Mark Scheme 4723
June 2006

1		Differentiate to obtain $k(4x+1)^{-\frac{1}{2}}$	**M1**		any non-zero constant *k*
		Obtain $2(4x+1)^{-\frac{1}{2}}$	**A1**		or equiv, perhaps unsimplified
		Obtain $\frac{2}{3}$ for value of first derivative	**A1**		or unsimplified equiv
		Attempt equation of tangent through (2, 3)	**M1**		using numerical value of first derivative provided derivative is of form $k'(4x+1)^n$
		Obtain $y = \frac{2}{3}x + \frac{5}{3}$ or $2x - 3y + 5 = 0$	**A1**	**5**	or equiv involving 3 terms
2		<u>Either</u>: Attempt to square both sides	**M1**		producing 3 terms on each side
		Obtain $3x^2 - 14x + 8 = 0$	**A1**		or inequality involving < or >
		Obtain correct values $\frac{2}{3}$ and 4	**A1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**5**	or correctly expressed equiv; allow ≤ signs
		<u>Or</u>: Attempt solution of two linear equations or inequalities	**M1**		one eqn with signs of 2*x* and *x* the same, second eqn with signs different
		Obtain value $\frac{2}{3}$	**A1**		
		Obtain value 4	**B1**		
		Attempt valid method for solving inequality	**M1**		implied by correct answer or plausible incorrect answer
		Obtain $\frac{2}{3} < x < 4$	**A1**	**(5)**	or correctly expressed equiv; allow ≤ signs
3	**(i)**	Attempt evaluation of cubic expression at 2 and 3	**M1**		
		Obtain –11 and 31	**A1**		
		Conclude by noting change of sign	**A1√**	**3**	or equiv; following any calculated values provided negative then positive
	(ii)	Obtain correct first iterate	**B1**		using x_1 value such that $2 \le x_1 \le 3$
		Attempt correct process to obtain at least 3 iterates	**M1**		using any starting value now
		Obtain 2.34	**A1**	**3**	answer required to 2 d.p. exactly; 2→2.3811→2.3354→2.3410; 2.5→2.3208→2.3428→2.3401; 3→2.2572→2.3505→2.3392

4	**(i)** State $\ln y = (x-1)\ln 5$	**B1**		whether following $\ln y = \ln 5^{x-1}$ or not; brackets needed
	Obtain $x = 1 + \frac{\ln y}{\ln 5}$	**B1**	**2**	**AG**; correct working needed; missing brackets maybe now implied
	(ii) Differentiate to obtain single term of form $\frac{k}{y}$	**M1**		any constant k
	Obtain $\frac{1}{y \ln 5}$	**A1**	**2**	or equiv involving y
	(iii) Substitute for y and attempt reciprocal	**M1**		or equiv method for finding derivative without using part **(ii)**
	Obtain $25 \ln 5$	**A1**	**2**	or exact equiv

5	**(i)** State $\sin 2\theta = 2 \sin\theta \cos\theta$	**B1**	**1**	or equiv; any letter acceptable here (and in parts **(ii)** and **(iii)**)
	(ii) Attempt to find exact value of $\cos \alpha$	**M1**		using identity attempt or right-angled triangle
	Obtain $\frac{1}{4}\sqrt{15}$	**A1**		or exact equiv
	Substitute to confirm $\frac{1}{8}\sqrt{15}$	**A1**	**3**	**AG**
	(iii) State or imply $\sec \beta = \frac{1}{\cos \beta}$	**B1**		
	Use identity to produce equation involving $\sin \beta$	**M1**		
	Obtain $\sin \beta = 0.3$ and hence 17.5	**A1**	**3**	and no other values between 0 and 90; allow 17.4 or value rounding to 17.4 or 17.5

6	**(i)** <u>Either</u>: Obtain f(–3) = –7	**B1**		maybe implied
	Show correct process for compn of functions	**M1**		
	Obtain –47	**A1**	**3**	
	<u>Or</u>: Show correct process for compn of functions	**M1**		using algebraic approach
	Obtain $2-(2-x^2)^2$	**A1**		or equiv
	Obtain –47	**A1**	**(3)**	
	(ii) Attempt correct process for finding inverse	**M1**		as far as $x = \ldots$ or equiv
	Obtain either one of $x = \pm\sqrt{2-y}$ or both	**A1**		or equiv perhaps involving x
	Obtain correct $-\sqrt{2-x}$	**A1**	**3**	or equiv; in terms of x now
	(iii) Draw graph showing attempt at reflection in $y = x$	**M1**		
	Draw (more or less) correct graph	**A1**		with end-point on x-axis and no minimum point in third quadrant
	Indicate coordinates 2 and $-\sqrt{2}$	**A1**	**3**	accept –1.4 in place of $-\sqrt{2}$
7	**(a)** Obtain integral of form $k(4x-1)^{-1}$	**M1**		any non-zero constant k

	Obtain $-\frac{1}{2}(4x-1)^{-1}$	**A1**		or equiv; allow + *c*
	Substitute limits and attempt evaluation	**M1**		for any expression of form $k'(4x-1)^n$
	Obtain $\frac{2}{21}$	**A1**	**4**	or exact equiv
(b)	Integrate to obtain ln *x*	**B1**		
	Substitute limits to obtain ln 2*a* – ln *a*	**B1**		
	Subtract integral attempt from attempt at area of appropriate rectangle	**M1**		or equiv
	Obtain 1 – (ln 2*a* – ln *a*)	**A1**		or equiv
	Show at least one relevant logarithm property	**M1**		at any stage of solution
	Obtain 1 – ln 2 and hence $\ln(\frac{1}{2}e)$	**A1**	**6**	**AG**; full detail required

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8 (i)	State *R* = 13	**B1**		or equiv
	State at least one equation of form *R* cos α = *k*, *R* sin α = *k*′, tan *α* = k''	**M1**		or equiv; allow sin / cos muddles; implied by correct α
	Obtain 67.4	**A1**	**3**	allow 67 or greater accuracy
(ii)	Refer to translation and stretch	**M1**		in either order; allow here equiv terms such as ‘move’, ‘shift’; with both transformations involving constants
	State translation in positive *x* direction by 67.4	**A1**√		or equiv; following their α; using correct terminology now
	State stretch in *y* direction by factor 13	**A1**√	**3**	or equiv; following their *R*; using correct terminology now
(iii)	Attempt value of $\cos^{-1}(2 \div R)$	**M1**		
	Obtain 81.15	**A1**√		following their *R*; accept 81
	Obtain 148.5 as one solution	**A1**		accept 148.5 or 148.6 or value rounding to either of these
	Add their α value to second value correctly attempted	**M1**		
	Obtain 346.2	**A1**	**5**	accept 346.2 or 346.3 or value rounding to either of these; and no other solutions

9	**(i)**	Attempt to express *x* in terms of *y*	***M1**		obtaining two terms
		Obtain $x = e^{\frac{1}{2}y} + 1$	**A1**		or equiv
		State or imply volume involves $\int \pi x^2$	**B1**		
		Attempt to express x^2 in terms of *y*	***M1**		dep ***M**; expanding to produce at least 3 terms
		Obtain $k\int(e^y + 2e^{\frac{1}{2}y} + 1)\,dy$	**A1**		any constant *k* including 1; allow if d*y* absent
		Integrate to obtain $k(e^y + 4e^{\frac{1}{2}y} + y)$	**A1**		
		Use limits 0 and *p*	**M1**		dep ***M *M**; evidence of use of 0 needed
		Obtain $\pi(e^p + 4e^{\frac{1}{2}p} + p - 5)$	**A1**	**8**	**AG**; necessary detail required
	(ii)	State or imply $\frac{dp}{dt} = 0.2$	**B1**		maybe implied by use of 0.2 in product
		Obtain $\pi(e^p + 2e^{\frac{1}{2}p} + 1)$ as derivative of *V*	**B1**		
		Attempt multiplication of values or expressions for $\frac{dp}{dt}$ and $\frac{dV}{dp}$	**M1**		
		Obtain $0.2\pi(e^4 + 2e^2 + 1)$	**A1√**		following their $\frac{dV}{dp}$ expression
		Obtain 44	**A1**	**5**	or greater accuracy