ADVANCED GCE **4753/01**

MATHEMATICS (MEI)

Methods for Advanced Mathematics (C3)

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

This document consists of **4** printed pages.

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Section A (36 marks)

1 Differentiate $\sqrt[3]{1+6x^2}$. [4]

2 The functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are defined for all real numbers x by

$$\mathrm{f}(x) = x^2, \qquad \mathrm{g}(x) = x - 2.$$

(i) Find the composite functions $\mathrm{fg}(x)$ and $\mathrm{gf}(x)$. [3]

(ii) Sketch the curves $y = \mathrm{f}(x)$, $y = \mathrm{fg}(x)$ and $y = \mathrm{gf}(x)$, indicating clearly which is which. [2]

3 The profit £P made by a company in its nth year is modelled by the exponential function

$$P = A\mathrm{e}^{bn}.$$

In the first year (when $n = 1$), the profit was £10 000. In the second year, the profit was £16 000.

(i) Show that $\mathrm{e}^b = 1.6$, and find b and A. [6]

(ii) What does this model predict the profit to be in the 20th year? [2]

4 When the gas in a balloon is kept at a constant temperature, the pressure P in atmospheres and the volume $V\,\mathrm{m}^3$ are related by the equation

$$P = \frac{k}{V},$$

where k is a constant. [This is known as Boyle's Law.]

When the volume is $100\,\mathrm{m}^3$, the pressure is 5 atmospheres, and the volume is increasing at a rate of $10\,\mathrm{m}^3$ per second.

(i) Show that $k = 500$. [1]

(ii) Find $\dfrac{\mathrm{d}P}{\mathrm{d}V}$ in terms of V. [2]

(iii) Find the rate at which the pressure is decreasing when $V = 100$. [4]

5 **(i)** Verify the following statement:

'$2^p - 1$ is a prime number for all prime numbers p less than 11'. [2]

(ii) Calculate 23×89, and hence disprove this statement:

'$2^p - 1$ is a prime number for all prime numbers p'. [2]

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6 Fig. 6 shows the curve $e^{2y} = x^2 + y$.

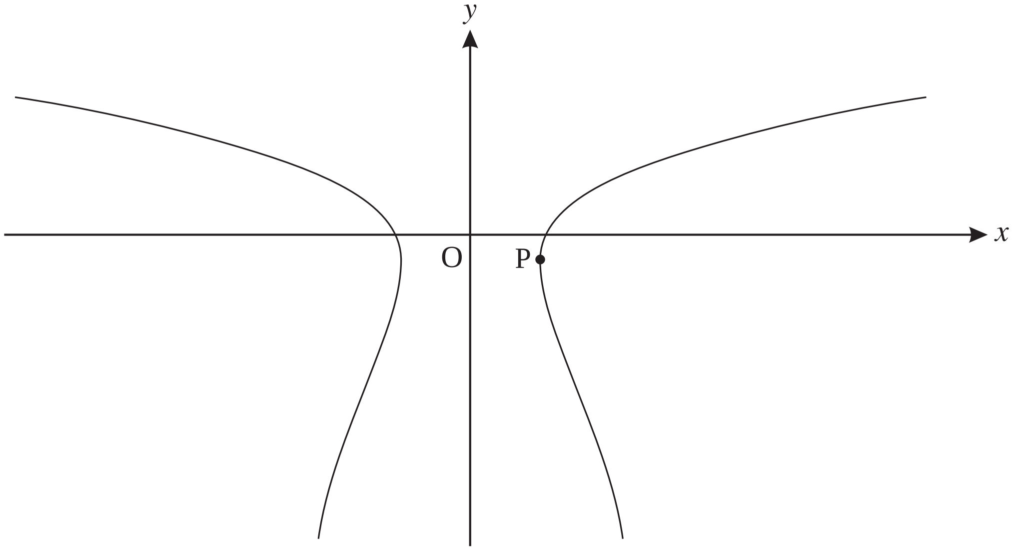

Fig. 6

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x}{2e^{2y} - 1}$. [4]

(ii) Hence find to 3 significant figures the coordinates of the point P, shown in Fig. 6, where the curve has infinite gradient. [4]

Section B (36 marks)

7 A curve is defined by the equation $y = 2x \ln(1 + x)$.

(i) Find $\dfrac{dy}{dx}$ and hence verify that the origin is a stationary point of the curve. [4]

(ii) Find $\dfrac{d^2y}{dx^2}$, and use this to verify that the origin is a minimum point. [5]

(iii) Using the substitution $u = 1 + x$, show that $\displaystyle\int \frac{x^2}{1+x}\,dx = \int \left(u - 2 + \frac{1}{u}\right) du$.

Hence evaluate $\displaystyle\int_0^1 \frac{x^2}{1+x}\,dx$, giving your answer in an exact form. [6]

(iv) Using integration by parts and your answer to part **(iii)**, evaluate $\displaystyle\int_0^1 2x \ln(1 + x)\,dx$. [4]

8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = 1 + \sin 2x$ for $-\frac{1}{4}\pi \leqslant x \leqslant \frac{1}{4}\pi$.

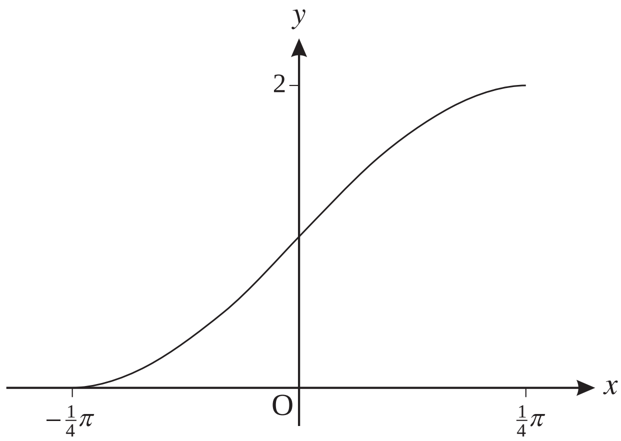

Fig. 8

(i) State a sequence of two transformations that would map part of the curve $y = \sin x$ onto the curve $y = f(x)$. [4]

(ii) Find the area of the region enclosed by the curve $y = f(x)$, the x-axis and the line $x = \frac{1}{4}\pi$. [4]

(iii) Find the gradient of the curve $y = f(x)$ at the point $(0, 1)$. Hence write down the gradient of the curve $y = f^{-1}(x)$ at the point $(1, 0)$. [4]

(iv) State the domain of $f^{-1}(x)$. Add a sketch of $y = f^{-1}(x)$ to a copy of Fig. 8. [3]

(v) Find an expression for $f^{-1}(x)$. [2]

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