

## Question 1

(i)	Faults are detected randomly and independently Uniform (mean) rate of occurrence	B1 B1	<b>2</b>
(ii)	(A) $P(X=0) = e^{-0.15} \frac{0.15^0}{0!} = 0.8607$  (B) $P(X \geq 2) = 1 - 0.8607 - e^{-0.15} \frac{0.15^1}{1!}$  $= 1 - 0.8607 - 0.1291 = 0.0102$	M1 for probability calc. M0 for tables unless interpolated A1  M1 A1	<b>4</b>
(iii)	$\lambda = 30 \times 0.15 = 4.5$ Using tables: $P(X \leq 3) = 0.3423$	B1 for mean (SOI) M1 attempt to find $P(X \leq 3)$ A1	<b>3</b>
(iv)	Poisson distribution with $\lambda = 10 \times (0.15 + 0.05) = 2$ $P(X=5) = e^{-2} \frac{2^5}{5!} = 0.0361$ (3 s.f.) or from tables $= 0.9834 - 0.9473 = 0.0361$	B1 for Poisson stated B1 for $\lambda = 2$ M1 for calculation or use of tables A1 FT	<b>4</b>
(v)	Mean no. of items in 200 days $= 200 \times 0.2 = 40$ Using Normal approx. to the Poisson, $X \sim N(40,40)$ : $P(X \geq 50) = P\left(Z > \frac{49.5 - 40}{\sqrt{40}}\right)$ $= P(Z > 1.502) = 1 - \Phi(1.502) = 1 - 0.9334$ $= 0.0666$ (3 s.f.)	B1 for Normal approx. (SOI) B1 for both parameters  B1 for continuity corr.  M1 for probability using correct tail A1 <b>cao</b> , (but FT wrong or omitted CC)	<b>5</b>
			<b>18</b>

## Question 2

(i) (A)	$X \sim N(42, 3^2)$ $P(X > 50.0) = P\left(Z > \frac{50.0 - 42.0}{3.0}\right)$ $= P(Z > 2.667)$ $= 1 - \Phi(2.667) = 1 - 0.9962$ $= 0.0038$	M1 for standardizing M1 for prob. calc. with correct tail A1 <b>NB answer given</b>	3
(i) (B)	$P(\text{not positive}) = 0.9962$ $P(\text{At least one is out of 7 is positive})$ $= 1 - 0.9962^7 = 1 - 0.9737$ $= 0.0263$	B1 for use of 0.9962 in binomial expression  M1 for correct method  A1 CAO	3
(i) (C)	If an innocent athlete is tested 7 times in a year there is a reasonable possibility (1 in 40 chance) of testing positive. Thus it is likely that a number of innocent athletes may come under suspicion and suffer a suspension so the penalty could be regarded as unfair. <i>Or</i> this is a necessary evil in the fight against performance enhancing drugs in sport.	E1 comment on their probability in (i) B  E1 for sensible contextual conclusion consistent with first comment	2
(ii) (A)	$B(1000, 0.0038)$	B1 for $B(, )$ or Binomial B1 <i>dep</i> for both parameters	2
(ii) (B)	A suitable approximating distribution is Poisson(3.8) $P(\text{at least 10 positive tests})$ $= P(X \geq 10) = 1 - P(X \leq 9)$  $= 1 - 0.9942$  $= 0.0058$ <i>NB Do not allow use of Normal approximation.</i>	B1 for Poisson soi B1FT <i>dep</i> for $\lambda = 3.8$ M1 for attempt to use $1 - P(X \leq 9)$ A1 FT	4
(iii)	$P(\text{not testing positive}) = 0.995$ From tables $z = \Phi^{-1}(0.995) = 2.576$  $\frac{h - 48.0}{2.0} = 2.576$  $h = 48.0 + 2.576 \times 2.0 = 53.15$	B1 for 0.995 seen (or implied by 2.576) B1 for 2.576 (FT their 0.995)  M1 for equation in $h$ and positive $z$ -value  A1 CAO	4
			<b>18</b>

## Question 3

(i)	<table border="1" data-bbox="334 281 995 457"> <tbody> <tr> <td>Rank <math>x</math></td> <td>1</td> <td>5</td> <td>4</td> <td>7</td> <td>6</td> <td>8</td> <td>10</td> <td>3</td> <td>9</td> <td>2</td> </tr> <tr> <td>Rank <math>y</math></td> <td>2</td> <td>4</td> <td>5</td> <td>8</td> <td>9</td> <td>7</td> <td>10</td> <td>6</td> <td>3</td> <td>1</td> </tr> <tr> <td><math>d</math></td> <td>-1</td> <td>1</td> <td>-1</td> <td>-1</td> <td>-3</td> <td>1</td> <td>0</td> <td>-3</td> <td>6</td> <td>1</td> </tr> <tr> <td><math>d^2</math></td> <td>1</td> <td>1</td> <td>1</td> <td>1</td> <td>9</td> <td>1</td> <td>0</td> <td>9</td> <td>36</td> <td>1</td> </tr> </tbody> </table> $r_s = 1 - \frac{6\sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 60}{10 \times 99}$ <p>= 0.636 (to 3 s.f.) [allow 0.64 to 2 s.f.]</p>	Rank $x$	1	5	4	7	6	8	10	3	9	2	Rank $y$	2	4	5	8	9	7	10	6	3	1	$d$	-1	1	-1	-1	-3	1	0	-3	6	1	$d^2$	1	1	1	1	9	1	0	9	36	1	<p>M1 for ranking (allow all ranks reversed)</p> <p>M1 for <math>d^2</math></p> <p>A1 CAO for <math>\sum d^2</math></p> <p>M1 for structure of <math>r_s</math> using their <math>\sum d^2</math></p> <p>A1 f.t. for <math> r_s  &lt; 1</math> NB No ranking scores zero</p>	5
Rank $x$	1	5	4	7	6	8	10	3	9	2																																					
Rank $y$	2	4	5	8	9	7	10	6	3	1																																					
$d$	-1	1	-1	-1	-3	1	0	-3	6	1																																					
$d^2$	1	1	1	1	9	1	0	9	36	1																																					
(ii)	<p><math>H_0</math>: no association between <math>x</math> and <math>y</math></p> <p><math>H_1</math>: positive association between <math>x</math> and <math>y</math></p> <p>Looking for positive association (one-tail test):</p> <p>Critical value at 5% level is 0.5636</p> <p>Since <math>0.636 &gt; 0.5636</math>, there is sufficient evidence to reject <math>H_0</math>, i.e. conclude that there appears to be positive association between temperature and nitrous oxide level.</p>	<p>B1 for <math>H_0</math></p> <p>B1 for <math>H_1</math></p> <p>NB <math>H_0 H_1</math> <u>not</u> ito rho</p> <p>B1 for <math>\pm 0.5636</math></p> <p>(FT their <math>H_1</math>)</p> <p>M1 for comparison with c.v., provided <math> r_s  &lt; 1</math></p> <p>A1 for conclusion in words f.t. their <math>r_s</math> and sensible cv</p>	5																																												
(iii)	<p>Underlying distribution must be bivariate normal.</p> <p>If the distribution is bivariate normal then the scatter diagram will have an elliptical shape.</p> <p>This scatter diagram is not elliptical and so a PMCC test would not be valid.</p> <p>(Allow comment indicating that the sample is too small to draw a firm conclusion on ellipticity and so on validity)</p>	<p>B1 CAO for bivariate normal</p> <p>B1 indep for elliptical shape</p> <p>E1 dep for conclusion</p>	3																																												
(iv)	<p><math>n=60</math>, PMCC critical value is <math>r = 0.2997</math></p> <p>So the critical region is <math>r \geq 0.2997</math></p>	<p>B1</p> <p>B1 FT their sensible c.v.</p>	2																																												
(v)	<p>Any three of the following:</p> <ul style="list-style-type: none"> <li>• Correlation does not imply causation;</li> <li>• There could be a third factor (causing the correlation between temperature and ozone level);</li> <li>• the claim could be true;</li> <li>• increased ozone could cause higher temperatures.</li> </ul>	<p>E1</p> <p>E1</p> <p>E1</p>	3																																												
			18																																												

## Question 4

(i)	<p><math>H_0</math>: no association between method of travel and type of school;  <math>H_1</math>: some association between method of travel and type of school;..</p>	B1 for both	<b>1</b>
(ii)	<p>Expected frequency = <math>120/200 \times 70 = 42</math>  Contribution = <math>(21 - 42)^2 / 42</math>  = 10.5</p>	<p>M1 A1  M1 for valid attempt at <math>(O-E)^2/E</math>  A1 FT their 42 provided <math>O = 21</math> (at least 1 dp)</p>	<b>4</b>
(iii)	<p><math>X^2 = 42.64</math>  Refer to <math>\chi_2^2</math>  Critical value at 5% level = 5.991  Result is significant  There appears to be some association between method of travel and year group.  NB if <math>H_0</math> <math>H_1</math> reversed, or 'correlation' mentioned, do not award first B1 or final E1</p>	<p>B1 for 2 deg of f(seen)  B1 CAO for cv  B1 for significant (FT their c.v. provided consistent with their d.o.f.  E1</p>	<b>4</b>
(iv)	<p><math>H_0: \mu = 18.3</math>; <math>H_1: \mu \neq 18.3</math>  Where <math>\mu</math> denotes the mean travel time by car for the whole population.  Test statistic <math>z = \frac{22.4 - 18.3}{8.0/\sqrt{20}} = \frac{4.1}{1.789} = 2.292</math>  10% level 2 tailed critical value of <math>z</math> is 1.645  <math>2.292 &gt; 1.645</math> so significant.  There is evidence to reject <math>H_0</math>  It is reasonable to conclude that the mean travel time by car is different from that by bus.</p>	<p>B1 for both correct  B1 for definition of <math>\mu</math>  M1 (standardizing sample mean)  A1 for test statistic  B1 for 1.645  M1 for comparison leading to a conclusion  A1 for conclusion in words and context</p>	<b>7</b>
(v)	<p>The test suggests that students who travel by bus get to school more quickly.  This may be due to their journeys being over a shorter distance.  It may be due to bus lanes allowing buses to avoid congestion.  It is possible that the test result was incorrect (ie implication of a Type I error).  More investigation is needed before any firm conclusion can be reached.</p>	E1, E1 for any two valid comments	<b>2</b>
			<b>18</b>

## Question 4 chi squared calculations

$H_0$ : no association between method of travel and type of school; $H_1$ : some association between method of travel and type of school;				
<b>Observed</b>		Type of school		<b>Row totals</b>
		Year 6	Year 11	
Method of travel	Bus	21	49	<b>70</b>
	Car	65	15	<b>80</b>
	Cycle/Walk	34	16	<b>50</b>
<b>Column totals</b>		<b>120</b>	<b>80</b>	<b>200</b>
<b>Expected</b>		Type of school		<b>Row totals</b>
		Year 6	Year 11	
Method of travel	Bus	42	28	<b>70</b>
	Car	48	32	<b>80</b>
	Cycle/Walk	30	20	<b>50</b>
<b>Column totals</b>		<b>120</b>	<b>80</b>	<b>200</b>
<b>Chi Squared Contribution</b>		Type of school		<b>Row totals</b>
		Year 6	Year 11	
Method of travel	Bus	10.50	15.75	<b>26.25</b>
	Car	6.02	9.03	<b>15.05</b>
	Cycle/Walk	0.53	0.80	<b>1.33</b>
<b>Column totals</b>		<b>17.05</b>	<b>25.58</b>	<b>42.64</b>