (a, b) B1√
 Radius $\sqrt{10}$ or their \sqrt{k} B1√ **3** √ provided $k > 0$

9 (i) Using $\begin{pmatrix} -8 \\ 1 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} -9 \\ 2 \\ -5 \end{pmatrix}$ as the relevant vectors M1 i.e. correct direction vectors
 Using $\cos \theta = \frac{a \cdot b}{|a||b|}$ AEF for any 2 vectors M1 Accept $\cos \theta = \frac{|a \cdot b|}{|a||b|}$
 Method for scalar product of any 2 vectors M1
 Method for finding magnitude of any vector M1
 $15^\circ (15.38\dots), 0.268 \text{ rad}$ A1 **5**

(ii) Produce (at least) 2 of the 3 eqns in t and s M1 e.g. $4 - 8t = -2 - 9s,$
 $-6 - 2t = -2 - 5s$
 Solve the (x) and (z) equations M1
 $t = 3$ or $s = 2$ A1 for first value found
 $s = 2$ or $t = 3$ f.t. A1√ for second value found
 Substituting their (t, s) into (y) equation M1
 $a = 1$ A1
 Substituting their t into L_1 or their (s, a)

into l_2

$$\begin{pmatrix} -20 \\ 5 \\ -12 \end{pmatrix}$$

M1

A1

8 Any format but not $\begin{pmatrix} \\ \\ \end{pmatrix} + \begin{pmatrix} \\ \\ \end{pmatrix}$