

### **General Certificate of Education**

## **Mathematics 6360**

MS2B Statistics 2B

# **Mark Scheme**

2008 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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#### Key to mark scheme and abbreviations used in marking

M	mark is for method					
m or dM	mark is dependent on one or more M marks and is for method					
A	mark is dependent on M or m marks and is for accuracy					
В	mark is independent of M or m marks and is for method and accuracy					
E	mark is for explanation					
$\sqrt{\text{or ft or F}}$	follow through from previous					
	incorrect result	MC	mis-copy			
CAO	correct answer only	MR	mis-read			
CSO	correct solution only	RA	required accuracy			
AWFW	anything which falls within	FW	further work			
AWRT	anything which rounds to	ISW	ignore subsequent work			
ACF	any correct form	FIW	from incorrect work			
AG	answer given	BOD	given benefit of doubt			
SC	special case	WR	work replaced by candidate			
OE	or equivalent	FB	formulae book			
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme			
–x EE	deduct x marks for each error	G	graph			
NMS	no method shown	c	candidate			
PI	possibly implied	sf	significant figure(s)			
SCA	substantially correct approach	dp	decimal place(s)			

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

#### MS2B

MS2B		1		
Q	Solution	Marks	Total	Comments
1	$H_0$ : $\mu = 5.0$	D.1		
	$H_1: \mu > 5.0$	B1		Both H <sub>0</sub> and H <sub>1</sub> : correct
	$z = \frac{5.5 - 5}{\sqrt{1 - 10^{-3}}}$			
	$z = \frac{5.5 - 5}{\sqrt{1.31} / \sqrt{40}}$	M1		
		A 1		
	2 – 2.70	A1		(AWFW 2.76 to 2.78)
	$z_{crit} = 2.3263$	B1ft		on their $H_1$ : $(t_{crit} = 2.426)$
				1 (50)
	Reject H <sub>0</sub>	A1		(dep M1)
	sufficient evidence to support David's	E1	6	
	claim at 1% level	171		
2(a)(i)	$\begin{array}{c} \text{Total} \\ V & \text{Po}(0,0) \rightarrow \end{array}$		6	
2(a)(i)	$X \sim \text{Po}(9.0) \Rightarrow$	B1	1	
	standard deviation $= 3$		-	
(ii)	P(6 < X < 12)			
	· · · · · · · · · · · · · · · · · · ·	3.51		
	$= P(X \le 11) - P(X \le 6)$	M1		
	= 0.8030 - 0.2068 $= 0.5962$	M1ft A1	2	
	0.0702	7.11	3	CAO
(b)(i)	$T = \mathbf{p}_{0}(11.5)$	D1	1	CAO
(b)(i)	$T \sim \text{Po}(11.5)$	B1	1	CAO
(ii)	$P(T \le 1) = P(T = 0) + P(T = 1)$	M1		Use of $T = 0$ and 1
	$=e^{-11.5}+11.5e^{-11.5}$	M1		Substitute correctly into formula
	= 0.000127	A1	3	AWFW 0.000126 and 0.00013
			3	
(c)	$\overline{x} = 12.0$ and $s^2 = 19.3$ (s = 4.40)	B1		$\sigma^2 = 17.4 \ (\sigma = 4.17)$
	2 2712 (2 11.0)			·
	Mean and variance very different	E1		dep on $s^2$ (or $\sigma^2$ )
	$\Rightarrow$ Po(12.0) not a suitable model	E1	3	(dep E1)
	Total		11	
	1000	l		

MS2B (cont)

MS2B (cont) Q	Solution	Marks	Total	Comments
3(a)(i)			3 55-52	
	$k = \frac{1}{a+b}$	B1	1	
(ii)	$E(T) = \int_{a}^{b} ktdt$	M1		
	-u	1411		
	$=\left\lceil \frac{kt^2}{2} \right\rceil^b$	A1		
	$\lfloor 2 \rfloor_{-a}$	AI		
	$= \frac{1}{2} \times \frac{1}{(a+b)} \times \left[b^2 - a^2\right]$			
	,			
	$= \frac{1}{2} \times \frac{1}{(a+b)} \times (b-a)(a+b)$	M1		Factorise
	$=\frac{1}{2}(b-a)$	A1	4	AG
<b>a</b> > <b>a</b>		D1	4	
(b)(i)	$\mathrm{E}\left(T\right) = 1$	B1	1	CAO
(ii)	D(T < 2  or  T > 2)			Alternative
(11)	P(T < -3  or  T > 3)			Attendative
	= P(T < -3) + P(T > 3) = 0.1 + 0.3	M1		$1 - P\left(-3 < T < 3\right)$
	= 0.4	A1	2	1 - (0.3 + 0.3) = 0.4
	Total		8	
4(a)	$\overline{v} = \frac{1179}{10} = 117.9$	B1		
	$s^2 = \frac{1014.9}{9} = 112.8 \implies s = 10.6$	B1		$\sigma^2 = 101.5  (\sigma = 10.08)$
	$t_{0.995} = 3.250$	B1		
	99% confidence interval:			or use of $\frac{\sqrt{101.5}}{3}$ = 3.359
	99% confidence interval.			or use of $\frac{1}{3}$ = 3.359
	$117.9 \pm \frac{10.6}{\sqrt{10}} \times 3.250$	M1		$(\text{their } \overline{v}) \pm \frac{(\text{their } s)}{\sqrt{10}} \times t_9$
	$\sqrt{10}$	1,11		$\sqrt{10}$
	$= 117.9 \pm 10.9$	A1ft		$(\operatorname{on} \overline{x}, s \text{ and } t_9 = 3.25)$
				,
	= (106.98, 128.82)	A1		AWRT (107, 129)
	= (107,129)			
	Assumption:			
	Speeds form a Normal Distribution	B1	7	
(b)	John's claim is unlikely since 130 mph	E1	1	
	lies outside the confidence interval.			
	Total		8	

MS2B (Cont)

MS2B (Cont	Solution	Marks	Total	Comments
5(a)	$P(X \ge 5) = P(X = 5) + P(X = 6)$			
	$= \frac{5}{20} + \frac{6}{24}$ $= \frac{1}{2}$	M1 A1	2	
(b)(i)	$E\left(\frac{1}{X}\right) = \sum \frac{1}{x} \times P(X = x) = \left(1 \times \frac{1}{20}\right) + \left(\frac{1}{2} \times \frac{2}{20}\right) + \left(\frac{1}{3} \times \frac{3}{20}\right) + \left(\frac{1}{4} \times \frac{4}{20}\right) + \left(\frac{1}{5} \times \frac{5}{20}\right) + \left(\frac{1}{6} \times \frac{6}{24}\right)$	M1		Use of $\sum \frac{1}{x} \times p$
(ii)	$= \frac{1}{4} + \frac{1}{24}$ $= \frac{7}{24}$ $E\left(\frac{1}{X^2}\right) = \frac{109}{900}$	A1 M1 A1	2	AG Use of $\sum \frac{1}{x^2} \times p$ or 0.21
	$Var\left(\frac{1}{X}\right) = \frac{109}{900} - \left(\frac{7}{24}\right)^2$ = 0.036 (3dp)	A1	3	AG
(c)	$A = \frac{1}{X}(X+3)$ $A = 1 + \frac{3}{X}$	B1		(either)
	$E(A) = 1 + 3E(X^{-1})$ $E(A) = 1 + 3 \times \frac{7}{24}$	M1		(either)
	$=1\frac{7}{8}$	A1		(1.875)
	$\operatorname{Var}(A) = \operatorname{Var}\left(1 + \frac{3}{X}\right) = 9\operatorname{Var}\left(X^{-1}\right)$	M1		
	$=9 \times \frac{173}{4800}$			allow 9×0.036
	$=0.324$ or $\frac{519}{1600}$	<b>A</b> 1	5	0.324375
	Total		12	

MS2B (cont)

MS2B (cont				<u></u>
Q	Solution	Marks	Total	Comments
6(a)	H <sub>0</sub> : no association between education and salary  O E $\alpha =  o - E  - 0.5$ $\alpha^2$	B1		
	78 70.2 7.3 0.7591 57 64.8 0.8224 52 59.8 0.8911 63 55.2 0.9654	M1 A1 M1		E attempted, correctly Yates' correction attempted $\alpha^{2}/_{E} \text{ attempted (final col)}$
	3.4380	M1 A1		AWRT 3.4
	$\chi^2 (10\%) = 2.706$	B1		
	Reject H <sub>o</sub> at 10% level	A1ft		
	Evidence to suggest an association between salary and having a university education.	E1ft	9	
(b)	Rejecting H <sub>o</sub> when H <sub>o</sub> correct Stating that there is an association	E1		
	between salary and education when there is not.	E1	2	
	Total		11	
7(a)(i)	F 1			B1 for axes 0 to 4 & 0 to 1 B1 for straight line $0 - \left(1, \frac{1}{2}\right)$
	$\frac{1}{2}$ 0 1 2 3 4 $x$	B4	4	B1 for convex curve from $\left(1, \frac{1}{2}\right)$ to $(4,1)$ B1 for at least the straight line for $x > 4$
(ii)	$F(q_1) = 0.25 \implies \frac{1}{2}q_1 = 0.25$ $\implies q_1 = \frac{1}{2}$	M1 A1	2	From sketch, or from $F(x)$ , Median = 1. $\frac{1}{2}x$ is linear on $(0, 0)$ to $(1, \frac{1}{2})$
	2			$\therefore q_1 = \frac{1}{2}$ AG

Q	Solution	Marks	Total	Comments
7(iii)	F(1.6) = 0.744	N/1		
	F(1.7) = 0.775	M1		
	$F(q_3) = 0.75$	M1		
	$\Rightarrow 1.6 < q_3 < 1.7$	<b>A</b> 1	3	AG
(b)(i)		M1		$f(1) = \alpha = 9\beta$ B1
	$\Rightarrow f(x) = \frac{1}{2} \text{ for } 0 \le x \le 1$	A1		$\int_{-\infty}^{\infty} f(x) dx = 1 \implies$
	$\Rightarrow \alpha = \frac{1}{2}$			$ \begin{bmatrix} f(1) = \alpha = 9\beta \\ \int_{-\infty}^{\infty} f(x) dx = 1 \Rightarrow \\ [\alpha x]_{0}^{1} + \left[\frac{\beta (x-4)^{3}}{3}\right]_{1}^{4} = 1 $ M1
	$\Rightarrow$ for $1 \le x \le 4$			
	$f(x) = \frac{1}{54} (3x^2 - 24x + 48)$	<b>A</b> 1		$\Rightarrow \boxed{\alpha + 9\beta = 1}$ A1 Solving: M1
	$=\frac{3}{54}(x^2-8x+16)$	M1		Solving: M1 $\alpha = \frac{1}{2} \text{ and } \beta = \frac{1}{18}$ A1
	$=\frac{1}{18}(x-4)^2$			2 18
	$\Rightarrow \beta = \frac{1}{18}$	A1	5	
(ii)	$E(X) = \int_{0}^{1} \frac{1}{2} x dx + \int_{1}^{4} \frac{1}{18} (x^{3} - 8x^{2} + 16x) dx$	M1		Both seen
	$= \left[ \frac{1}{4} x^2 \right]_0^1 + \frac{1}{18} \left[ \frac{x^4}{4} - \frac{8x^3}{3} + 8x^2 \right]_0^4$	A1A1		
		m1		Dependent on M1
	$=\frac{1}{4}+\frac{7}{8}$	A1	5	CAO
	$=1\frac{1}{8}$			
	Total		19	
	TOTAL		75	