OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)   **4761**

Mechanics 1

Wednesday 27 January 2010
Afternoon

Duration: 1 hour 30 minutes

4 7 6 1

INSTRUCTIONS TO CANDIDATES

*   Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
*   Use black ink. Pencil may be used for graphs and diagrams only.
*   Read each question carefully and make sure that you know what you have to do before starting your answer.
*   Answer **all** the questions.
*   Do **not** write in the bar codes.
*   You are permitted to use a graphical calculator in this paper.
*   Final answers should be given to a degree of accuracy appropriate to the context.
*   The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

*   The number of marks is given in brackets **[ ]** at the end of each question or part question.
*   You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
*   The total number of marks for this paper is **72**.
*   This document consists of **8** pages. Any blank pages are indicated.

Section A (36 marks)

1   A ring is moving up and down a vertical pole. The displacement, s m, of the ring above a mark on the pole is modelled by the displacement-time graph shown in Fig. 1. The three sections of the graph are straight lines.
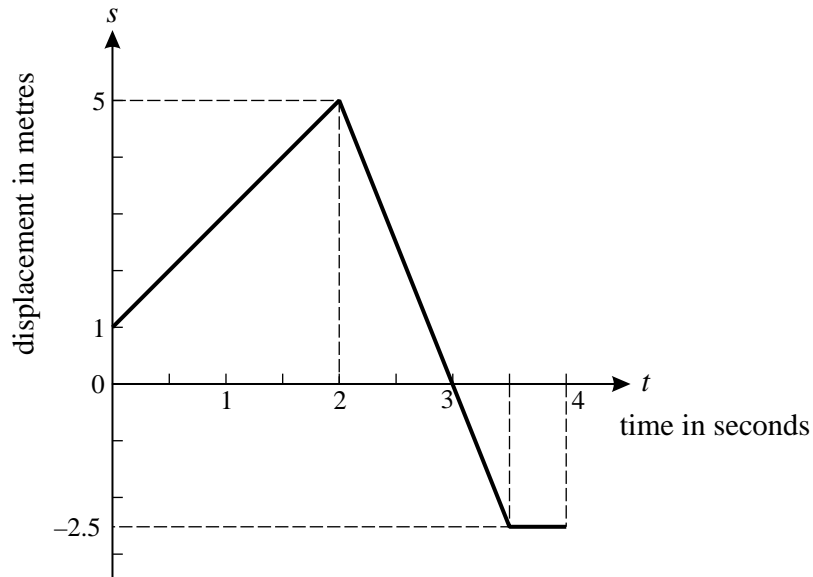

Fig. 1

   (i) Calculate the velocity of the ring in the interval $0 < t < 2$ and in the interval $2 < t < 3.5$.   **[2]**

   (ii) Sketch a velocity-time graph for the motion of the ring during the 4 seconds.   **[2]**

   (iii) State the direction of motion of the ring when

      (*A*) $t = 1$,

      (*B*) $t = 2.75$,

      (*C*) $t = 3.25$.   **[1]**

2   A particle of mass 5 kg has constant acceleration. Initially, the particle is at $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ m with velocity $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ m s^{-1}; after 4 seconds the particle has velocity $\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ m s^{-1}.

   (i) Calculate the acceleration of the particle.   **[2]**

   (ii) Calculate the position of the particle at the end of the 4 seconds.   **[3]**

   (iii) Calculate the force acting on the particle.   **[2]**

3 In this question, **i** is a horizontal unit vector and **j** is a unit vector pointing vertically upwards.

A force **F** is −**i** + 5**j**.

   (i) Calculate the magnitude of **F**.

   Calculate also the angle between **F** and the upward vertical. **[4]**

Force **G** is 2*a***i** + *a***j** and force **H** is −2**i** + 3*b***j**, where *a* and *b* are constants. The force **H** is the resultant of forces 4**F** and **G**.

   (ii) Find **G** and **H**. **[4]**

4 A box of mass 2.5 kg is on a smooth horizontal table, as shown in Fig. 4. A light string AB is attached to the table at A and the box at B. AB is at an angle of 50° to the vertical. Another light string is attached to the box at C; this string is inclined at 15° above the horizontal and the tension in it is 20 N. The box is in equilibrium.
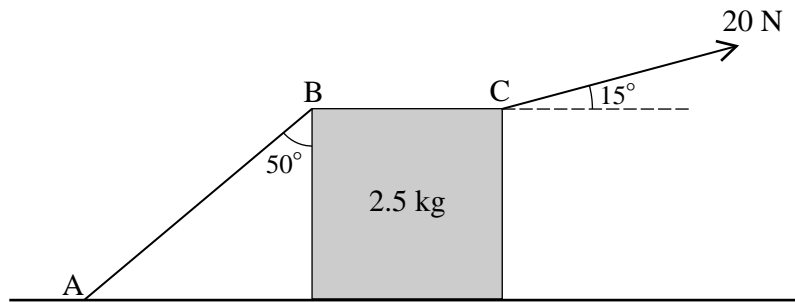

Fig. 4

   (i) Calculate the horizontal component of the force exerted on the box by the string at C. **[1]**

   (ii) Calculate the tension in the string AB. **[2]**

   (iii) Calculate the normal reaction of the table on the box. **[4]**

The string at C is replaced by one inclined at 15° below the horizontal with the same tension of 20 N.

   (iv) Explain why this has no effect on the tension in string AB. **[1]**

5 The velocity, $v\,\text{m s}^{-1}$, of a particle moving along a straight line is given by

$$v = 3t^2 − 12t + 14,$$

where *t* is the time in seconds.

   (i) Find an expression for the acceleration of the particle at time *t*. **[2]**

   (ii) Find the displacement of the particle from its position when *t* = 1 to its position when *t* = 3. **[4]**

   (iii) You are *given* that *v* is always positive. Explain how this tells you that the distance travelled by the particle between *t* = 1 and *t* = 3 has the same value as the displacement between these times. **[2]**

Section B (36 marks)

6

barge A
mass 1.5 kg
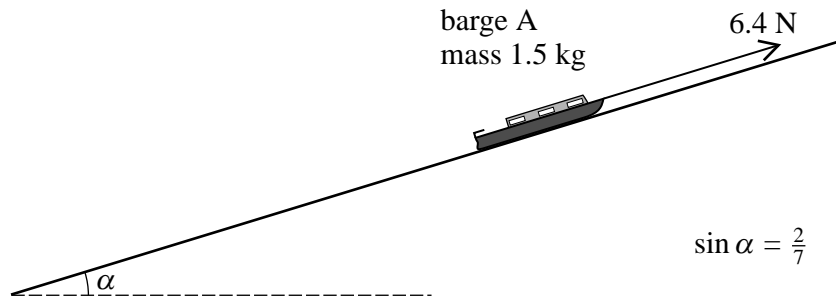
6.4 N

$\sin \alpha = \frac{2}{7}$

α

Fig. 6.1

Fig. 6.1 shows a toy barge A of mass 1.5 kg on a rough plane. The plane is at an angle α to the horizontal where $\sin \alpha = \frac{2}{7}$.

(i) Show that the component of the weight of the barge down the slope is 4.2 N. **[2]**

The barge is held in equilibrium by a force of 6.4 N acting up and parallel to the plane.

(ii) Determine the frictional force on the barge and state whether it acts up or down the plane. **[2]**

The force of 6.4 N is removed and the barge now slides down the plane with acceleration $1.2 \, \mathrm{m \, s^{-2}}$.

(iii) Calculate the new frictional force on the barge. **[4]**

(iv) Determine how far the barge travels while its speed increases from $0.8 \, \mathrm{m \, s^{-1}}$ to $2 \, \mathrm{m \, s^{-1}}$. **[3]**

Fig. 6.2 shows barge A on the same slope with a second barge B of mass 2 kg attached to it by means of a light rigid coupling parallel to the plane. The frictional force on barge B is 0.7 N and the frictional force on barge A is now 2.3 N. At one stage of the motion the two barges are being pulled up the plane by a force of 10 N parallel to the plane.
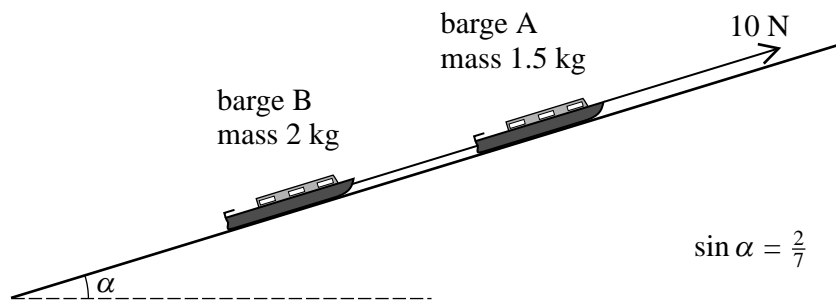

barge A
mass 1.5 kg

10 N

barge B
mass 2 kg

$\sin \alpha = \frac{2}{7}$

α

Fig. 6.2

(v) Draw diagrams showing the forces acting on each barge.

Calculate the acceleration of the barges and clearly indicate its direction.

Find the force in the coupling, stating whether this is a tension or a thrust (compression). **[7]**
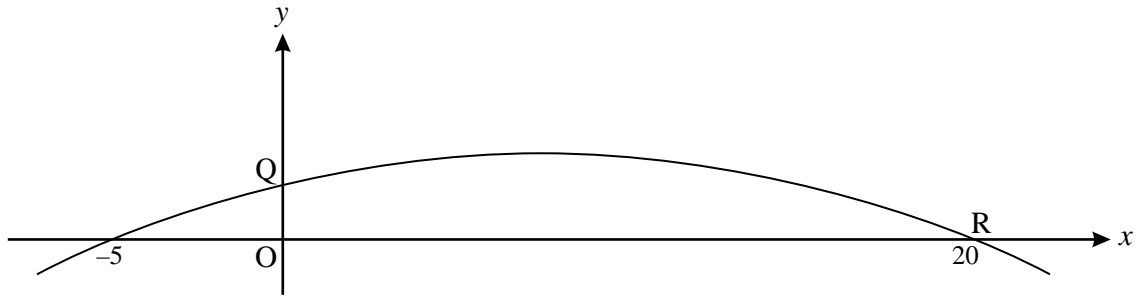
7

Fig. 7

Fig. 7 shows the graph of $y = \frac{1}{100}(100 + 15x - x^2)$.

For $0 \leqslant x \leqslant 20$, this graph shows the trajectory of a small stone projected from the point Q where y m is the height of the stone above horizontal ground and x m is the horizontal displacement of the stone from O. The stone hits the ground at the point R.

(i) Write down the height of Q above the ground. **[1]**

(ii) Find the horizontal distance from O of the highest point of the trajectory and show that this point is 1.5625 m above the ground. **[5]**

(iii) Show that the time taken for the stone to fall from its highest point to the ground is 0.565 seconds, correct to 3 significant figures. **[3]**

(iv) Show that the horizontal component of the velocity of the stone is $22.1\,\text{m s}^{-1}$, correct to 3 significant figures. Deduce the time of flight from Q to R. **[5]**

(v) Calculate the speed at which the stone hits the ground. **[4]**