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## Level 2 Certificate FURTHER MATHEMATICS

Paper 2 Calculator

Monday 17 June 2019

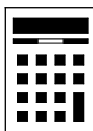
Afternoon

Time allowed: 2 hours

### Materials

For this paper you must have:

- a calculator
- mathematical instruments.



### Instructions

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.
- In all calculations, show clearly how you work out your answer.

### Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 105.
- You may ask for more answer paper, graph paper and tracing paper. These must be tagged securely to this answer book.
- The use of a calculator is expected but calculators with a facility for symbolic algebra must **not** be used.

For Examiner's Use	
Pages	Mark
3	
4–5	
6–7	
8–9	
10–11	
12–13	
14–15	
16–17	
18–19	
20–21	
22–23	
24–25	
26–27	
28–29	
30	
<b>TOTAL</b>	

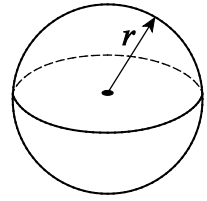


## Formulae Sheet

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outside the  
box

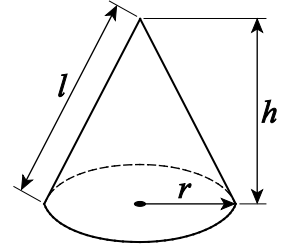
$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$

$$\text{Surface area of sphere} = 4\pi r^2$$



$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Curved surface area of cone} = \pi r l$$



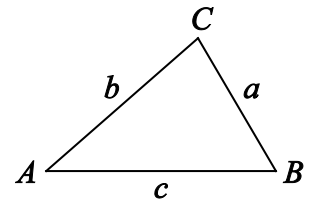
In any triangle  $ABC$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

$$\text{Sine rule} \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Cosine rule} \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$



### The Quadratic Equation

The solutions of  $ax^2 + bx + c = 0$ , where  $a \neq 0$ , are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

### Trigonometric Identities

$$\tan \theta \equiv \frac{\sin \theta}{\cos \theta} \quad \sin^2 \theta + \cos^2 \theta \equiv 1$$



Answer **all** questions in the spaces provided.

Do not write  
outside the  
box

1 (a)  $a \begin{pmatrix} 3 \\ 5 \end{pmatrix} = 4 \begin{pmatrix} 2a+3 \\ b \end{pmatrix}$

Work out the values of  $a$  and  $b$ .

[3 marks]

$a =$  \_\_\_\_\_  $b =$  \_\_\_\_\_

1 (b)  $\begin{pmatrix} m & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & 2 \\ -2 & -1 \end{pmatrix} = \mathbf{I}$  where  $\mathbf{I}$  is the identity matrix.

Work out the value of  $m$ .

[2 marks]

Answer \_\_\_\_\_

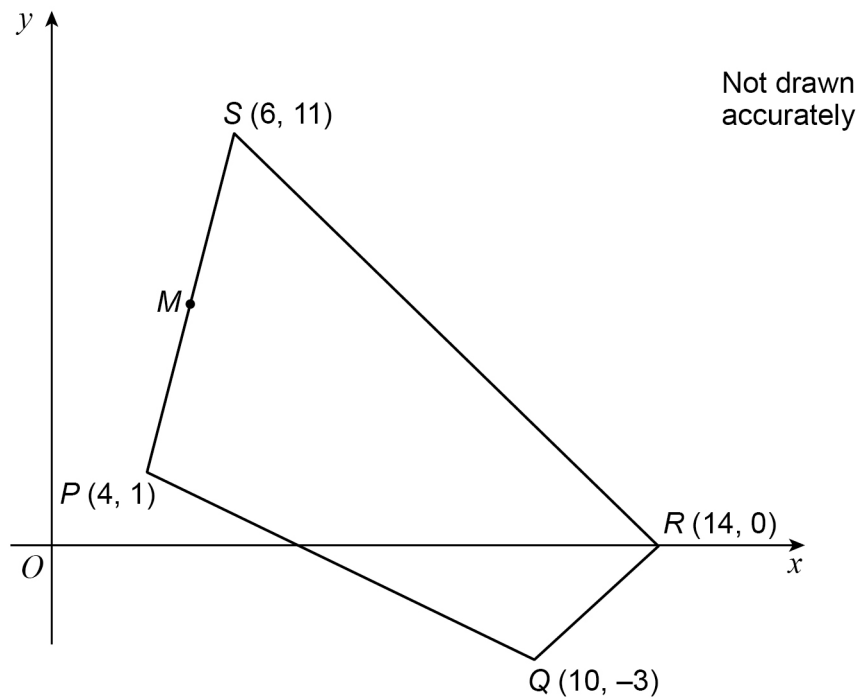
5

Turn over ►



2 Here is a sketch of quadrilateral  $PQRS$ .

$M$  is the midpoint of  $PS$ .



Use gradients to show that  $MR$  is parallel to  $PQ$ .

[3 marks]

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**3**  $-2 < a < 0$  and  $-1 < b < 1$

Tick the correct box for each statement.

[4 marks]

	Always true	Sometimes true	Never true
$a^2 < 0$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$-1 < b^3 < 1$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$\frac{b}{a} < 0$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
$a - b > 0$	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Turn over for the next question

Turn over ►



- 4**  $P$  is a point on a curve.  
The curve has **gradient function**  $\frac{x^5 - 17}{10}$   
The tangent to the curve at  $P$  is parallel to the line  $3x - 2y = 9$   
Work out the  $x$ -coordinate of  $P$ . **[4 marks]**

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Answer \_\_\_\_\_



5 (a) Write  $\sqrt[4]{a \times a^{-9}}$  as an integer power of  $a$ .

[2 marks]

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Answer \_\_\_\_\_

5 (b) Simplify fully  $\frac{(4cd^2)^3}{2cd^4}$

[3 marks]

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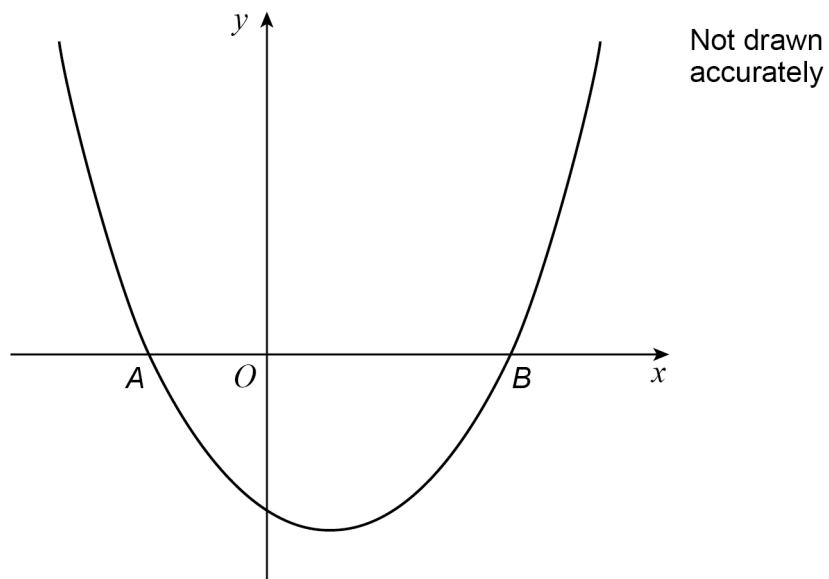
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Answer \_\_\_\_\_

Turn over for the next question



- 6** Here is a sketch of the curve  $y = (2x + 3)(x - 2)$   
The curve intersects the  $x$ -axis at  $A$  and  $B$ .



- 6 (a)** Complete the coordinates of  $A$  and  $B$ .

[2 marks]

$A$  ( \_\_\_\_\_ , 0 )

$B$  ( \_\_\_\_\_ , 0 )

- 6 (b)** Write down the range of values for  $x$  for which  $(2x + 3)(x - 2) < 0$

[1 mark]

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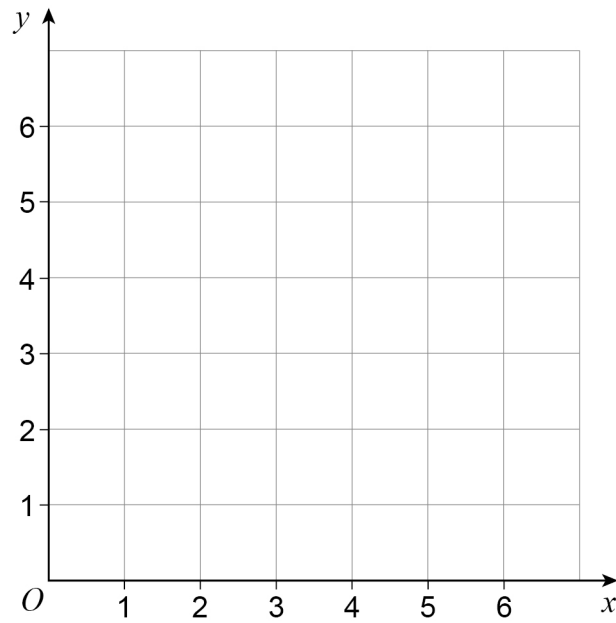
Answer \_\_\_\_\_





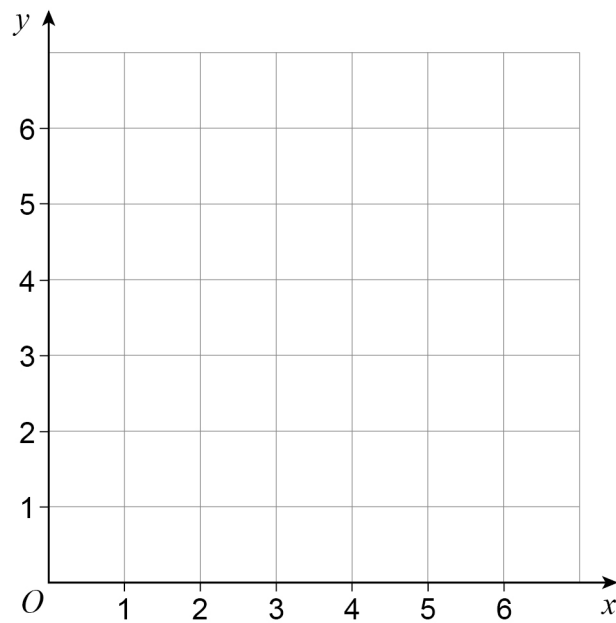
- 7 (a)** On the grid, sketch a graph for which  
the rate of change of  $y$  with respect to  $x$  is always zero.

[1 mark]



- 7 (b)** On the grid, sketch a graph for which  
the rate of change of  $y$  with respect to  $x$  is always a positive constant.

[1 mark]



**8 (a)** A linear sequence has first term  $7 + 12\sqrt{5}$

The term-to-term rule is

add  $9 - 2\sqrt{5}$

One term of the sequence is an integer.

Work out the value of this integer.

**[2 marks]**

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Answer \_\_\_\_\_

**8 (b)** The  $n$ th term of a different sequence is  $\frac{3n^2 - 1}{n^2 + 1}$

Work out the sum of the first three terms.

**[2 marks]**

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Answer \_\_\_\_\_



**8 (c)** The first four terms of a quadratic sequence are

-3          3          13          27

Work out an expression for the  $n$ th term.

**[3 marks]**

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Answer \_\_\_\_\_

**Turn over for the next question**



- 9** Factorise fully  $(p + 6)^{11} - (p + 6)^{10}$  **[2 marks]**

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Answer \_\_\_\_\_

- 10 (a)**  $f(x) = x^3 - 2$   
The domain of  $f(x)$  is  $x \leq 3$   
Work out the range of  $f(x)$ . **[2 marks]**

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Answer \_\_\_\_\_

- 10 (b)**  $g(x) = 5 - x^2$   
The domain of  $g(x)$  is  $-2 \leq x \leq 1$   
Work out the range of  $g(x)$ . **[2 marks]**

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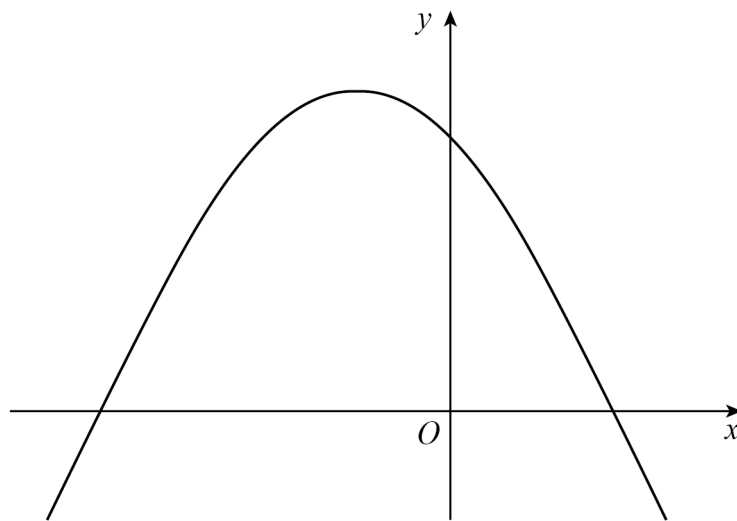
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Answer \_\_\_\_\_



11

Here is a sketch of a quadratic curve which has a maximum point at  $(-2, 5)$



Not drawn  
accurately

What is the equation of the normal to the curve at the maximum point?

Circle your answer.

[1 mark]

$x = -2$

$y = 5$

$x = 5$

$y = -2$

Turn over for the next question

Turn over ►

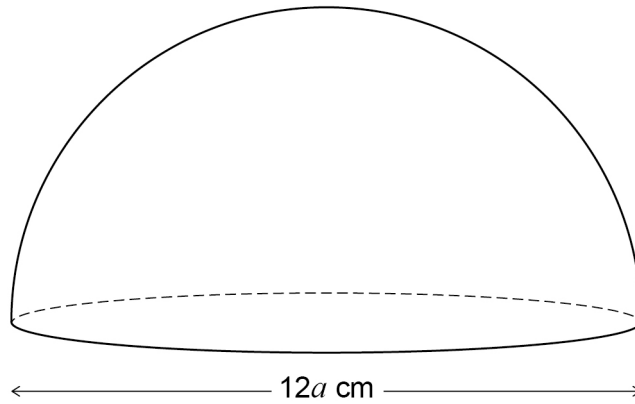


12

The diagram shows a solid hemisphere.

The diameter is  $12a$  cm

The volume is  $486\pi$  cm<sup>3</sup>



Work out the value of  $a$ .

[3 marks]

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Answer \_\_\_\_\_



13 Simplify fully  $\frac{x - x^3}{2x + 2x^2}$

You **must** show your working.

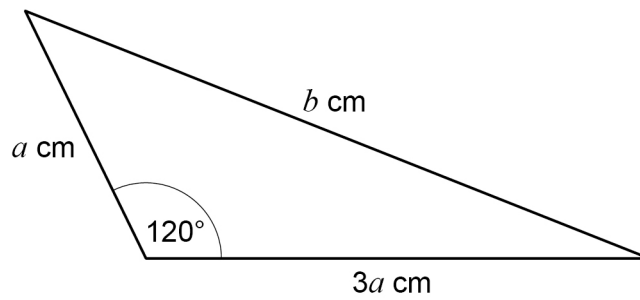
[4 marks]

Answer \_\_\_\_\_

**Turn over for the next question**



14 Here is a triangle.



Not drawn  
accurately

Use the cosine rule to work out the ratio  $b^2 : a^2$

[3 marks]

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Answer \_\_\_\_\_ :





- 15 Rearrange  $m = \frac{2p+1}{p} + \frac{p+5}{3p}$  to make  $p$  the subject.

[4 marks]

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Answer \_\_\_\_\_

- 16 The curve  $y = 2\sqrt{x-a} + 5$  passes through the point (1, 8)

Work out the value of  $a$ .

[3 marks]

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Answer \_\_\_\_\_



**17** Show that  $(x + 1)(x + 3)(x + 4) - x(x^2 + 7x + 11)$   
can be written in the form  $(x + a)(x + b)$  where  $a$  and  $b$  are positive integers.

**[5 marks]**

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- 18** Solve  $4(x - 5)^2 = k^2$  where  $k$  is a constant.  
Give your answers in their simplest form in terms of  $k$ .

**[3 marks]**

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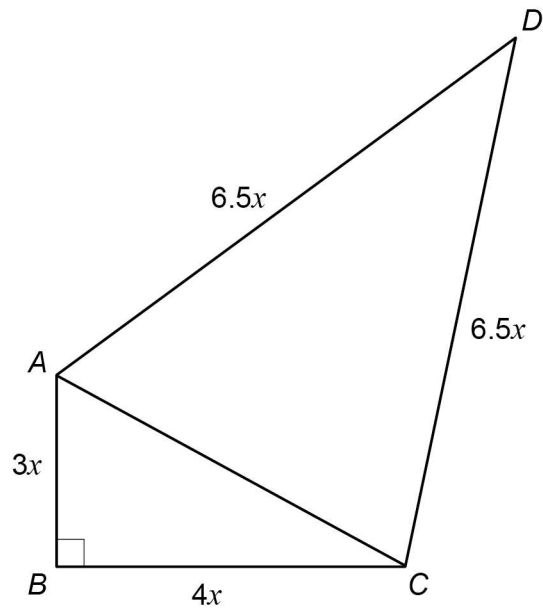
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Answer \_\_\_\_\_

**Turn over for the next question****Turn over ►**

- 19**  $ABC$  is a right-angled triangle.  
 $ACD$  is an isosceles triangle.  
All dimensions are in centimetres.



Not drawn  
accurately

- 19 (a)** Show that  $AC = 5x$

[1 mark]

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**19 (b)** Work out an expression, in  $\text{cm}^2$ , for the area of quadrilateral  $ABCD$ .

Give your answer in the form  $px^2$  where  $p$  is an integer.

**[5 marks]**

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Answer \_\_\_\_\_  $\text{cm}^2$

**Turn over for the next question**

Turn over ►



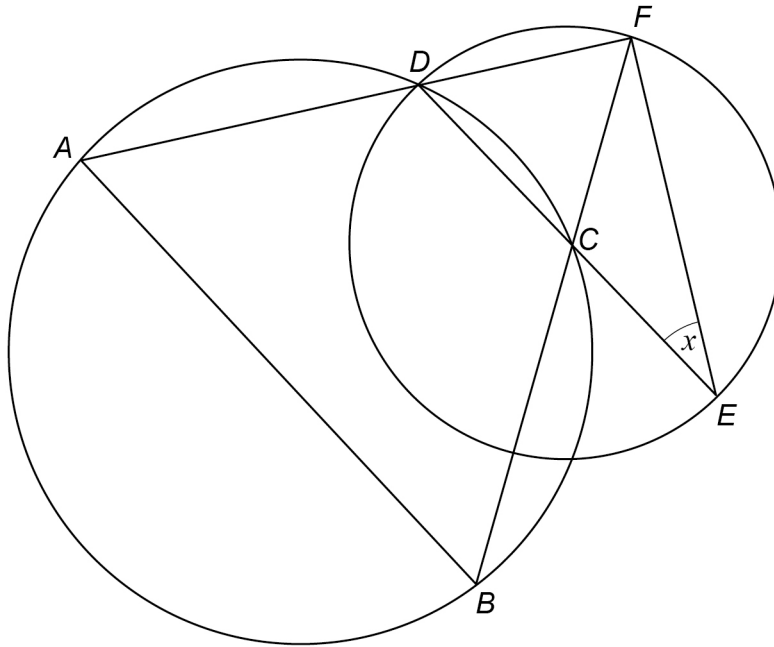
20

$A, B, C$  and  $D$  are points on a circle.

$D, E$  and  $F$  are points on a different circle, centre  $C$ .

$DCE, ADF$  and  $BCF$  are straight lines.

angle  $DEF = x$



Not drawn  
accurately

20 (a) Prove that angle  $BAD = 2x$

[3 marks]

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**20 (b)** In the case when  $AB$  is parallel to  $DE$ , work out the size of angle  $x$ .

**[2 marks]**

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Answer \_\_\_\_\_ degrees

**Turn over for the next question**



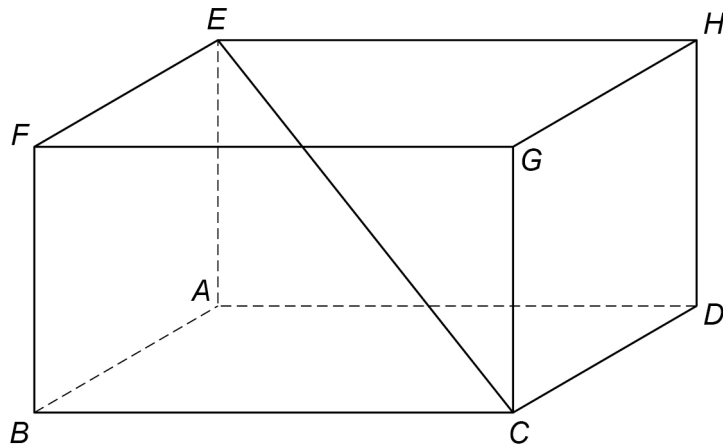
21

 $ABCDEFGH$  is a cuboid.

$BC = 15 \text{ cm}$

$CD = 12 \text{ cm}$

$DH = 8 \text{ cm}$

Work out the size of the angle between the line  $CE$  and the plane  $CDHG$ .**[4 marks]**


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Answer \_\_\_\_\_ degrees





22 (a) Show that  $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x}$  is equivalent to  $\tan x$

[3 marks]

22 (b) Hence solve  $\frac{2\sin^2 x - 1 + \cos^2 x}{\sin x \cos x} = -1$  for  $0^\circ \leq x \leq 360^\circ$

[2 marks]

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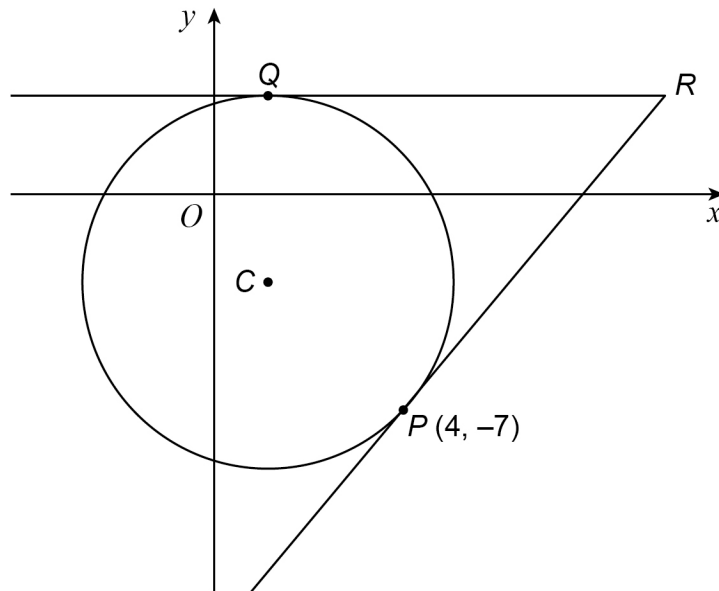
Answer \_\_\_\_\_



- 23** A circle has centre  $C$  and equation  $(x - 1)^2 + (y + 3)^2 = 25$   
 $P(4, -7)$  and  $Q$  are points on the circle.

The tangent at  $Q$  is parallel to the  $x$ -axis.

The tangents at  $P$  and  $Q$  intersect at point  $R$ .



Not drawn  
accurately

- 23 (a)** Write down the coordinates of  $C$ .

**[1 mark]**

Answer \_\_\_\_\_



**23 (b)** Show that the equation of the tangent at Q is  $y = 2$

**[1 mark]**

**23 (c)** Work out the  $x$ -coordinate of  $R$ .

**[4 marks]**

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Answer \_\_\_\_\_

      
**6**

**Turn over ►**



- 24 Show that the curve  $y = \frac{3}{5}x^5 + x^4$  has **exactly two** stationary points. **[4 marks]**

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- 25  $f(x) = x^3 - 10x - c$  where  $c$  is a positive integer.  
 $(x + c)$  is a factor of  $f(x)$ .

Use the factor theorem to work out the value of  $c$ .

**[3 marks]**

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Answer \_\_\_\_\_



**26**  $f(x)$  is a function with domain all values of  $x$ .

$$f(x) = \sqrt{x^2 + 6x - a} \quad \text{where } a \text{ is a constant.}$$

Work out the possible values of  $a$ .

Give your answer as an inequality.

**[4 marks]**

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Answer \_\_\_\_\_

**Turn over for the next question**



27 The curve  $y = f(x)$  has  $\frac{dy}{dx} = (x + 2)^6 + (x + 2)^4$

The curve has exactly one stationary point at  $P$  where  $x = -2$

Use the expression for  $\frac{dy}{dx}$  to show that  $P$  is a point of inflection.

[3 marks]

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END OF QUESTIONS



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