RECOGNISING ACHIEVEMENT

GCE

Mathematics

Advanced Subsidiary GCE

Unit **4721:** Core Mathematics 1

Mark Scheme for June 2011

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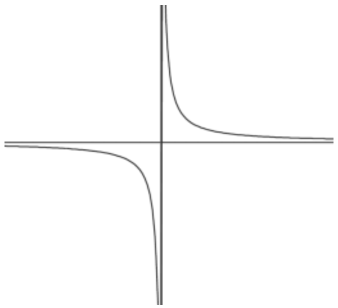
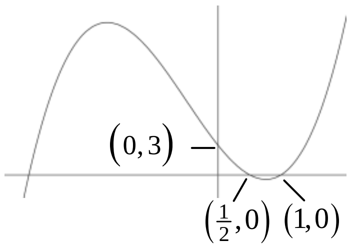
1	$3(x^2-6x)+4$ $=3\left[(x-3)^2-9\right]+4$ $=3(x-3)^2-23$	**B1** **B1** **M1** **A1**	4 **4**	$p=3$ $(x-3)^2$ seen or $q=-3$ $4-3q^2$ or $\frac{4}{3}-q^2$ (their q) $r=-23$	If *p*, *q*, *r* found correctly, then **ISW** slips in format. $3(x-3)^2+23$ **B1 B1 M0 A0** $3(x-3)-23$ **B1 B1 M1 A1 (BOD)** $3(x-3x)^2-23$ **B1 B0 M1 A0** $3(x^2-3)^2-23$ **B1 B0 M1 A0** $3(x+3)^2-23$ **B1 B0 M1 A1 (BOD)** $3x(x-3)^2-23$ **B0 B1M1A1**
2 (i)		**B1** **B1**	 2	Reasonably correct curve for $y=\frac{1}{x}$ in 1st and 3rd quadrants only Very good curves for $y=\frac{1}{x}$ in 1st and 3rd quadrants **SC** If 0, very good single curve in either 1st or 3rd quadrant and nothing in other three quadrants. **B1**	N.B. Ignore 'feathering' now that answers are scanned. Reasonably correct shape, not touching axes more than twice. Correct shape, not touching axes, asymptotes clearly the axes. Allow slight movement away from asymptote at one end but not more. Not finite.
(ii)	Translation 4 units parallel to *y* axis	**B1** **B1**	2 **4**	**Must** be translation/translated – not shift, move etc. Or $\begin{pmatrix}0\\4\end{pmatrix}$	For "parallel to the y axis" allow "vertically", "up", "in the (positive) y direction". **Do not accept** "in/on/across/up/along the y axis"
3 (i)	$\frac{16x^2\times 2x^3}{x}$ $=32x^4$	**B1** **B1**	 2	32 x^4	
(ii)	$\frac{1}{6}x$	**M1** **A1** **B1**	 3 **5**	6 or $\frac{1}{36^{\frac{1}{2}}}$ or $\frac{1}{\sqrt{36}}$ seen $\frac{1}{6}$ in final answer x (Allow x^1) in final answer	$\frac{1}{\frac{1}{\sqrt{36}}}$ is M0 $\pm\frac{1}{6}$ is **A0**

4	$2x^2 - 8x + 8 = 26 - 3x$	**M1**		Attempt to eliminate *x* or *y*	Must be a clear attempt to reduce to one variable. Condone poor algebra for first mark.
	$2x^2 - 5x - 18 (= 0)$	**A1**		Correct 3 term quadratic (not necessarily all in one side)	<u>If x eliminated:</u>
	$(2x - 9)(x + 2)(= 0)$	**M1**		Correct method to solve quadratic	$y = 2(\frac{26 - y}{3} - 2)^2$
	$x = \frac{9}{2}, x = -2$	**A1**		*x* values correct	Leading to $2y^2 - 89y + 800 = 0$
	$y = \frac{25}{2}, y = 32$	**A1**	5	*y* values correct	(2y – 25)(y – 32) =0 etc.
			5	**SR** If A0 A0, one correct pair of values, spotted or from correct factorisation **www B1**	
5 (i)	$10\sqrt{3} - 4\sqrt{3}$	**M1**		Attempt to express both surds in terms of $\sqrt{3}$	e.g. $\sqrt{3\text{x}100} - \sqrt{3\text{x}16}$
		B1		One term correct	
	$= 6\sqrt{3}$	**A1**	3	Fully correct (not $\pm 6\sqrt{3}$)	
(ii)	$\frac{\sqrt{5}(15 + \sqrt{40})}{5}$	**M1**		Multiply numerator and denominator by $\sqrt{5}$ or - $\sqrt{5}$ **or** attempt to express both terms of numerator in terms of $\sqrt{5}$ (e.g. dividing both terms by $\sqrt{5}$)	Check both numerator and denominator have been multiplied
	$= \frac{15\sqrt{5} + 10\sqrt{2}}{5}$	**B1**		One of a, b correctly obtained	
	$= 3\sqrt{5} + 2\sqrt{2}$	**A1**	3 **6**	Both a = 3 and b=2 correctly obtained	

Question	Answer	Marks		Guidance	
6	$k = x^{\frac{1}{4}}$	M1*		Use a substitution to obtain a quadratic or factorise into 2 brackets each containing $x^{\frac{1}{4}}$	**No marks** unless evidence of substitution (quadratic seen or square rooting or squaring of roots found). = 0 may be implied.
	$3k^2 - 8k + 4 = 0$ $(3k-2)(k-2) = 0$	DM1		Correct method to solve a quadratic	Allow $x = x^{\frac{1}{4}}$ as a substitution.
	$k = \frac{2}{3}$ or $k = 2$	A1			**No marks** if straight to quadratic formula to get x = "$\frac{2}{3}$" x = "2" and no further working
	$x = \left(\frac{2}{3}\right)^4$ or $x = 2^4$	M1		Attempt to calculate k^4	**No marks** if $k = x^{\frac{1}{4}}$ then $3k - 8k^2 + 4 = 0$
	$x = \frac{16}{81}$ or $x = 16$	A1	5 **5**		**SC** If **M0** Spotted solutions **www B1** each Justifies 2 solutions exactly **B3**
	If candidates use $k = x^{\frac{1}{2}}$ and rearrange:				
	$3k - 8\sqrt{k} + 4 = 0$ $8\sqrt{k} = 3k + 4$ $64k = 9k^2 + 24k + 16$	M1*		Substitute, rearrange and square both sides	
	$9k^2 - 40k + 16 = 0$ $(9k-4)(k-4) = 0$	DM1		Correct method to solve quadratic	
	$k = \frac{4}{9}$ or $k = 4$	A1			
	$x = \left(\frac{4}{9}\right)^2$ or $x = 4^2$	M1		Attempt to calculate k^2	
	$x = \frac{16}{81}$ or $x = 16$	A1			
7 (i)	$-14 \le 6x \le -5$	M1		2 equations or inequalities both dealing with all 3 terms resulting in $a \le 6x \le b$, $a \ne -9$, $b \ne 0$	**Do not ISW** after correct answer if contradictory inequality seen.
		A1		-14 and -5 seen **www**	
	$-\frac{7}{3} \le x \le -\frac{5}{6}$	A1	3	Accept as two separate inequalities provided not linked by "or" (must be $\le$)	Allow $-\frac{14}{6} \le x \le -\frac{5}{6}$
(ii)	$0 < x^2 - 4x - 12$	M1		Rearrange to collect all terms on one side	**Do not ISW** after correct answer if contradictory inequality seen.
	$(x-6)(x+2)$	M1		Correct method to find roots	
		A1		6, -2 seen	
		M1	5	Correct method to solve quadratic inequality i.e. $x >$ their higher root, $x <$ their lower root	
	$x > 6$, $x < -2$	A1	**8**	(not wrapped, strict inequalities, no 'and')	e.g. for last two marks, $-2 > x > 6$ scores **M1 A0**

8 (i)	$\frac{dy}{dx} = 6x + 6x^{-2}$	**M1** **A1**		Attempt to differentiate (one non-zero term correct) Completely correct	**NB** – $x = -1$ (and therefore possibly $y = 7$) can be found from equating the incorrect differential
	$6x + \frac{6}{x^2} = 0$	**M1**		Sets their $\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 6x + 6$ to 0. This could score **M1A0 M1A0A1 ft**
	$x = -1$	**A1**		Correct value for x - **www**	
	$y = 7$	**A1 ft**	5	Correct value of y for *their* value of x	If more than one value of x found, allow **A1 ft** for one correct value of y
(ii)	$\frac{d^2y}{dx^2} = 6 - 12x^{-3}$	**M1**		Correct method e.g. substitutes their x from (i) into their $\frac{d^2y}{dx^2}$ (must involve x) and considers sign.	Allow comparing signs of their $\frac{dy}{dx}$ either side of their "– 1", comparing values of y to their "7"
	When $x = -1$, $\frac{d^2y}{dx^2} > 0$ so minimum pt	**A1 ft**	2 **7**	**ft** from their $\frac{dy}{dx}$ differentiated correctly and correct substitution of *their* value of x and consistent final conclusion **NB** If second derivate evaluated, it must be correct (18 for $x = -1$). If more than one value of x used, max **M1 A0**	**SC** $\frac{d^2y}{dx^2}$ = a constant correctly obtained from their $\frac{dy}{dx}$ and correct conclusion (ft) **B1**

9 (i)	Gradient of $AB = \frac{1-3}{7-1} = -\frac{1}{3}$	**M1***		Uses $\frac{y_2 - y_1}{x_2 - x_1}$ for any 2 points	
	Gradient of $AC = \frac{-9-3}{-3-1} = 3$	**A1**		One correct gradient (may be for gradient of *BC* =1)	
		A1		Gradients for both *AB* and *AC* found correctly	Do not allow final mark if vertex A found from wrong working. (Dependent on 1st M 1 A1 A1)
		M1		Attempts to show that $m_1 \times m_2 = -1$ oe, accept "negative reciprocal"	
	Vertex *A*	**DB1**			Accept BÂC etc for vertex A or "between AB and AC" Allow if marked on diagram.
	OR:				
	Length of $AB = \sqrt{(7-1)^2 + (1-3)^2} = \sqrt{40}$				
	$AC = \sqrt{(-3-1)^2 + (-9-3)^2} = \sqrt{160}$	**M1***		Correct use of Pythagoras, square rooting not needed	
	$BC = \sqrt{(-3-7)^2 + (-9-1)^2} = \sqrt{200}$	**A1**		Any length or length squared correct	
	Shows that $AB^2 + AC^2 = BC^2$	**A1**		All three correct	
	Vertex *A*	**M1** **DB1**	5	Correct use of Pythagoras to show $AB^2 + AC^2 = BC^2$	i.e must add squares of shorter two lengths
9 (ii)	Midpoint of *BC* is $\left(\frac{7+-3}{2}, \frac{1+-9}{2}\right)$ = (2, -4)	**M1***		Uses $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ o.e. for BC, AB or AC (3 out of 4 subs correct)	Substitution method 1 (into $x^2 + y^2 + ax + by + c = 0$) Substitutes all 3 points to get 3 equations in *a*,*b*,*c* **M1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of a, b, c correct **A1** Correct method to find other values **M1** All values correct **A1** Correct equation in required form **A1**
		A1		Correct centre (**cao**)	
	Length of *BC* = $\sqrt{(-3-7)^2 + (-9-1)^2} = \sqrt{200} = 10\sqrt{2}$	**M1****		Correct method to find *d* or *r* or d^2 or r^2 o.e. for BC, AB or AC (must be consistent with their midpoint if found)	Alternative markscheme for last 4 marks with *f*,*g*, *c* method: $x^2 - 4x + y^2 + 8y$ for their centre **DM1*** $c = (\pm 2)^2 + 4^2 - 50$ **DM1**** c = – 30 **A1** Correct equation in required form **A1**
	Radius = $5\sqrt{2}$				
	$(x-2)^2 + (y+4)^2 = (5\sqrt{2})^2$	**DM1***	7	$(x-a)^2 + (y-b)^2$ seen for their centre	
	$(x-2)^2 + (y+4)^2 = 50$	**DM1****	**12**	$(x-a)^2 + (y-b)^2$ = their r^2	
	$x^2 + y^2 - 4x + 8y - 30 = 0$	**A1** **A1**		Correct equation Correct equation in required form	Ends of diameter method (*p*, *q*) to (*c*, *d*): Attempts to use $(x-p)(x-c) + (y-q)(y-d) = 0$ for BC,AC or AB **M2** $(x-7)(x+3) + (y-1)(y+9) = 0$ **A2** for both *x* brackets correct, **A2** for both *y* brackets correct $x^2 + y^2 - 4x + 8y - 30 = 0$ **A1** **SC** If **M2 A0 A0** then **B1** if both x brackets correct and **B1** if both y brackets correct for **AC** or **AB**

					<u>Substitution method 2</u>into $(x-p)^2 + (y-q)^2$ = their r^2 Correct method to find d or r or d^2 or r^2 ***M1** Substitutes all 3 points to get 3 equations in p,q **DM1** At least 2 equations correct **A1** Correct method to find one variable **M1** One of p, q correct **A1** Correct equation [$(x-2)^2+(y+4)^2=50$]**A1** Correct equation in required form [$x^2+y^2-4x+8y-30=0$]**A1**
10(i)		**B1** **B1** **B1**	 3	+ve cubic with 3 distinct roots (0, 3) labelled or indicated on y-axis (-3, 0), ($\frac{1}{2}$, 0) and (1, 0) labelled or indicated on x-axis and no other x- intercepts	For first **B1**, left end of curve must finish below x axis and right end must end above x axis. Allow slight wrong curvature at one end but not both ends. No cusp at either turning point. No straight lines drawn with a ruler. Condone (0, 3) as maximum point. To gain second and third **B** marks, there must be an attempt at a curve, not just points on axes. Final **B1** can be awarded for a negative cubic.
(ii)	$2x^2 + 5x - 3$, $x^2 + 2x - 3$, $2x^2 - 3x + 1$ $(2x^2 + 5x - 3)(x - 1)$ $2x^3 + 3x^2 - 8x + 3$ $\frac{dy}{dx} = 6x^2 + 6x - 8$ When $x = 1$, gradient = 4	**B1** **M1** **A1** **M1** **A1** **A1**	 6	Obtain one quadratic factor (can be unsimplified) Attempt to multiply a quadratic by a linear factor Attempt to differentiate (one non-zero term correct) Fully correct expression **www** Confirms gradient = 4 at x = 1 ****AG**	<u>Alternative for first 3 marks:</u> Attempt to expand all 3 brackets with an appropriate number of terms (including an x^3 term) **M1** Expansion with at most 1 incorrect term **A1** Correct, answer (can be unsimplified) **A1** Allow if done in part(i) please check.
(iii)	Gradient of l = 4 On curve, when x = -2, y = 15 $y - 15 = 4(x + 2)$ $y = 4x + 23$	**B1** **B1** **M1** **A1**	 4	May be embedded in equation of line Correct y coordinate Correct equation of line using their values Correct answer **in correct form**	 **M** mark is for any equation of line with any non-zero numerical gradient through (-2, their evaluated y)
(iv)	Attempt to find gradient of curve when x = -2 $6(-2)^2 + 6(-2) - 8 = 4$ So line is a tangent	**M1** **A1** **A1**	 3 **16**	Substitute x = -2 into their $\frac{dy}{dx}$ Obtain gradient of 4 **CWO** Correct conclusion	<u>Alternatives</u> **1)** Equates equation of l to equation of curve and attempts to divide resulting cubic by $(x + 2)$ **M1** Obtains $(x + 2)^2(2x - 5)$ (=0) **A1** Concludes repeated root implies tangent at x = -2 **A1** **2)** Equates their gradient function to 4 and uses correct method to solve the resulting quadratic **M1** Obtains $(x + 2)(x - 1) = 0$ oe **A1** Correctly concludes gradient = 4 when x = -2 **A1**

Allocation of method mark for solving a quadratic

e.g. $2x^2 - 5x - 18 = 0$

1) If the candidate attempts to solve by factorisation, their attempt when expanded must produce the **correct quadratic term** and **one other correct term** (with correct sign):

$(2x+2)(x-9)=0$	**M1**	$2x^2$ and -18 obtained from expansion
$(2x+3)(x-4)=0$	**M1**	$2x^2$ and $-5x$ obtained from expansion
$(2x-9)(x-2)=0$	**M0**	only $2x^2$ term correct

2) If the candidate attempts to solve by using the formula

a) If the formula is quoted incorrectly then **M0.**

b) If the formula is quoted correctly then one **sign** slip is permitted. Substituting the wrong numerical value for a or b or c scores **M0**

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times 2}$ earns **M1** (minus sign incorrect at start of formula)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$ earns **M1** (18 for *c* instead of −18)

$\dfrac{-5 \pm \sqrt{(-5)^2 - 4\times 2\times 18}}{2\times 2}$ **M0** (2 sign errors: initial sign and *c* incorrect)

$\dfrac{5 \pm \sqrt{(-5)^2 - 4\times 2\times -18}}{2\times -5}$ **M0** (2*b* on the denominator)

Notes – for equations such as $2x^2 - 5x - 18 = 0$, then $b^2 = 5^2$ would be condoned in the discriminant and would not be counted as a sign error. Repeating the sign error for *a* in both occurrences in the formula would be two sign errors and score **M0**.

c) If the formula is not quoted at all, substitution must be completely correct to earn the **M1**

3) If the candidate attempts to complete the square, they must get to the "square root stage" involving ±; we are looking for evidence that the candidate knows a quadratic has two solutions!

$$2x^2 - 5x - 18 = 0$$

$$2\left(x^2 - \frac{5}{2}x\right) - 18 = 0$$

$$2\left[\left(x - \frac{5}{4}\right)^2 - \frac{25}{16}\right] - 18 = 0$$

$$\left(x - \frac{5}{4}\right)^2 = \frac{169}{16}$$

$$x - \frac{5}{4} = \pm\sqrt{\frac{169}{16}}$$

This is where the **M1** is awarded – arithmetical errors may be condoned provided $x - \frac{5}{4}$ seen or implied

If a candidate makes repeated attempts (e.g. fails to factorise and then tries the formula), mark only what you consider to be their last full attempt.

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