

# Monday 4 October 2021 – Afternoon

## A Level Further Mathematics A

**Y540/01** Pure Core 1

### Time allowed: 1 hour 30 minutes



#### You must have:

- the Printed Answer Booklet
- the Formulae Booklet for A Level Further
- Mathematics A
- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the Printed Answer Booklet. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . When a numerical value is needed use g = 9.8 unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

### INFORMATION

- The total mark for this paper is 75.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

### ADVICE

• Read each question carefully before you start your answer.

#### Answer all the questions.

- 1 (a) Sketch on a single Argand diagram the loci given by
  - (i) |z-1+2i| = 3, [2]

(ii) 
$$|z+1| = |z-2|$$
. [2]

(b) Indicate, by shading, the region of the Argand diagram for which  $|z-1+2i| \le 3$  and  $|z+1| \le |z-2|$ . [2]

### 2 You are given that $f(x) = \tan^{-1}(1+x)$ .

- (a) (i) Find the value of f(0). [1]
  - (ii) Determine the value of f'(0). [2]

(iii) Show that 
$$f''(0) = -\frac{1}{2}$$
. [3]

(b) Hence find the Maclaurin series for f(x) up to and including the term in  $x^2$ . [2]

3 A function f(z) is defined on all complex numbers z by  $f(z) = z^3 - 3z^2 + kz - 5$  where k is a real constant. The roots of the equation f(z) = 0 are  $\alpha$ ,  $\beta$  and  $\gamma$ . You are given that  $\alpha^2 + \beta^2 + \gamma^2 = -5$ .

- (a) Explain why f(z) = 0 has only one real root. [3]
  - (b) Find the value of k.
  - (c) Find a cubic equation with integer coefficients that has roots  $\frac{1}{\alpha}$ ,  $\frac{1}{\beta}$  and  $\frac{1}{\gamma}$ . [2]

[3]

### 4 Points A, B and C have coordinates (4, 2, 0), (1, 5, 3) and (1, 4, -2) respectively. The line l passes through A and B.

(a) Find a cartesian equation for *l*.[3]

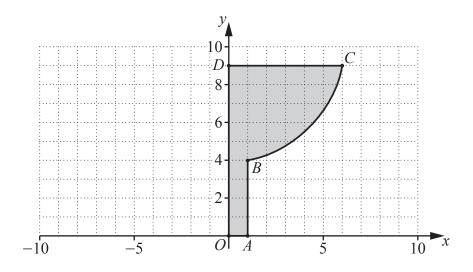
*M* is the point on *l* that is closest to *C*.

- (b) Find the coordinates of M. [4]
- (c) Find the exact area of the triangle ABC. [4]

- 5 Use de Moivre's theorem to find the constants A, B and C in the identity  $\sin^5 \theta \equiv A \sin \theta + B \sin 3\theta + C \sin 5\theta$ .
- 6 *O* is the origin of a coordinate system whose units are cm. The points *A*, *B*, *C* and *D* have coordinates (1, 0), (1, 4), (6, 9) and (0, 9) respectively. The arc *BC* is part of the curve with equation  $x^2 + (y-10)^2 = 37$ .

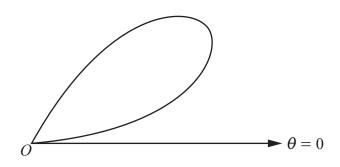
The closed shape OABCD is formed, in turn, from the line segments OA and AB, the arc BC and the line segments CD and DO (see diagram).

A funnel can be modelled by rotating *OABCD* by  $2\pi$  radians about the *y*-axis.

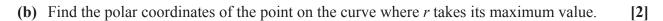


Find the volume of the funnel according to the model.

7 The diagram below shows the curve with polar equation  $r = \sin 3\theta$  for  $0 \le \theta \le \frac{1}{3}\pi$ .



(a) Find the values of  $\theta$  at the pole.



### (c) In this question you must show detailed reasoning.

Find the exact area enclosed by the curve. [4]

(d) Given that  $\sin 3\theta = 3\sin \theta - 4\sin^3 \theta$ , find a cartesian equation for the curve. [2]

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Turn over

[1]

[3]

[4]

- 8 You are given that  $f(x) = 4 \sinh x + 3 \cosh x$ .
  - (a) Show that the curve y = f(x) has no turning points. [3]
  - (b) Determine the exact solution of the equation f(x) = 5. [5]
- 9 You are given that the matrix  $\begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix}$  represents a transformation T.

(a) You are given that the line with equation y = kx is invariant under T.Determine the value of k.

(b) Determine whether the line with equation y = kx in part (a) is a line of invariant points under T. [1]

[4]

10 Using an algebraic method, determine the least value of *n* for which  $\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} \ge 0.49$ . [8]

11 The displacement of a door from its equilibrium (closed) position is measured by the angle,  $\theta$  radians, which the door makes with its closed position. The door can swing either side of the equilibrium position so that  $\theta$  can take positive and negative values. The door is released from rest from an open position at time t = 0.

A proposed differential equation to model the motion of the door for  $t \ge 0$  is

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \lambda \frac{\mathrm{d}\theta}{\mathrm{d}t} + 3\theta = 0 \text{ where } \lambda \text{ is a constant and } \lambda \ge 0.$$

- (a) (i) According to the model, for what value of  $\lambda$  will the motion of the door be simple harmonic? [1]
  - (ii) Explain briefly why modelling the motion of the door as simple harmonic is unlikely to be realistic. [1]
- (b) Find the range of values of  $\lambda$  for which the model predicts that the door will never pass through the equilibrium position. [2]
- (c) Sketch a possible graph of  $\theta$  against t when  $\lambda$  lies **outside** the range found in part (b) but the motion is not simple harmonic. [1]

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