AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

# A-level MATHEMATICS

## Unit Mechanics 2B

Monday 27 June 2016 Morning Time allowed: 1 hour 30 minutes

**Materials**
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

**1** A stone, of mass $0.3\,\text{kg}$, is thrown with a speed of $8\,\text{m s}^{-1}$ from a point at a height of $5$ metres above a horizontal surface.

**(a)** Calculate the initial kinetic energy of the stone.

**[2 marks]**

**(b) (i)** Find the kinetic energy of the stone when it hits the surface.

**[3 marks]**

**(ii)** Hence find the speed of the stone when it hits the surface.

**[2 marks]**

**(iii)** State one modelling assumption that you have made.

**[1 mark]**

*QUESTION PART REFERENCE*

**Answer space for question 1**

| *QUESTION PART REFERENCE* | **Answer space for question 1** |
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**2** A particle moves in a horizontal plane under the action of a single force, $\mathbf{F}$ newtons.

The unit vectors **i** and **j** are directed east and north respectively.

At time $t$ seconds, the velocity of the particle, $\mathbf{v}\,\mathrm{m\,s^{-1}}$, is given by

$$\mathbf{v} = (8t - t^4)\mathbf{i} + 6\mathrm{e}^{-3t}\,\mathbf{j}$$

**(a)** Find an expression for the acceleration of the particle at time $t$.

**[2 marks]**

**(b)** The mass of the particle is $2\,\mathrm{kg}$.

**(i)** Find an expression for the force $\mathbf{F}$ acting on the particle at time $t$.

**[2 marks]**

**(ii)** Find the magnitude of $\mathbf{F}$ when $t = 1$.

**[3 marks]**

**(c)** Find the value of $t$ when $\mathbf{F}$ acts due south.

**[2 marks]**

**(d)** When $t = 0$, the particle is at the point with position vector $(3\mathbf{i} - 5\mathbf{j})$ metres.

Find an expression for the position vector, $\mathbf{r}$ metres, of the particle at time $t$.

**[4 marks]**

QUESTION PART REFERENCE

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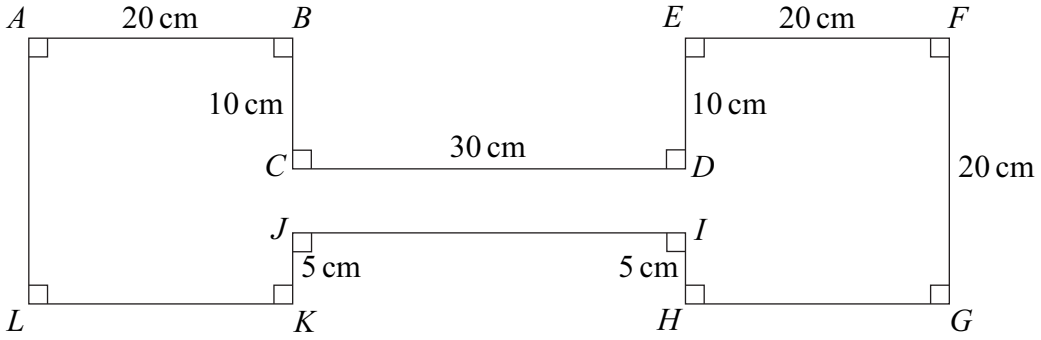
**3** The diagram shows a uniform lamina $ABCDEFGHIJKL$.




**(a)** Explain why the centre of mass of the lamina is $35\,\text{cm}$ from $AL$.

**[1 mark]**

**(b)** Find the distance of the centre of mass from $AF$.

**[4 marks]**

**(c)** The lamina is freely suspended from $A$.

Find the angle between $AB$ and the vertical when the lamina is in equilibrium.

**[3 marks]**

**(d)** Explain, briefly, how you have used the fact that the lamina is uniform.

**[1 mark]**

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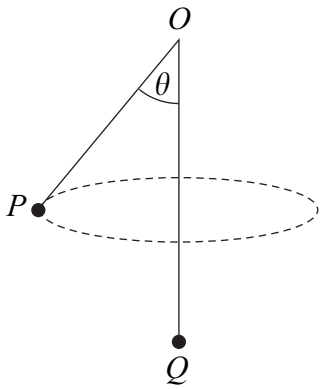
**4** A particle $P$, of mass $6\,\text{kg}$, is attached to one end of a light inextensible string. The string passes through a small smooth ring, fixed at a point $O$. A second particle $Q$, of mass $8\,\text{kg}$, is attached to the other end of the string.

The particle $Q$ hangs at rest vertically below the ring, and the particle $P$ moves with speed $5\,\text{m s}^{-1}$ in a horizontal circle, as shown in the diagram.

The angle between $OP$ and the vertical is $\theta$.




**(a)** Find the tension in the string.

**[1 mark]**

**(b)** Find $\theta$.

**[3 marks]**

**(c)** Find the radius of the horizontal circle.

**[4 marks]**

QUESTION PART REFERENCE

**Answer space for question 4**

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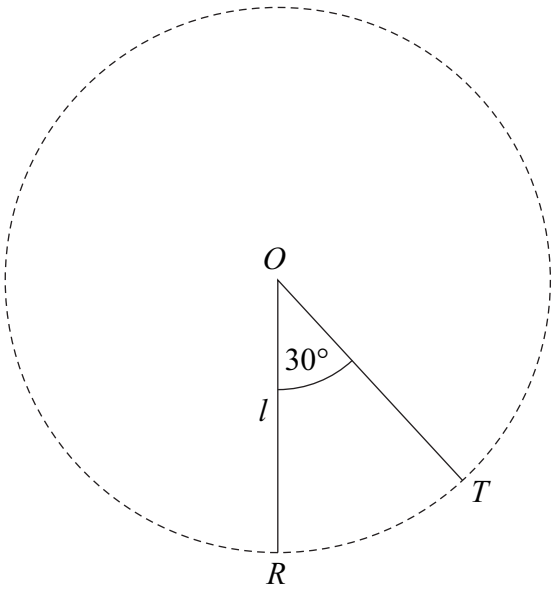
**Answer space for question 4**

**5** A particle of mass $m$ is suspended from a fixed point $O$ by a light inextensible string of length $l$. The particle hangs in equilibrium at the point $R$ vertically below $O$.

The particle is set into motion with a horizontal velocity $u$ so that it moves in a complete vertical circle with centre $O$.

The point $T$ on the circle is such that angle $ROT$ is $30°$, as shown in the diagram.




**(a)** Find, in terms of $g$, $l$ and $u$, the speed of the particle at the point $T$.

**[3 marks]**

**(b)** Find, in terms of $g$, $l$, $m$ and $u$, the tension in the string when the particle is at the point $T$.

**[3 marks]**

**(c)** Find, in terms of $g$, $l$, $m$ and $u$, the tension in the string when the particle returns to the point $R$.

**[2 marks]**

**(d)** The particle makes complete revolutions.

Find, in terms of $g$ and $l$, the minimum value of $u$.

**[4 marks]**

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**6** A stone, of mass $m$, falls vertically downwards under gravity through still water. At time $t$, the stone has speed $v$ and it experiences a resistance force of magnitude $\lambda mv$, where $\lambda$ is a constant.

**(a)** Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - \lambda v$$

**[2 marks]**

**(b)** The initial speed of the stone is $u$.

Find an expression for $v$ at time $t$.

**[6 marks]**

QUESTION PART REFERENCE

**Answer space for question 6**

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**Answer space for question 6**

**7** A uniform ladder, of weight $W$, rests with its top against a rough vertical wall and its base on rough horizontal ground.

The coefficient of friction between the wall and the ladder is $\mu$ and the coefficient of friction between the ground and the ladder is $2\mu$.

When the ladder is on the point of slipping, the ladder is inclined at an angle of $\theta$ to the horizontal.

**(a)** Draw a diagram to show the forces acting on the ladder.

**[2 marks]**

**(b)** Find $\tan\theta$ in terms of $\mu$.

**[7 marks]**

QUESTION PART REFERENCE

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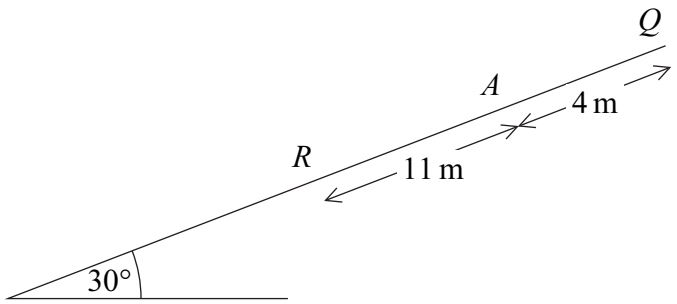
**8** A particle, $P$, of mass $5\,\text{kg}$ is placed at the point $A$ on a rough plane which is inclined at $30°$ to the horizontal.

The points $Q$ and $R$ are also on the surface of the inclined plane, with $QR = 15$ metres. The point $A$ is between $Q$ and $R$ so that $AQ = 4$ metres and $AR = 11$ metres.

The three points $Q$, $A$ and $R$ are on a line of greatest slope of the plane.




The particle is attached to two light elastic strings, $PQ$ and $PR$.

One of the strings, $PQ$, has natural length $4$ metres and modulus of elasticity $160\,\text{N}$, the other string, $PR$, has natural length $6$ metres and modulus of elasticity $120\,\text{N}$.

The particle is released from rest at the point $A$.

The coefficient of friction between the particle and the plane is $0.4$ .

Find the distance of the particle from $Q$ when it is next at rest.

**[8 marks]**

QUESTION PART REFERENCE

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**END OF QUESTIONS**

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