A-level

FURTHER MATHEMATICS
7367/3S

Paper 3 Statistics

Mark scheme

June 2021

Version: 1.0 Final Mark Scheme

2 1 6 A 7 3 6 7 / 3 S / M S

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
A	mark is dependent on M marks and is for accuracy
B	mark is independent of M marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

AS/A-level Maths/Further Maths assessment objectives

AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	1.1b	B1	0.4
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Circles correct answer	1.2	B1	$a^2\text{Var}(X)$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Models the situation with a discrete random variable and an unknown to represent the relationships between probabilities and uses total probability = 1 to form an equation in terms of that unknown Condone slips in representing the relationships between probabilities.	3.3	M1	<table><tr><td>x</td><td>10</td><td>20</td><td>30</td></tr><tr><td>$P(X = x)$</td><td>$4a$</td><td>$2a$</td><td>a</td></tr></table> $4a + 2a + a = 1$ $7a = 1$ $a = \frac{1}{7}$
	Uses their model and the formula for the mean of a discrete random variable	3.4	M1	$E(X) = 10 \times 4 \times \frac{1}{7} + 20 \times 2 \times \frac{1}{7} + 30 \times \frac{1}{7}$
	Obtains correct mean points AWRT 15.71	3.2a	A1	Mean = 15.71 points
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
3(b)	Uses the formula $E(AX + B) = AE(X) + B$ Condone sign error or adjusts their discrete random variable and attempts to calculate the mean	3.1b	M1	$E(5X - 10) = 5E(X) - 10$ $= 5 \times 15.71 - 10$ = 69 pence
	Obtains correct expected value Follow through their mean points from part **(a)** AWRT 69	3.2a	A1F	
	Total		**2**	

	Question total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Obtains correct z value AWRT 1.96 PI by correct value of standard deviation	1.1b	B1	$z = 1.96$ $0.4 = 1.96 \times \frac{s}{\sqrt{150}}$ $s = 2.50$ °C
	Uses formula for the full or half width of a confidence interval using their z-value to obtain an equation and attempts to solve, obtaining a value for standard deviation PI	3.1a	M1	
	Obtains the correct value of standard deviation AWRT 2.50 Accept 2.5	1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(b)	Deduces the correct upper limit of the confidence interval	2.2a	B1	25.3 + 0.4 = 25.7 26 > 25.7 So the confidence interval does not support Oscar's claim
	Infers the confidence interval does not support Oscar's claim as 26 is not in the confidence interval FT their upper limit of the confidence interval	2.2b	E1F	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(c)	Explains how Oscar could refine the model in order to reduce the width of his confidence interval	3.5c	B1	Oscar can take a larger random sample
	Total		**1**	

	Question total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Differentiates to find either $\frac{1}{10}$ or $\frac{1}{45}x$ or equivalent	1.1a	M1	$f(x) = \begin{cases} \frac{1}{10} & 1 < x \le 6 \\ \frac{1}{45}x & 6 < x \le 9 \\ 0 & \text{otherwise} \end{cases}$
	Obtains fully defined $f(x)$	1.1b	A1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(b)	Uses integrals of $xf(x)$ to find E(X)	1.1a	M1	E(X) = $\int_1^6 \frac{1}{10}x\,dx + \int_6^9 \frac{1}{45}x^2\,dx$ = 5.55
	Uses integrals of $x^2f(x)$ to find E(X^2)	1.1a	M1	E(X^2) = $\int_1^6 \frac{1}{10}x^2\,dx + \int_6^9 \frac{1}{45}x^3\,dx$
	Obtains correct value of E(X) or the correct value of E(X^2) Accept AWRT 36.4 for E(X^2) PI	1.1b	A1	$= \frac{437}{12}$ Var (X) = $\frac{437}{12} - (5.55)^2$
	Completes rigorous argument to find the given value of Var(X) using Var(X) = E(X^2) – (E(X))^2	2.1	R1	$= \frac{6737}{1200}$
	Total		**4**	

	Question total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	States both hypotheses using correct language (oe) Variables need to be stated in at least the null hypothesis	2.5	B1	H_0: There is no association between town and number of speeding offences per year H_1: There is an association between town and number of speeding offences per year
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)(i)	Explains that model will need to be refined by merging the column (or row) that the expected frequency less than 5 is in with another suitable column (or row)	3.5c	E1	The column that the expected frequency less than 5 is in will need to be merged with another column for both the expected frequencies and the observed frequencies.
	Explains that model will need to be refined for both the observed and the expected frequencies	3.5c	E1	
	Total		**2**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(b)(ii)	States the correct test statistic	1.2	B1	$\sum \frac{(O-E)^2}{E}$
	Total		**1**	

Q	Marking Instructions	AO	Marks	Typical Solution
6(c)	Evaluates χ^2-test statistic by comparing the correct critical value with the test statistic	3.5a	R1	χ^2 cv for 25 df = 44.314 45.22 > 44.314 Reject H_0 Some evidence to suggest that there is an association between town and number of speeding offences per year
	Infers H_0 rejected	2.2b	E1	
	Concludes in context (The conclusion must not be definite.) FT their incorrect acceptance of H_0 if stated or 'their' comparison if not	3.2a	E1F	
	Total		**3**	

	Question total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(a)	Uses $\lim\limits_{k\to\infty}\int_0^k \lambda x e^{-\lambda x}\ \mathrm{d}x$ to find E(X) Condone use of $\int_0^\infty \lambda x e^{-\lambda x}\ \mathrm{d}x$ Condone use of –∞ instead of 0 or missing limits Condone missing $\mathrm{d}x$ Condone sign error on power	1.1a	M1	E(X) = $\lim\limits_{k\to\infty}\int_0^k \lambda x e^{-\lambda x}\ \mathrm{d}x$ = $\lim\limits_{k\to\infty}\left[-xe^{-\lambda x}\right]_0^k + \lim\limits_{k\to\infty}\int_0^k e^{-\lambda x}\ \mathrm{d}x$ = $\lim\limits_{k\to\infty}\int_0^k e^{-\lambda x}\ \mathrm{d}x$ = $\lim\limits_{k\to\infty}\left[-\frac{1}{\lambda}e^{-\lambda x}\right]_0^k$ = $0-\left(-\frac{1}{\lambda}\right)$ = $\frac{1}{\lambda}$
	Uses integration by parts to simplify the integral Condone missing $\mathrm{d}x$ FT integration of $\lambda x e^{\lambda x}$	1.1b	A1F	
	Integrates $\int e^{-\lambda x}\ \mathrm{d}x$	1.1a	M1	
	Completes rigorous argument to find the correct expression for E(X) Condone absence of limiting process throughout	2.1	R1	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
7(b)	Uses $\lim_{k\to\infty}\int_0^k \lambda x^2 e^{-\lambda x}\,dx$ to find E(X^2) Condone use of $\int_0^\infty \lambda x^2 e^{-\lambda x}\,dx$ Condone use of $-\infty$ instead of 0 or missing limits Condone missing dx Condone sign error on power	1.1a	M1	E(X^2) = $\lim_{k\to\infty}\int_0^k \lambda x^2 e^{-\lambda x}\,dx$
	Uses integration by parts once to simplify the integral Condone missing dx FT integration of $\lambda x^2 e^{\lambda x}$	1.1b	A1F	$= \lim_{k\to\infty}\left[-x^2 e^{-\lambda x}\right]_0^k + \lim_{k\to\infty}\int_0^k 2xe^{-\lambda x}\,dx$ $= \lim_{k\to\infty}\int_0^k 2xe^{-\lambda x}\,dx$
	Uses integration by parts twice to simplify the integral Condone missing dx FT integration of $\lambda x^2 e^{\lambda x}$	1.1b	A1F	$= \lim_{k\to\infty}\left[-\frac{2}{\lambda}xe^{-\lambda x}\right]_0^k + \lim_{k\to\infty}\int_0^k \frac{2}{\lambda}e^{-\lambda x}\,dx$ $= \lim_{k\to\infty}\int_0^k \frac{2}{\lambda}e^{-\lambda x}\,dx$
	Integrates $\int_0^\infty \frac{2}{\lambda}e^{-\lambda x}\,dx$	1.1a	M1	$= \lim_{k\to\infty}\left[-\frac{2}{\lambda^2}e^{-\lambda x}\right]_0^k$ $= 0 - \left(-\frac{2}{\lambda^2}\right)$
	Obtains the correct expression for E(X^2)	1.1b	A1	$= \frac{2}{\lambda^2}$ Var(X) = $\frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$
	Substitutes into the formula Var(X) = E(X^2) – (E(X))^2	1.1a	M1	$= \frac{1}{\lambda^2}$
	Completes rigorous argument to find the correct expression for Var(X) Condone absence of limiting process throughout	2.1	R1	
	Total		**7**	

	Question total		**11**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Obtains correct sample mean	1.1b	B1	$\bar{x} = \frac{102}{60} = 1.7$ $s^2 = \frac{103.25}{59} = 1.75$ It is appropriate to model the number of complaints in a day with a Poisson distribution with mean 1.7 as the sample mean is approximately equal to the sample variance
	Obtains correct sample variance	1.1b	B1	
	Explains that it is appropriate to model the number of complaints in a day with a Poisson distribution with mean 1.7 as the sample mean is approximately equal to the sample variance Condone use of σ^2 but values of both must be seen	2.4	E1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(b)	States both hypotheses using correct language with λ = 6.9, 1.7 or 5.2	2.5	B1	H_0: $\lambda = 6.9$ H_1: $\lambda \neq 6.9$ X~Po(6.9) P($X \leq 3$) = 0.087 0.087 > 0.05 Accept H_0 No evidence to suggest that the mean total number of complaints and enquiries received per day has changed
	Uses Poisson model with λ = 6.9	3.3	M1	
	Uses model to obtain correct value of P($X \leq 3$) AWRT 0.087 PI by comparing subset of P($X \leq 3$) that is greater than 0.05 with 0.05	3.4	A1	
	Evaluates the Poisson model by comparing the probability with 0.05	3.5a	R1	
	Infers H_0 not rejected	2.2b	E1	
	Concludes in context (The conclusion must not be definite.) FT their incorrect rejection of H_0 if stated or 'their' comparison if not	3.2a	E1F	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(c)	Finds the correct value of $P(X \le 2)$ or $P(X \ge 12)$ PI	1.1a	M1	$P(X \le 2) = 0.031$ $P(X \ge 12) = 0.049$
	Identifies the correct critical region (or acceptance region)	1.1b	A1	Reject hypothesis test if $X \le 2$ or $X \ge 12$ $Y \sim Po(6.1)$
	Uses Poisson model with $\lambda = 6.1$ to find a probability	1.1a	M1	Power of test = $P(Y \le 2) + P(Y \ge 12)$ $= 0.05765\ldots + 0.02244\ldots$
	Obtains the correct power of the test AWRT 0.0801	1.1b	A1	$= 0.0801$
	Total		**4**	

	Question total		**13**	

	Paper total		**50**	