4724 Core Mathematics 4

Q		Answer	Marks		Guidance
1		Long division method			
		Correct leading term x^2 in quotient	B1		
		Evidence of correct div process	M1		Sufficient to convince
		(Quotient =) x^2+6x-4	A1		
		(Remainder =) $11x+9$	A1		
		Identity method			
		$x^4+11x^3+28x^2+3x+1=Q(x^2+5x+2)+R$	M1		
		$Q=ax^2+bx+c$ or x^2+bx+c ; $R=dx+e$ & ≥ 3 ops	M1		N.B. $a=1 \Rightarrow$ 1 of the 3 ops
		$a=1, b=6, c=-4, d=11, e=9$ (for all 5)	A2		S.R. B1 for 3 of these
			4		
2	(i)	Find at least 2 of $(\overrightarrow{AB}$ or $\overrightarrow{BA})$, $(\overrightarrow{BC}$ or $\overrightarrow{CB})$, $(\overrightarrow{AC}$ or $\overrightarrow{CA})$	M1		irrespect of label; any notation
		Use correct method to find scal prod of any 2 vectors	M1		or use corr meth for modulus
		Use $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\frac{\overrightarrow{AB}.\overrightarrow{BC}}{\lvert AB\rvert\lvert BC\rvert}=0$	M1		or use $\lvert\overrightarrow{AB}\rvert^2+\lvert\overrightarrow{BC}\rvert^2=\lvert\overrightarrow{AC}\rvert^2$
		Obtain $p=1$ (dep 3 @ M1)	A1	**4**	
	(ii)	Use equal ratios of appropriate vectors	M1		or scalar product method
		Obtain $p=-8$	A1	**2**	
			6		
3		Use $\cos 2x = a\cos^2 x + b\ /\ \pm\cos^2 x - \sin^2 x\ /\ 1-2\sin^2 x$	*M1		
		Obtain $\lambda + \mu\sec^2 x$	dep*M1		using 'reasonable' Pythag attempt
		$\int \lambda + \mu\sec^2 x\,dx = \lambda x + \mu\tan x$	A1		(λ or μ may be 0 here/prev line)
		Obtain correct result $2x-\tan x$	A1		no follow-through
		$\frac{1}{6}\pi-\sqrt{3}+1$ ISW	A1		exact answer required
			5		
4		Attempt to connect du and dt or find $\frac{du}{dt}$ or $\frac{dt}{du}$	M1		not du = dt but no accuracy
		$du=\frac{1}{t}dt$ or $\frac{du}{dt}=\frac{1}{t}$ or $dt=e^{u-2}du$ or $\frac{dt}{du}=e^{u-2}$	A1		
		Indef int $\rightarrow \int\frac{1}{u^2}(du)$	A1		no t or dt in evidence
		$=-\frac{1}{u}$	A1		
		Attempt to change limits if working with $f(u)$	M1		or re-subst & use 1 and e
		$\frac{1}{6}$ ISW	A1		ln e must be changed to 1, ln 1 to 0
			6		

5 (i)	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + ...$	B1		
	$... \ -\frac{1}{9}x^2$	B1	**2**	$-\frac{2}{18}x^2$ acceptable
(ii) (a)	$(8+16x)^{\frac{1}{3}} = 8^{\frac{1}{3}}(1+2x)^{\frac{1}{3}}$	B1		not $16^{\frac{1}{3}}\left(\frac{1}{2}+x\right)^{\frac{1}{3}}$
	$(1+2x)^{\frac{1}{3}}$ = their **(i)** expansion with $2x$ replacing x	M1		not dep on prev B1
	$= 1+\frac{2}{3}x-\frac{4}{9}x^2 + ...$	√A1		$-\frac{8}{18}x^2$ acceptable
	Required expansion = 2 (expansion just found)	√B1	**4**	accept equiv fractions

N.B. If not based on part **(i)**, award M1 for $8^{1/3} + \frac{1}{3}.8^{-2/3}(16x) + \frac{\frac{1}{3}.-\frac{2}{3}}{1.2}8^{-5/3}(16x)^2$, allowing $16x^2$ for $(16x)^2$, with 3 @ A1 for $2 ... + \frac{4}{3}x ... - \frac{8}{9}x^2$, accepting equivalent fractions & ISW

(ii) (b)	$-\frac{1}{2} < x < \frac{1}{2}$ or $\lvert x \rvert < \frac{1}{2}$	B1	**1**	no equality
		7		
6	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$	M1		quoted/implied
	$\frac{dx}{dt} = 9 - \frac{9}{9t}$ ISW	B1		
	$\frac{dy}{dt} = 3t^2 - \frac{3t^2}{t^3}$ ISW	B1		
	Stating/implying $\frac{3t^2 - \frac{3}{t}}{9 - \frac{1}{t}} = 3 \Rightarrow t^2 = 9$ or $t^3 - 9t = 0$	A1		WWW, totally correct at this stage
	$t = 3$ as final ans with clear log indication of invalidity of -3; ignore (non) mention of $t = 0$	A2		**S.R.** A1 if $t = \pm 3$ or $t = -3$ or ($t = 3$ **&** wrong/no indication)
		6		
7	Treat $\frac{d}{dx}(x^2 y)$ as a product	M1		
	$\frac{d}{dx}(y^3) = 3y^2 \frac{dy}{dx}$	B1		
	$3x^2 + 2x^2\frac{dy}{dx} + 4xy = 3y^2\frac{dy}{dx}$	A1		Ignore $\frac{dy}{dx} =$ if not used
	Subst (2, 1) and solve for $\frac{dy}{dx}$ or vice-versa	M1		
	$\frac{dy}{dx} = -4$ WWW	A1		
	grad normal = $-\frac{1}{\text{their } \frac{dy}{dx}}$	√A1		stated or used
	Find eqn of line, through (2, 1), with either gradient	M1		using their $\frac{dy}{dx}$ or $-\frac{1}{\text{their } \frac{dy}{dx}}$
	$x - 4y + 2 = 0$	A1		AEF with integral coefficients
		8		

8 (i)	$-\sin x\, e^{\cos x}$	B1	**1**	
(ii)	$\int \sin x\, e^{\cos x} dx = -e^{\cos x}$	B1		anywhere in part **(ii)**
	Parts with split $u = \cos x, dv = \sin x\, e^{\cos x}$	M1		result $f(x) +/- \int g(x)\,dx$
	Indef Integ, 1st stage $-\cos x\, e^{\cos x} - \int \sin x\, e^{\cos x} dx$	A1		accept $\ldots - \int -e^{\cos x}.-\sin x\,dx$
	Second stage = $-\cos x\, e^{\cos x} + e^{\cos x}$	*A1		
	Final answer = 1	dep*A2	**6**	
		7		
9 (i)	P is $\begin{pmatrix}3\\1\\1\end{pmatrix}+\begin{pmatrix}1\\-1\\2\end{pmatrix}=\begin{pmatrix}4\\0\\3\end{pmatrix}$	B1		
	direction vector of ℓ is $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and of $\overrightarrow{OP}$ is their P	√B1		
	Use $\cos\theta = \dfrac{\mathbf{a}.\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$ for $\begin{pmatrix}1\\-1\\2\end{pmatrix}$ and their OP	M1		
	$\theta = 35.3$ or better (0.615… rad)	A1	**4**	
(ii)	Use $\begin{pmatrix}1\\-1\\2\end{pmatrix}.\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}=0$	M1		
	$1(3+t)-1(1-t)+2(1+2t)=0$	A1		
	$t=-\frac{2}{3}$	A1		
	Subst. into $\begin{pmatrix}3+t\\1-t\\1+2t\end{pmatrix}$ to produce $\begin{pmatrix}7/3\\5/3\\-1/3\end{pmatrix}$ ISW	A1	**4**	
(iii)	Use $\sqrt{x^2+y^2+z^2}$ where $\begin{pmatrix}x\\y\\z\end{pmatrix}$ is part **(ii)** answer	M1		
	Obtain $\sqrt{\frac{75}{9}}$ AEF, 2.89 or better (2.8867513….)	A1	**2**	
		10		

10 (i)	$\dfrac{\frac{1}{3}}{3-x}$ ……. $-\dfrac{\frac{1}{3}}{6-x}$	B1+1 **2**	
(ii) **(a)**	Separate variables $\int \dfrac{1}{(3-x)(6-x)}\,\mathrm{d}x = \int k\,\mathrm{d}t$	M1	or invert both sides
	<u>Style</u>: For the M1, dx & dt must appear on correct sides <u>or</u> there must be $\int$ sign on both sides		
	Change $\dfrac{1}{(3-x)(6-x)}$ into partial fractions from **(i)**	√B1	
	$\int \dfrac{A}{3-x}\,\mathrm{d}x = \left(-A \text{ or } -\dfrac{1}{A}\right)\ln(3-x)$	B1	or $\int \dfrac{B}{6-x}\,\mathrm{d}x = \left(-B \text{ or } -\dfrac{1}{B}\right)\ln(6-x)$
	$-\dfrac{1}{3}\ln(3-x)+\dfrac{1}{3}\ln(6-x) = kt\ (+c)$	√A1	f.t. from wrong multiples in **(i)**
	Subst (x = 0, t = 0) & (x = 1, t = 1) into eqn with ‘c’	M1	and solve for ‘k’
	Use $\ln a + \ln b = \ln ab$ or $\ln a - \ln b = \ln \frac{a}{b}$	M1	
	Obtain $k = \dfrac{1}{3}\ln\dfrac{5}{4}$ with sufficient working & WWW	A1 **7**	**AG**
(b)	Substitute $k = \dfrac{1}{3}\ln\dfrac{5}{4}$, $t = 2$ & their value of ‘c’	*M1	
	Reduce to an eqn of form $\dfrac{6-x}{3-x} = \lambda$	dep*M1	where λ is a const
	Obtain $x = \dfrac{27}{17}$ or 1.6 or better (1.5882353…)	A2 **4**	S.R. A1√ for $x = \dfrac{3\lambda-6}{\lambda-1}$
		13	