GCE

Further Mathematics B MEI

Y436/01: Further pure with technology

A Level

Mark Scheme for June 2022

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✘	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
E	Explanation mark 1
SC	Special case
^	Omission sign
MR	Misread
BP	Blank page
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *. The * may be omitted if only previous M mark.
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for AS/A Level Further Mathematics B (MEI)

a   Annotations must be used during your marking. For a response awarded zero (or full) marks a single appropriate annotation (cross, tick, M0 or ^) is sufficient, but not required.

For responses that are not awarded either 0 or full marks, you must make it clear how you have arrived at the mark you have awarded and all responses must have enough annotation for a reviewer to decide if the mark awarded is correct without having to mark it independently.

It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

Award NR (No Response)
if there is nothing written at all in the answer space and no attempt elsewhere in the script
OR if there is a comment which does not in any way relate to the question (e.g. 'can't do', 'don't know')
OR if there is a mark (e.g. a dash, a question mark, a picture) which isn't an attempt at the question.
Note: Award 0 marks only for an attempt that earns no credit (including copying out the question).

    If a candidate uses the answer space for one question to answer another, for example using the space for 8(b) to answer 8(a), then give benefit of doubt unless it is ambiguous for which part it is intended.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not always be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c   The following types of marks are available.

M

A suitable method has been selected and applied in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A method mark may usually be implied by a correct answer unless the question includes the DR statement, the command words "Determine" or "Show that", or some other indication that the method must be given explicitly.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

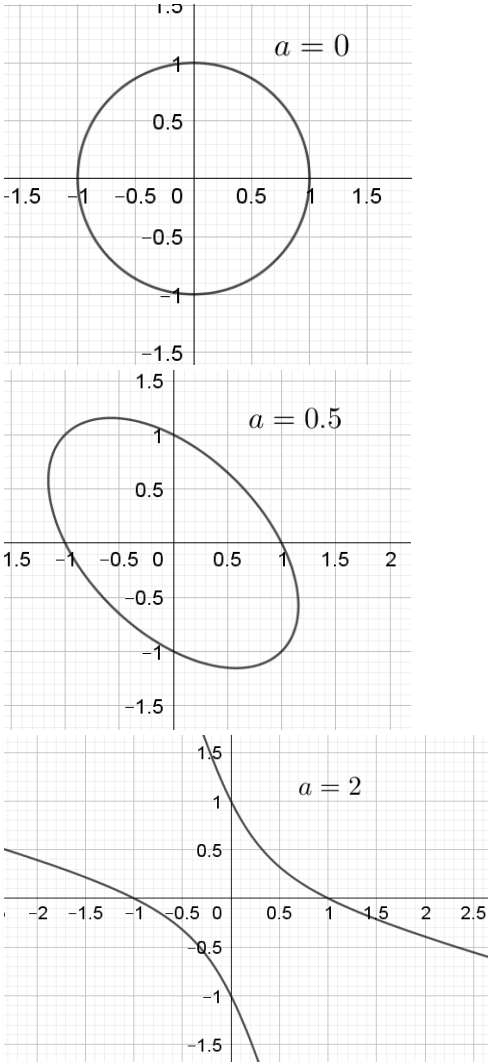Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

         We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.

         When a value is **given** in the paper only accept an answer correct to at least as many significant figures as the given value.

         When a value is **not given** in the paper accept any answer that agrees with the correct value to **2 s.f.** unless a different level of accuracy has been asked for in the question, or the mark scheme specifies an acceptable range.

         NB for Specification A the rubric specifies 3 s.f. as standard, so this statement reads "3 s.f"

         Follow through should be used so that only one mark in any question is lost for each distinct accuracy error.

         Candidates using a value of 9.80, 9.81 or 10 for g should usually be penalised for any final accuracy marks which do not agree to the value found with 9.8 which is given in the rubric.

g   Rules for replaced work and multiple attempts:

         If one attempt is clearly indicated as the one to mark, or only one is left uncrossed out, then mark that attempt and ignore the others.

         If more than one attempt is left not crossed out, then mark the last attempt unless it only repeats part of the first attempt or is substantially less complete.

         if a candidate crosses out all of their attempts, the assessor should attempt to mark the crossed out answer(s) as above and award marks appropriately.

h   For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A or B mark in the question. Marks designated as cao may be awarded as long as there are no other errors. If a candidate corrects the misread in a later part, do not continue to follow through. E marks are lost unless, by chance, the given results are established by equivalent working. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i   If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers provided that there is nothing in the wording of the question specifying that analytical methods are required such as the bold "In this question you must show detailed reasoning", or the command words "Show" and "Determine. Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j   If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1	(a)	(i)	$a=0$	**B1**	**1.1b**	Shape relative to axes, points on axes.	
			$a=0.5$	**B1**	**1.1b**	Shape relative to axes, points on axes.	
			$a=2$	**B1** **[3]**	**1.1b**	Shape relative to axes, points on axes.	

Question			Answer	Marks	AOs		Guidance
		(ii)	Closed curve	**B1** [1]	1.2	Bounded also allowed.	'Maximum' B0 'Continuous' B0
		(iii)	When a = 1 the curve is the points (x, y) which satisfy $x^2 + y^2 + 2xy = 1$ $\Leftrightarrow (x+y)^2 = 1$ $\Leftrightarrow x + y = 1$ or $x + y = -1$ $\Leftrightarrow y = 1 - x$ or $y = -1 - x$	**M1**	1.1b	Working must be shown for M1A1, otherwise M0A0.	
			Therefore the two straight lines are y = - x + 1 and y = -x – 1.	**A1** [2]	2.2a		
	(b)	(i)	$x^2 + y^2 + 2axy = 1$ Setting $x = r\cos\theta$ and $y = r\sin\theta$ gives	**M1**	3.1a		
			$r^2 + 2ar^2\sin\theta\cos\theta = 1.$	**M1**	1.1b	Allow correct formula for r^2. Allow formula lwith +/- in front of the brackets. Allow formula with $2\sin\theta\cos\theta$ rather than $\sin 2\theta$ for A1.	
			Therefore a polar form for the curve is $r^2 = \dfrac{1}{1 + a\sin 2\theta}$ $\left(\text{So } r = (\pm)\sqrt{\dfrac{1}{1 + a\sin 2\theta}}\right)$.	**A1** [3]	2.2a	Note entire curve is given for $0 \le \theta < 2\pi$ (taking values of θ for which $1 + a\sin 2\theta > 0$	

Question			Answer	Marks	AOs		Guidance
		(ii)	When a = 2 the area is $$\frac{1}{2}\int_0^{\frac{\pi}{2}} r^2 \, d\theta = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{1}{1+2\sin(2\theta)} \, d\theta =$$	M1	1.1a	Need integration formula with correct limits.	
			$$-\frac{\sqrt{3}}{12}\ln\left(7-4\sqrt{3}\right) = \frac{\sqrt{3}}{12}\ln\left(7+4\sqrt{3}\right)$$	A1 [2]	1.1b	Answer given is equivalent to $\frac{\sqrt{3}}{6}\ln\left(2+\sqrt{3}\right)$. Note that exact form is required.	$\frac{\sqrt{3}}{6}\ln\left(\frac{2-\sqrt{3}}{7-4\sqrt{3}}\right)$ is also equivalent to the answer.

(c)		Solving				
		$y = mx$ $x^2 + y^2 + 2axy = 1$ as a pair of simultaneous equations gives solution pairs (x, y) of	**M1**	1.1a	Sub mx as y to get $x^2 + (mx)^2 + 2ax(mx) = 1$ or $x^2 + m^2x^2 + 2amx^2 = 1$ (**) or better	
		$\left(\dfrac{\sqrt{m^2+2am+1}}{m^2+2am+1}, \dfrac{m\sqrt{m^2+2am+1}}{m^2+2am+1} \right)$ and $\left(-\dfrac{\sqrt{m^2+2am+1}}{m^2+2am+1}, -\dfrac{m\sqrt{m^2+2am+1}}{m^2+2am+1} \right)$	**M1**	2.5	Only x-coordinates required. $x^2(m^2 + 2am + 1) = 1$ or better is needed for this M1	
		These two points only exist if $m^2 + 2am + 1 > 0$, in which case they are distinct.	**M1**	3.1a	Commenting on existence of the points (soi). Allow use of discriminant	Putting discriminant > 0 for (**) as a quadratic in x is another way to get to $m^2 + 2am + 1 > 0$
		$m^2 + 2am + 1 > 0$ $\Leftrightarrow (m+a)^2 + 1 - a^2 > 0$ $\Leftrightarrow (m+a)^2 > a^2 - 1$	**A1**	3.1a	For finding conditions for existence of the points (implies previous M1)	
		If $-1 < a < 1$ then this is true for all values of m.	**B1**	1.1a		
		If $a = 1$ this is true only when $m \neq -a$, so when $m \neq -1$	**B1**	1.1a		
		If $a = -1$ this is true only when $m \neq -a$, so when $m \neq 1$	**B1**	1.1a		

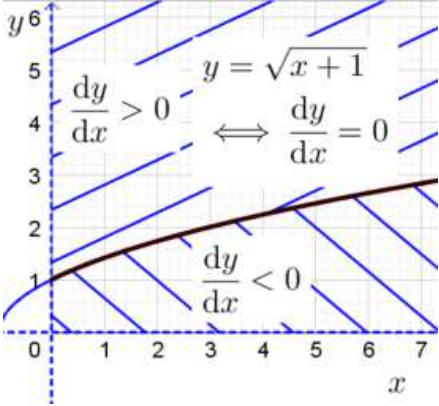| Question	Answer	Marks	AOs		Guidance
	If $a < -1$ or $a > 1$ then require that $$m+a > \sqrt{a^2-1} \quad \text{or} \quad m+a < -\sqrt{a^2-1}$$ i.e $m > -a + \sqrt{a^2-1}$ or $m < -a - \sqrt{a^2-1}$	**M1** **A1** **[9]**	**3.2a** **3.2a**	For case descriptor and at least one inequality (allow reasonable FT (e.g. sign errors or minor coefficient errors) from earlier work. For both inequalities. Note that if this final case appears with descriptor "$a \le -1$ or $a \ge 1$" then the previous two B1 marks are implied.	Must be strict inequalities for M1A1 but for non-strict award M1A0.

	Question		Answer	Marks	AOs	Guidance	

2	(a)	(i)	59
			191
			309
			441
			559
			691
			809
			941
			
			No need to state program example given on the right
				B1
				[1]
					2.5	for i in range(0,1000):
						  if i*i%1000==481:
						    print(i)
						
						Need all the values for the mark	

		(ii)	481 is a quadratic residue modulo 1000 because there is an integer x such that $x^2 \equiv 481 \ (\text{mod} \ 1000)$. For example $x = 59$.	**B1** **[1]**	2.4	Need an actual example or reference to working in 2(a)(i) to get this mark.	
		(iii)	Quadratic residues modulo 11 are $1 \equiv 1^2 \ (\text{mod} \ 11)$ $4 \equiv 2^2 \ (\text{mod} \ 11)$ $9 \equiv 3^2 \ (\text{mod} \ 11)$ $5 \equiv 4^2 \ (\text{mod} \ 11)$ $3 \equiv 5^2 \ (\text{mod} \ 11)$ (squaring 6, 7, 8, 9, 10 give values already in this list)	**M1** **A1** **[2]**	2.5 1.2	Accept appropriate code instead, must be stated SC1 for 1, 3, 4, 5 and 9.	

Question			Answer	Marks	AOs	Guidance	
		(iv)	Quadratic residues modulo 13 are $1 \equiv 1^2$ (mod 13) $4 \equiv 2^2$ (mod 13) $9 \equiv 3^2$ (mod 13) $3 \equiv 4^2$ (mod 13) $12 \equiv 5^2$ (mod 13) $10 \equiv 6^2$ (mod 13)	M1	2.5	Accept appropriate code instead	
			(squaring 7, 8, 9, 10, 11, 12 give values already in this list)	A1 [2]	1.2	SC1 for 1, 3, 4, 9, 10 and 12.	
		(v)	$(n-m)^2 = n^2 - 2mn + m^2 (= m^2 + n(n-2m))$	M1	1.1a	Multiplying out the brackets	
			So $(n-m)^2$ and m^2 differ by a multiple of n. Therefore $m^2 \equiv (n-m)^2$ (mod n)	A1 [2]	2.1	Explanation or see n(n – 2m), and consideration modulo n.	Allow e.g. $n^2 - 2mn + m^2 = 0 \pm 0 + m^2$ (mod n)
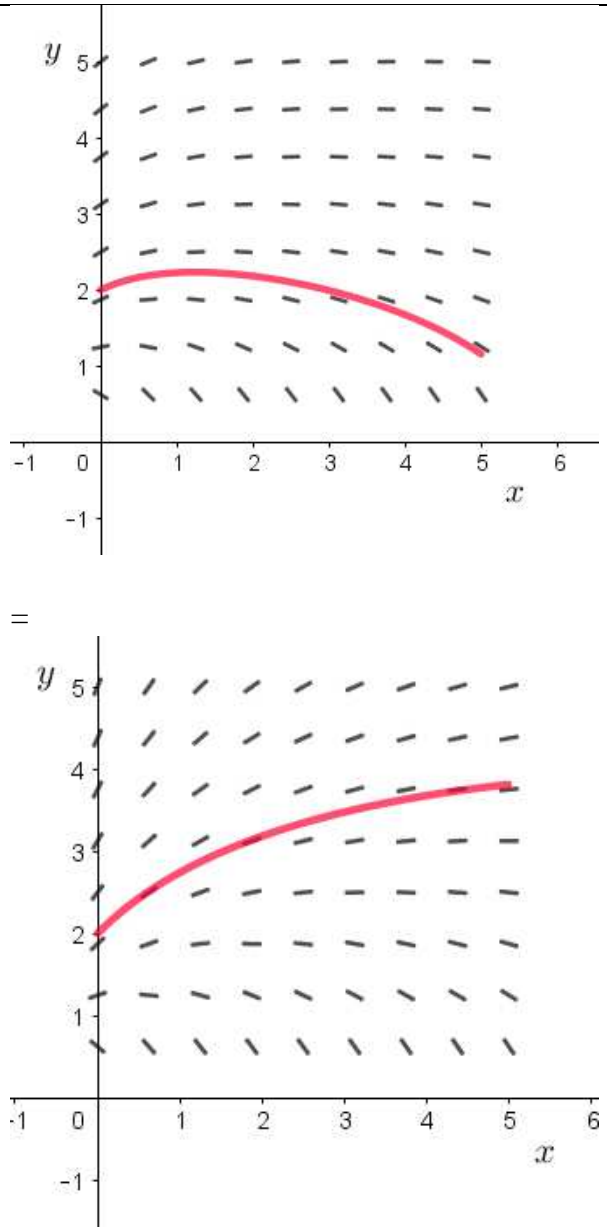
Question			Answer	Marks	AOs	Guidance	
		(vi)	By part (v) $m^2 \equiv (p-m)^2 \bmod p$	M1	3.1a	Use of previous result with n replaced by p.	m may be replaced by a specific value.
			$1^2 \equiv (p-1)^2 \bmod p$ $2^2 \equiv (p-2)^2 \bmod p$ ... $\left(\dfrac{p-1}{2}\right)^2 \equiv \left(\dfrac{p+1}{2}\right)^2 \bmod p$ Therefore the number of quadratic residues mod p is at most $\dfrac{p-1}{2}$.	A1	2.1	Explanation that pairing up p – 1 values gives $\dfrac{p-1}{2}$ pairs.	
			Suppose n, m are such that $1 \le m \le n \le \dfrac{p-1}{2}$ and that $m^2 = n^2 \bmod p$. Then p divides $n^2 - m^2 = (n-m)(n+m)$ and so p divides either $n - m$ or $n + m$ (since p is prime). But $0 \le n-m < n+m < p$ and so $n - m = 0$ Therefore m = n.	M1	3.1a		
			Therefore each distinct integer between 1 and $\dfrac{p-1}{2}$ has a distinct square mod p and so there are $\dfrac{p-1}{2}$ quadratic residues mod p.	A1 [4]	2.1		

Question			Answer	Marks	AOs	Guidance	
	(b)	(i)	If 91 were prime then $2^{90} \equiv 1 \bmod 91$ by Fermat's little theorem. However $2^{90} \equiv 64 \bmod 91$	**M1** **A1** **[2]**	2.2a 3.2a	soi. Need to see idea that $2^{90} - 1$ is not a multiple of 91 implies 91 is not prime.	Can use values of a other than 2 here.

Question			Answer	Marks	AOs	Guidance	
		(ii)	561 by the program on the right	**M1**	**3.3**	Pseudo code accepted, condone lack of syntax, give reasonable BOD on possible transcription errors	
				M1	**3.3**		
			Appropriate structure program	**M1**	**3.4**		
				M1	**2.1**		
				M1	**2.1**		

			Loops with correct range dependent on m, n. Check for common divisors and divisors with if statement (and tracking greatest one found in case of coprime function)	**A1**	**2.3**	`def isprime(n):`
						  `prime=1`
						  `for i in range(2,n):`
						    `if n%i==0:`
						      `prime=0`
						  `return prime`
							M1 for prime test

				[6]			

			Fully correct program.			`def coprime(m,n):`
						  `coprime=1`
						  `for i in range(2,m+1):`
						    `if n%i==0 and m%i==0:`
						      `coprime=i`
						  `return coprime`
							M1 for coprime test

							M1 for test over range (can be implied by any stated output in this range).

						`for i in range(500,600):`
						  `candidate = 1`
						  `if isprime(i)==1:`
						    `candidate = 0`
						  `if isprime(i)==0:`
						    `for j in range(1,i):`
						      `if coprime(j,i)==1 and`
						          `((j**(i-1))%i)!=1:`
						        `candidate = 0`
						  `if candidate == 1:`
						    `print(i)`
							M1 for rejecting primes

							M1 for testing equation for appropriate values.
							A1 for value 561.

Question			Answer	Marks	AOs	Guidance	
3	(a)	(i)		**B1**	1.2	For sketch of $y=\sqrt{x+1}$ only needed for $x\geq 0$.	Need general shape correct (decreasing positive gradient and includes (0,1) and other coordinates approximately correct). Must be correct in upper right quadrant. Ignore anything in other quadrants.
				B1	1.1a	For region $\frac{dy}{dx}>0 \Leftrightarrow y>\sqrt{x+1}$	Only award if general shape correct (see above)
				B1 [3]	1.1a	For region $\frac{dy}{dx}<0 \Leftrightarrow y<\sqrt{x+1}$	Only award if general shape correct (see above)
		(ii)	Solution is $y=\sqrt{(b^2-2)x^2+2(b^2-1)x+b^2}$	**B1** [1]	1.2	oe	

Question			Answer	Marks	AOs	Guidance	
		(iii)	With y as in (ii) $$\frac{dy}{dx} = \frac{\left(b^2 - 2\right)x + b^2 - 1}{\sqrt{\left(b^2 - 2\right)x^2 + 2\left(b^2 - 1\right)x + b^2}}$$	M1	1.1a	Could also use $y^2 = x+1$ direct from (*). Numerator $= 0$ or equivalent.	
			So $\frac{dy}{dx} = 0 \Leftrightarrow x = \frac{1-b^2}{b^2-2}$	A1	1.1b	For x in terms of b from $\frac{dy}{dx} = 0$	
			This has a solution with $x \geq 0$ if and only if $1 \leq b < \sqrt{2}$	A1	3.2a	FT using their x	
			When $x = \frac{1-b^2}{b^2-2}$ in the solution in (ii) $$y = \frac{1}{\sqrt{2-b^2}}$$ **[4]**	B1	1.1b	Allow anything equivalent to this y and FT using their x	
(b)	(i)		Around a = 0.	B1 **[1]**	1.1b	Allow $-0.5 \leq a \leq 0.5$	
		(ii)	Around a = 0.6	B1 **[1]**	1.1b	Allow $0.5 \leq a \leq 1$ (has to be different to answer to (b)(i) to be awarded).	

| | | (iii) | | **B1** | **1.1b** | Needs to have a maximum and negative second derivative throughout. Must extend to at least x = 4.5. | |
| | | | = | **B1** **[2]** | **1.1b** | No turning point and negative second derivative throughout. Must extend to at least x = 4.5. | |

Question			Answer	Marks	AOs	Guidance	
		(iv)	One curve has a turning point (local max between 0 and 5), the other doesn't.	**B1** **[1]**	1.2	Also allow 'not strictly increasing' or 'intersects the x -axis' or 'stationary point'.	Not 'asymptote'.
	(c)	(i)				Give reasonable BOD on possible transcription errors and consider correct answers to 3(c)(ii), 3(c)(iii) as evidence of correct formulae in the spreadsheet.	
			Insert 0 and 2 resp into cells A1 and B1. Using cells I1 and I2 for the values of h and a respectively other formulae then could be	**B1**	1.1a	Cols for x and y and initialised x and y values.	
				B1	3.1a	Allows for a and h to be varied (doesn't need cell reference, can be implied)	
			Cell C1: ==I\$1*((B1^I\$2)/(A1+1)-1/B1) Cell A2: =A1+I\$1 Cell B2: =B1+C1	**B1**	2.1		
			And copy down.	**[3]**		Formulae for x_{n+1} and y_{n+1}.	
		(ii)	Spreadsheet gives that y is approximately 2.222487439 (to no. of dec places shown) when x = 3.	**B1** **[1]**	1.1b	Allow any value between 2.2 and 2.23 inclusive.	

Question			Answer	Marks	AOs	Guidance	
		(iii)	Using the spreadsheet with h = 0.01 and a = -0.2 gives (layout as in c(i), rows 81 to 86 shown below)				
			This suggest that the local maximum has x-coordinate between 0.82 and 0.84 and so would be a value which rounds to 0.8 to one decimal place. Smaller values of h (and use of other numerical methods confirms this)	**B1**	**1.1a**	For stating existence of increasing then decreasing y values (as x increases).	Or stating existence of positive then negative values of $\dfrac{dy}{dx}$ (as x increases).
				M1	**3.1a**	For analysis with values of h less than or equal to 0.05.	
				A1 **[3]**	**2.2b**	For correct answer of 0.8.	Note that h = 0.01 to be sure of accuracy of correct answer (but not required to state this to award A1).

The spreadsheet values shown:

81	0.8	2.12599303	7.39745E-05
82	0.81	2.126067	4.77092E-05
83	0.82	2.12611471	2.16878E-05
84	0.83	2.1261364	-4.094E-06
85	0.84	2.1261323	-2.96408E-05
86	0.85	2.12610266	-5.49568E-05

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