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General Certificate of Education

Mathematics 6360

MPC4 Pure Core 4

Mark Scheme

2007 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC4

Q	Solution	Marks	Total	Comments
1(a)	$2\left(-\frac{1}{2}\right)^2+\left(-\frac{1}{2}\right)-3=-3$	M1A1	2	use of $\pm\frac{1}{2}$ SC NMS –3 1/2 No ISW, so subsequent answer "3" AO
	Alt algebraic division: $\begin{array}{r} x \\ 2x+1\overline{)2x^2+x-3} \\ 2x^2+x \\ \hline -3 \end{array}$	(M1)		complete division with integer remainder
		(A1)	(2)	remainder = -3 stated, or –3 highlighted
	Alt $\frac{x(2x+1)-3}{2x+1}$	(M1)		attempt to rearrange numerator with $(2x+1)$ as a factor
		(A1)	(2)	remainder $=-3$ stated, or –3 highlighted
(b)	$\frac{(2x+3)(x-1)}{(x+1)(x-1)}$	B1 B1		$\left.\begin{array}{l}\text{numerator}\\ \text{denominator}\end{array}\right\}$ not necessarily in fraction
	$=\frac{2x+3}{x+1}$	B1	3	CAO in this form. Not $\frac{2x+3}{x+1}\frac{\cancel{x-1}}{\cancel{x-1}}$
(b)	**Alternative** $\frac{2x^2-2+x-1}{x^2-1}$			
	$=2+\frac{x-1}{x^2-1}$	(M1)		
	$=2+\frac{x-1}{(x-1)(x+1)}$	(B1)		
	$=2+\frac{1}{x+1}$	(A1)	(3)	
	Total		**5**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$(1+x)^{-1}=1+(-1)x+px^2+qx^3$	M1		$p\neq 0,\ q\neq 0$
	$=1-x+x^2-x^3$	A1	2	SC 1/2 for $=1-x+px^2$
(ii)	$(1+3x)^{-1}=1-3x+(3x)^2-(3x)^3$	M1		x replaced by $3x$ in candidate's (a)(i);condone missing brackets
	$=1-3x+9x^2-27x^3$ **Alt (starting again)**	A1	2	CAO SC x^3-term : $1-3x+\substack{3\\9}x^2$ 1/2
	$(1+3x)^{-1}=1-(3x)+$			
	$\frac{(-1)(-2)(3x)^2}{2!}+\frac{(-1)(-2)(-3)(3x)^3}{3!}$	(M1)		condone missing brackets accept 2 for 2!, 3.2 for 3!
	$=1-3x+9x^2-27x^3$	(A1)	(2)	CAO
(b)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$	M1		correct partial fractions form, and multiplication by denominator
	$1+4x=A(1+3x)+B(1+x)$			
	$x=-1,\ x=-\frac{1}{3}$	m1		Use (any) two values of x to find A and B
	$A=\frac{3}{2}\ ,\ B=-\frac{1}{2}$	A1	3	A and B both correct
	Alt:			
	$\frac{1+4x}{(1+x)(1+3x)}=\frac{A}{1+x}+\frac{B}{1+3x}$	(M1)		correct partial fractions form, and multiplication by denominator
	$1+4x=A(1+3x)+B(1+x)$			
	$A+B=1,\ 3A+B=4$	(m1)		Set up and solve
	$A=\frac{3}{2}\ ,\ B=-\frac{1}{2}$	(A1)	(3)	A and B both correct
(c)(i)	$\frac{1+4x}{(1+x)(1+3x)}=\frac{3}{2(1+x)}-\frac{1}{2(1+3x)}$	M1		
	$=\frac{3}{2}\left(1-x+x^2-x^3\right)-\frac{1}{2}\left(1-3x+9x^2-27x^3\right)$	m1		multiply candidate's expansions by A and B, and expand and simplify
	$=1-3x^2+12x^3$	A1	3	CAO SC A and B interchanged, treat as miscopy. $\left(1-4x+13x^2-40x^3\right)$
	Alt:			
	$=\frac{1+4x}{(1+x)(1+3x)}=(1+4x)(1+x)^{-1}(1+3x)^{-1}$			
	$=(1+4x)\left(1-x+x^2-x^3\right)\left(1-3x+9x^2-27x^3\right)$	(M1)		write as product, using expansions condone missing brackets on $(1+4x)$ only
	$=1-4x+13x^2-40x^3+4x-16x^2+52x^3$	(m1)		attempt to multiply the three expansions up to terms in x^3
	$=1-3x^2+12x^3$	(A1)	(3)	CAO
(ii)	$\lvert x\rvert<1$ and $\lvert 3x\rvert<1$	M1		OE and nothing else incorrect
	$\lvert x\rvert<\frac{1}{3}$ (0.33)	A1	2	OE Condone $\leq$
	Total		**12**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$R = 5$ $\tan\alpha = \frac{3}{4}$ (OE) $\alpha = 36.9°$ (ISW 216.9)	B1 M1A1	3	SC1 $\tan\alpha = \frac{4}{3}, \alpha = 53.1°$ R, α PI in (b)
(b)	$\cos(x-\alpha) = \frac{2}{R}$ $x - \alpha = 66.4°$ $x = 103.3°$ $x = 330.4°$	M1 A1 A1F A1F	4	accept $330.5°$, -1 each extra ft on acute α
(c)	minimum value $= -5$ $\cos(x - 36.9) = -1$ $x = 216.9°$	B1F M1 A1	3	ft on R SC $\cos(x + 36.9)$ treat as miscopy 216.9 or better accept graphics calculator solution to this accuracy SC Find max: max = 5 at $(x + 36.9)$ stated 1/3 Max 8/10 for work in radians
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$t=0:\ x=3$	B1	1	
(ii)	$t=14:\ x=15-12e^{-1}$	M1		or $15-12\,e^{\frac{-14}{14}}$
	$=10.6$	A1	2	CAO
(b)(i)	$-5=-12e^{-\frac{t}{14}}$	M1		substitute $x=10$; rearrange to form $p=qe^{-\frac{t}{14}}$
	$\ln\left(\frac{5}{12}\right)=-\frac{t}{14}$ (OE)	m1		take lns correctly
	$t=14\ln\left(\frac{12}{5}\right)$	A1	3	must come from correct working
(ii)	$t=12.256...\approx 12$ days	B1F	1	ft on *a, b* if $a>b$; accept t = 12 NMS Accept 12 from incorrect working in b(i) Accept 13 if 12.2 or 12.3 seen
(c)(i)	$\frac{dx}{dt}=-\frac{1}{14}\times-12e^{-\frac{t}{14}}$	M1		differentiate; allow sign error condone $\frac{dy}{dx}$ used consistently
	$=-\frac{1}{14}(x-15)$	m1		Or $\frac{1}{14}\left(12e^{-\frac{t}{14}}\right)$ and $12e^{-\frac{t}{14}}=15-x$ seen
	$=\frac{1}{14}(15-x)$	A1	3	AG – be convinced CSO
	Alt: $t=-14\ln\left(\frac{15-x}{12}\right)$	(M1)		attempt to solve given equation for *t*
	$\frac{dt}{dx}=\frac{-14\left(-\frac{1}{12}\right)}{\left(\frac{15-x}{12}\right)}$	(m1)		differentiate wrt *x*, with $\frac{1}{\frac{15-x}{12}}$ seen; OE
	$\frac{dt}{dx}=\frac{14}{15-x}\Rightarrow\frac{dx}{dt}=\frac{1}{14}(15-x)$	(A1)	(3)	AG – be convinced
	Alt: (backwards)			
	$\int\frac{dx}{15-x}=\int\frac{dt}{14}=\pm14\ln(15-x)=t+c$	(M1)		
	Use $(0,3):-14\ln(15-x)+14\ln12=t$	(m1)		
	Solve for x: $x=15-12e^{-\frac{t}{14}}$	(A1)	(3)	All steps shown
(ii)	rate of growth = 0.5 (cm per day)	B1	1	Accept $\frac{7}{14}$
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$x = 1,\ 5a^2 - a - 4 = 0$	M1		condone y for a
	$(5a+4)(a-1) = 0, \quad a = 1$	A1	2	AG – be convinced, both factors seen or $a = -\frac{4}{5}$ or $1 \Rightarrow a = 1$ A0 for 2 positive roots (substitute $(1, 1) \Rightarrow 5 = 5$ no marks)
(b)	$\frac{dy}{dx} + 4$	B1B1		(Ignore '$\frac{dy}{dx} =$' if not used, otherwise loses final A1)
	$= 10xy^2 + 10x^2 y \frac{dy}{dx}$	M1		attempt product rule, see two terms added
		M1		chain rule, $\frac{dy}{dx}$ attached to one term only
		A1		condone 5 × 2 for 10
	$x = 1, y = 1 \quad \frac{dy}{dx} + 4 = 10 + 10\frac{dy}{dx}$	M1		two terms, or more, in $\frac{dy}{dx}$
	$\frac{dy}{dx} = -\frac{6}{9} = \left(-\frac{2}{3}\right)$	A1	7	CSO
	Alt (for last two marks)			
	$\frac{dy}{dx} = \frac{10xy^2 - 4}{1 - 10x^2 y}$	(M1)		find $\frac{dy}{dx}$ in terms of x, y and substitute $x = 1,\ y = 1$ must be from expression with two terms or more in $\frac{dy}{dx}$
	$(1,1) \Rightarrow \frac{10-4}{1-10} = -\frac{6}{9}$	(A1)		
(c)	$\frac{y-1}{x-1} = -\frac{2}{3}$ (OE)	B1F	1	ft on gradient ISW after any correct form
	Total		**10**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta \quad \dfrac{\mathrm{d}y}{\mathrm{d}\theta}=2\cos 2\theta$	B1 B1	2	
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{2\cos 2\theta}{\sin\theta},\ \dfrac{\mathrm{d}y}{\mathrm{d}x}=-\dfrac{2\cos\frac{\pi}{3}}{\sin\frac{\pi}{6}}=-2$	M1		use chain rule $\text{their }\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\Big/\text{their }\dfrac{\mathrm{d}x}{\mathrm{d}\theta}$ and
				substitute $\theta=\dfrac{\pi}{6}$
		A1	2	
(b)	$y=2\sin\theta\cos\theta=2\sqrt{1-\cos^2\theta}\cos\theta$	B1 B1		use $\sin 2\theta=2\sin\theta\cos\theta$ use $\sin^2\theta=1-\cos^2\theta$
	$y=2\sqrt{1-x^2}\ x$	M1		$\sin\theta, \cos\theta$ in terms of x
	$y^2=4x^2\left(1-x^2\right)$	A1	4	all correct CSO
	Alt			
	$y^2=\sin^2 2\theta=\left(2\sin\theta\cos\theta\right)^2$	(B1)		use of double angle formula
	$=(4)\sin^2\theta\cos^2\theta=(4)\left(1-\cos^2\theta\right)\cos^2\theta$	(B1)		use of $\mathrm{s}^2+\mathrm{c}^2=1$ to eliminate $\sin\theta$
	$=(4)\left(1-x^2\right)x^2$	(M1)		Substitute $\cos\theta$ for x
	$=4\left(1-x^2\right)x^2$	(A1)	(4)	CSO
	Total		**8**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$\begin{bmatrix}3\\-3\\-1\end{bmatrix}\bullet\begin{bmatrix}1\\2\\-3\end{bmatrix}=3-6+3=0$	M1		attempt at sp, 3 terms, added
	$=0\Rightarrow$ perpendicular	A1	2	$=0\Rightarrow$ perpendicular seen $\left(\text{or }\cos\theta=0\Rightarrow\theta=90^\circ\right)$ Allow $\begin{array}{c}3\\-6\\\underline{3}\\0\end{array}$ but not $\begin{bmatrix}3\\-6\\3\end{bmatrix}=0$
(b)	$8+3\lambda=-4+\mu$ $6-3\lambda=2\mu$ $-9-\lambda=11-3\mu$	M1		set up any two equations
	$\lambda=-2,\mu=6$	m1 A1		solve for λ and μ
	verify third equation	m1		substitute λ,μ in third equation
	intersect at $(2,12,-7)$	A1	5	CAO
	Alt (for last two marks) substitute λ into l_1 and μ into l_2	(m1)		
	intersect at $(2,12,-7)$, condone $\begin{pmatrix}2\\12\\-7\end{pmatrix}$	(A1)		$(2,12,-7)$ found from both lines Note: working for (b) done in (a): award marks in (b)
7(c)	$\overrightarrow{AP}=\begin{pmatrix}6\\12\\-18\end{pmatrix}$	M1		$\overrightarrow{AP}=\pm\left\{\text{their }\overrightarrow{OP}-\begin{pmatrix}-4\\0\\11\end{pmatrix}\right\}$
	$AP^2=504$	A1F		ft on P
	$AB^2=2AP^2$	M1		Calculate AB^2
	$AB = 12\sqrt{7}$	A1	4	OE accept 31.7 or better
	Total		**11**	

MPC4 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$\int \frac{1}{\sqrt{1+2y}}\,\mathrm{d}y = \int \frac{1}{x^2}\,\mathrm{d}x$	M1		attempt to separate and integrate
	$\int \frac{1}{\sqrt{1+2y}}\,\mathrm{d}y = k\sqrt{1+2y}$	m1		
	$\sqrt{1+2y} = -\frac{1}{x}(+c)$	A1 A1		OE A1 for $\sqrt{1+2y}$ depends on both Ms A1 for $-\frac{1}{x}$ depends on first M1 only
	$x=1, y=4 \Rightarrow c=4$	m1		$+c$ must be seen on previous line
		A1F	6	ft on k and $\pm\frac{1}{x}$ only
(b)	$1+2y = \left(4-\frac{1}{x}\right)^2$	m1		need $k\sqrt{1+2y} =$ 'x expression with $+ c$' **and** attempt to square both sides
	$2y = 15 + \frac{1}{x^2} - \frac{8}{x}$	A1	2	terms on RHS in any order AG – be convinced CSO
	Total		**8**	
	TOTAL		**75**	