Mark Scheme 4754
January 2007

Paper A – Section A

1 $\frac{1}{x}+\frac{x}{x+2}=1$ $\Rightarrow$ $x+2+x^2=x(x+2)$ $=x^2+2x$ $\Rightarrow$ $x=2$	M1 A1 DM1 A1 [4]	Clearing fractions solving cao
2(i) <table><tr><td>x</td><td>0</td><td>0.5</td><td>1</td><td>1.5</td><td>2</td></tr><tr><td>y</td><td>1</td><td>1.0607</td><td>1.4142</td><td>2.0917</td><td>3</td></tr></table> $A \approx 0.5[(1+3)/2 + 1.0607 + 1.4142 + 2.0917]$ $= 3.28$ (3 s.f.)	B1 M1 A1 [3]	At least one value calculated correctly or 13.13…or 6.566… seen
(ii) 3.25 (or Chris) area should decrease with the number of strips used.	B1 B1 [2]	ft (i) or area should decrease as concave upwards
3(i) sin 60 = √3/2, cos 60 = 1/2 , sin 45 = 1/√2, cos 45 = 1/√2 sin(105°) = sin(60°+45°) = sin60°cos45° + cos60°sin45° $=\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}+\frac{1}{2}.\frac{1}{\sqrt{2}}$ $=\frac{\sqrt{3}+1}{2\sqrt{2}}$ *	M1 M1 A1 E1 [4]	 splitting into 60° and 45°, and using the compound angle formulae
(ii) Angle B = 105° By the sine rule: $\frac{AC}{\sin B}=\frac{1}{\sin 30}$ $\Rightarrow$ AC = $\frac{\sin 105}{\sin 30}=\frac{\sqrt{3}+1}{2\sqrt{2}}.2$ $=\frac{\sqrt{3}+1}{\sqrt{2}}$ *	 M1 A1 E1 [3]	 Sine rule with exact values www
4 $\frac{1+\tan^2\theta}{1-\tan^2\theta}=\frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$ $=\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta-\sin^2\theta}$ $=\frac{1}{\cos 2\theta}$ $=\sec 2\theta$ $\sec 2\theta = 2 \Rightarrow \cos 2\theta = ½$ $\Rightarrow$ $2\theta = 60°, 300°$ $\Rightarrow$ $\theta = 30°, 150°$	M1 M1 M1 E1 M1 B1 B1 [7]	$\tan\theta=\frac{\sin\theta}{\cos\theta}$ or $1+\tan^2\theta=\sec^2\theta$ used simplifying to a simple fraction in terms of $\sin\theta$ and/or $\cos\theta$ only $\cos^2\theta-\sin^2\theta=\cos 2\theta$ oe used or $1+\tan^2\theta=2(1-\tan^2\theta) \Rightarrow \tan\theta=\pm1/\sqrt{3}$oe 30° 150° and no others in range

5 $(1+3x)^{\frac{1}{3}} =$ $= 1+\frac{1}{3}(3x)+\frac{\frac{1}{3}.(-\frac{2}{3})}{2!}(3x)^2+\frac{\frac{1}{3}(-\frac{2}{3})(-\frac{5}{3})}{3!}(3x)^3+...$ $=1+x-x^2+\frac{5}{3}x^3+...$ Valid for $-1 < 3x < 1 \Rightarrow -1/3 < x < 1/3$	M1 B1 A2,1,0 B1 [5]	binomial expansion (at least 3 terms) correct binomial coefficients (all) x, $-x^2$, $5x^3/3$
6(i) $\frac{1}{(2x+1)(x+1)}=\frac{A}{2x+1}+\frac{B}{x+1}$ $\Rightarrow$ $1 = A(x + 1) + B(2x + 1)$ $x = -1$: $1 = -B \Rightarrow B = -1$ $x = -½ : 1 = ½ A \Rightarrow A = 2$	M1 A1 A1 [3]	or cover up rule for either value
(ii) $\frac{dy}{dx}=\frac{y}{(2x+1)(x+1)}$ $\Rightarrow$ $\int\frac{1}{y}dy=\int\frac{1}{(2x+1)(x+1)}dx$ $=\int(\frac{2}{2x+1}-\frac{1}{x+1})dx$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + c$ When $x = 0$, $y = 2$ $\Rightarrow$ $\ln 2 = \ln 1 - \ln 1 + c \Rightarrow c = \ln 2$ $\Rightarrow$ $\ln y = \ln(2x + 1) - \ln(x + 1) + \ln 2$ $=\ln\frac{2(2x+1)}{x+1}$ $\Rightarrow$ $y=\frac{4x+2}{x+1}$ *	M1 A1 B1ft M1 E1 [5]	separating variables correctly condone omission of c. ft A,B from (i) calculating c , no incorrect log rules combining lns www

Section B

7(i) At A, cos $\theta = 1 \Rightarrow \theta = 0$ At B, cos $\theta = -1 \Rightarrow \theta = \pi$ At C $x = 0, \Rightarrow$ cos $\theta = 0 \Rightarrow \theta = \pi/2$ $\Rightarrow y = \sin\frac{\pi}{2} - \frac{1}{8}\sin\pi = 1$	B1 B1 M1 A1 [4]	or subst in both x and y allow 180°
(ii) $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$ $= \frac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$ $= \frac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$ dy/dx = 0 when $\cos 2\theta - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$ $\Rightarrow 2\cos^2\theta - 4\cos\theta - 1 = 0$ *	M1 A1 A1 M1 E1 [5]	finding $dy/d\theta$ and $dx/d\theta$ correct numerator correct denominator =0 or their num=0
(iii) $\cos\theta = \frac{4 \pm \sqrt{16+8}}{4} = 1 \pm \frac{1}{2}\sqrt{6}$ (1 + ½ √6 > 1 so no solution) $\Rightarrow \theta = 1.7975$ $y = \sin\theta - \frac{1}{8}\sin 2\theta = 1.0292$	M1 A1ft A1 cao M1 A1 cao [5]	$1 \pm \frac{1}{2}\sqrt{6}$ or (2.2247,-.2247) both or –ve their quadratic equation 1.80 or 103° their angle 1.03 or better
(iv) $V = \int_{-1}^{1} \pi y^2 dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2)(1 - x^2)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4)dx$ $= \frac{1}{16}\pi\int_{-1}^{1}(16 - 8x - 15x^2 + 8x^3 - x^4)dx$ * $= \frac{1}{16}\pi\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{1}{5}x^5\right]_{-1}^{1}$ $= \frac{1}{16}\pi(32 - 10 - \frac{2}{5})$ = 1.35π = 4.24	M1 M1 E1 B1 M1 A1cao [6]	correct integral and limits expanding brackets correctly integrated substituting limits

8 (i) $\sqrt{(40-0)^2+(0+40)^2+(-20-0)^2}$ = 60 m	M1 A1 [2]	
(ii) $\overrightarrow{BA}=\begin{pmatrix}-40\\-40\\20\end{pmatrix}=20\begin{pmatrix}-2\\-2\\1\end{pmatrix}$ $\cos\theta=\dfrac{\begin{pmatrix}-2\\-2\\1\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}}{\sqrt{9}\sqrt{26}}=-\dfrac{13}{3\sqrt{26}}$ $\Rightarrow\quad\theta=148°$	M1 A1 A1 A1 [4]	or $\overrightarrow{AB}$ −13 oe eg -260 √9√26 oe eg 60√26 cao (or radians)
(iii) $\mathbf{r}=\begin{pmatrix}40\\0\\-20\end{pmatrix}+\lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ At C, $z=0\ \Rightarrow\lambda=20$ $\Rightarrow\quad a=40+3\times20=100$ $b=0+4\times20=80$	B1 B1 M1 A1 A1 [5]	$\begin{pmatrix}40\\0\\-20\end{pmatrix}+\ldots$ $\ldots+\lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$ or….$+\lambda\begin{pmatrix}a-40\\b\\20\end{pmatrix}$ 100 80
(iv) $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}-2\\-2\\1\end{pmatrix}=-12+10+2=0$ $\begin{pmatrix}6\\-5\\2\end{pmatrix}.\begin{pmatrix}3\\4\\1\end{pmatrix}=18-20+2=0$ $\Rightarrow\begin{pmatrix}6\\-5\\2\end{pmatrix}$ is perpendicular to plane. Equation of plane is $6x-5y+2z=c$ At B (say) $6\times40-5\times0+2\times-20=c$ $\Rightarrow c=200$ so $6x-5y+2z=200$	B1 B1 M1 M1 A1 [5]	(alt. method finding vector equation of plane M1 eliminating both parameters DM1 correct equation A1 stating Normal hence perpendicular B2)

Paper B Comprehension

<table>
<tr><td>1(i)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6</td><td>4</td><td>2</td><td>2</td><td>2</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
</table>
</td><td>B1 Table</td></tr>
<tr><td>(ii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>7</td><td>3</td><td>2</td><td>3</td><td>1</td><td>2</td><td>1</td><td>1</td><td>0</td></tr>
</table>
</td><td>M1 A1 Table</td></tr>
<tr><td>(iii)</td><td>
<table>
<tr><td>Leading digit</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr>
<tr><td>Frequency</td><td>6.0</td><td>3.5</td><td>2.5</td><td>1.9</td><td>1.6</td><td>1.3</td><td>1.2</td><td>1.0</td><td>0.9</td></tr>
</table>
</td><td>B1any 4 correct
B1 other 4 correct</td></tr>
<tr><td>(iv)</td><td>Any sensible comment such as:
• The general pattern of the frequencies/results is the same for all three tables.
• Due to the small number of data items we cannot expect the pattern to follow Benford’s Law very closely.</td><td>E1</td></tr>
<tr><td>2</td><td>Evidence of 4+3+4+2+2 from Table 4 frequencies is the same as 15 in Table 6</td><td>B1</td></tr>
<tr><td>3</td><td>$p_1 = p_3 + p_4 + p_5$: on multiplication by 3, numbers with a leading digit of 1 will be mapped to numbers with a leading digit of 3, 4 or 5 and no other numbers have this property.</td><td>B1 Multiplication

B1… by 3</td></tr>
<tr><td>4</td><td>$\log_{10}(n+1) - \log_{10} n = \log_{10}\left(\frac{n+1}{n}\right) = \log_{10}\left(\frac{n}{n} + \frac{1}{n}\right) = \log_{10}\left(1 + \frac{1}{n}\right)$</td><td>M1
E1</td></tr>
<tr><td>5</td><td>Substitute $\mathrm{L}(4) = 2\times\mathrm{L}(2)$ and $\mathrm{L}(6) = \mathrm{L}(3) + \mathrm{L}(2)$ in
$\mathrm{L}(8) - \mathrm{L}(6) = \mathrm{L}(4) - \mathrm{L}(3)$:
this gives $\mathrm{L}(8) = \mathrm{L}(6) - \mathrm{L}(3) + \mathrm{L}(4) = \mathrm{L}(2) + 2\times\mathrm{L}(2) = 3\times\mathrm{L}(2)$</td><td>M1
M1 subst
E1
(or alt
M1 for 2 or more Ls used
M1 use of at least 2 given results oe
E1)</td></tr>
<tr><td>6</td><td>$a = 28$. All entries with leading digit 2 or 3 will, on multiplying by 5, have leading digit 1. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td>$b = 9$. Similarly, all entries with leading digit 8 or 9 will, on multiplying by 5, have leading digit 4. None of the other original daily wages would have this property.</td><td>B1 B1</td></tr>
<tr><td></td><td></td><td>Total 18</td></tr>
</table>

4754 - Applications of Advanced Mathematics (C4)

General Comments

This paper was of a similar standard to that of last January. Candidates found it much more straightforward than the June 2006 paper. There was a wide range of responses but all questions were answered well by some candidates. There were some excellent scripts.
Candidates should be advised to read questions carefully. There were instances, particularly in the Comprehension, where instructions were not followed.
There was also some use of inefficient methods. Those that were competent at algebra and surds and were familiar with manipulating trigonometric formulae generally achieved good results. Some of the arithmetic in the trapezium rule and the integration of the polynomial was disappointing.
There was some evidence of shortage of time as a small proportion of candidates failed to complete question 8.

Comments on Individual Questions

Paper A

Section A

1 The algebraic fraction equation was almost always answered correctly.

2 (i) There seemed to be a lack of familiarity with the trapezium rule formula. Common errors were use of A=0.5h(y_0+y_4) +2(y_1+y_2 +y_3) but omission of the other brackets. Or alternatively omitting y_0 and using A=0.5h((y_1+y_4)+2 (y_2 +y_3)). Most obtained at least one ordinate correctly but there were many errors in the calculation of the answer.

(ii) Those without correct, or almost correct, answers in the first part could not make a valid comment about which of Chris or Dave was correct in their calculations. There were some poor explanations given, such as 'the trapezium rule always overestimates results'.

3 Most candidates correctly used the compound angle formula as the first stage. Those that used *sin/cos* 45° as $\sqrt{2}/2$ rather than $1/\sqrt{2}$ could not always deal with cancelling $(\sqrt{6}+\sqrt{2})/4$. The sine rule was usually correct.

4 There were some efficient solutions but weaker candidates found it difficult to see ahead to what was needed. In some cases poor knowledge of trigonometric identities and their rearrangement was the problem. Some tried to work on both sides simultaneously - some more clearly than others.
There were some confused starts using incorrect identities in the second part but many did obtain the first solution. The solution θ=150° was often lost - in some cases due to missing the negative square root.

5 This was well answered. The improvement seen in the binomial expansion was pleasing although this was possibly due to the first number in the bracket being a 1. There were still some candidates who used *x* rather than *3x* throughout the calculation and many could not deal successfully with the range for the validity.

6 The partial fractions were almost always correct.
The second part was less successful. Some separated the variables to *ydy*=…. Many integrated 2/ (2*x*+1) as 2ln (2*x*+1) and there were many instances of the omission of the constant. Poor use of the laws of logarithms meant that c was often not found correctly. For example, lny=ln(2x+1) – ln(x+1) +c leading to y= (2x+1)/(x+1) +c was common. Those that found c before combining their logs were more successful. 2ln(2x+1) – ln(x+1) = ln 2(2x+1)/(x+1) was also a common error.

Section B

7 (i) This was usually correctly answered although some candidates used long methods to show that $\theta=0$ at A and others gave the value of θ at B in degrees.

(ii) There were many errors in *dy/dθ*- usually the coefficient of *cos 2θ* being incorrect - and there were also sign errors. Most knew that they had to equate *dy/dx* to zero but made errors in their simplification to the given equation.

(iii) Some omitted this or tried to factorise and then abandoned the attempt. Of those that did use the formula, a common mistake was to solve the quadratic equation for *cos θ* but then to use this as *θ* in the expression for *y*.

(iv) This was disappointing. The first part was usually correct but a significant number failed to integrate the polynomial. Of those that did integrate, many surprisingly made numerical errors when substituting the limits.

8 (i) Most candidates correctly found the distance AB.

(ii) Many failed to find the required angle ABC.

(iii) This proved to be very successful for many. Those that gave the required vector equation in terms of *a* and *b*, however, could rarely make progress. A few found *a* and *b* successfully without explicitly writing down the equation of the line.

(iv) Once again too many candidates failed to realise that in order to prove that a vector is perpendicular to a plane it is necessary to show it is perpendicular to **two** vectors in the plane. Others did not evaluate their dot product, merely stating it was zero. Most used the Cartesian form of the equation with success. There were still some candidates who approached this from the vector equation of the plane and they were more likely to make errors.

Paper B

Comprehension

1 The tables in (i) and (ii) were usually correct but there were occasional slips. In (iii) candidates often failed to calculate using Benford's Law. It was unclear what their methods were in (iii) but they may have been trying to use Fig.9.

2 This was often successful but it was not always clear which tables the candidates were referring to.

3 Some failed to explain about the multiplication of leading digits. For those that did, the multiplication factor quoted did not always work for the complete range. Multiplying 3,4 and 5 by 3.5 or 4 was commonly seen.

4	Usually correct although log (n+1) –log n=log(n+1)/log n was seen.
5	The approach encouraged by the question was not always used. There were some very long and often confused solutions involving changing all ‘L’ expressions to strings of ‘p’ equations and eliminating.
6	Candidates often seemed not to have read this question carefully. There were many good solutions, but too often the proportions were calculated rather than using the frequencies in the table.

GCE

Mathematics (MEI)

Unit **4754B**: Applications of Advanced Mathematics: Paper B

Advanced GCE

Mark Scheme for June 2017

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question		Answer	Marks	Guidance
1.	(i)	(All x_0 for which) $0 < x_0 < 1$	B1	Condone as separate inequalities – allow x, x_n, etc. for x_0 Accept (0, 1) (i.e. correct set-notation) – must be strict inequalities
			[1]	
	(ii)	$x_0 = 0$ or 1	B1	Both required - allow x, x_n, etc. for x_0 - condone 0, 1 stated without x_0 (0 and 1 must not appear as part of a range of values)
			[1]	

2.	**(i)**	$x = 1.6x(1-x)$	**M1**	For correctly replacing x_{n+1} **and** x_n with x - must be using 1.6
		$x(1.6x - 0.6) = 0$		
		$(x = 0 \text{ or})\ \ x = \dfrac{0.6}{1.6} = 0.375 \text{ or } \dfrac{3}{8}$	**A1**	Using an iterative approach or stating the correct answer with no working - M0 A0 Accept as a minimum the correct equation stated in terms of x only followed by the correct answer for both marks
			[2]	
2.	**(ii)**	$x^2 - x + 2 = 0$		
		Discriminant $= (-1)^2 - 4\times1\times2 = -7$ or $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(2)}}{2}$	**M1**	Re-arranging **and** correct attempt at either the discriminant or solving using the correct formula/completing the square for their 3-term quadratic equation. Allow consideration of $\sqrt{b^2 - 4ac}$ with correct substitution of their values only
		so no (real) roots (oe)	**A1**	Correct working with a correct conclusion (so if discriminant stated must be -7 but may be awarded for $1 - 4(2) \Rightarrow$ no (real) roots/no points of equilibrium) No marks for non-algebraic approach
			[2]	

PLEASE NOTE THAT THE MARKS FOR BOTH 3(i) and 3(ii) MUST BE AWARDED TOGETHER (MAXIMUM OF 6)

3. **(i)**

$$x_{n+1} = kx_n(1 - x_n) - 0.25$$

	k = 2	k = 3	k = 4	k = 5
x_0	0.55	0.55	0.55	0.55
x_1	0.245	0.4925	0.74	0.9875
x_2	0.1199...	0.4998...	0.5196	-0.1882...
x_3	-0.0388...	0.4999...	0.7484...	...
x_4	...	0.5	0.5030...	...
x_5	...	...	0.7499...	...
x_6	...	...	0.5000...	...
x_7	...	...	0.7499...	...
x_8	...	...	0.5000...	...
x_9			0.75	
x_{10}			0.5	
...	...	...	...	...

There may be some variation in the columns for k = 3 and k = 4. Some calculators will round to the final values earlier than others. **In general answers must be given to at least 2dp**

Allow correct truncated or rounded answers

One mark for each column taken far enough

B1	$k = 2$ correct to at least x_3
B1	$k = 3$ correct to at least x_3 (accept 0.5)
B1	$k = 4$ correct to at least x_6 (accept 0.5)
B1	$k = 5$ correct to at least x_2

isw after reaching the values for each column as stated above

3.	**(ii)**	For k = 2 it is **unbounded** or '**dying out**' For k = 3 it **converges** (to x = 0.5) or **stable** or **equilibrium** For k = 4 it **oscillates** (between x = 0.5 and x = 0.75) or **alternates** but not **fluctuates** For k = 5, it is **unbounded** or '**dying out**'	**B2**	All 4 correct Allow B1 for 2 correct Allow equivalent to 'dying out' provided unambiguous Must be one of these three words Must be one of these two words Allow equivalent to 'dying out' provided unambiguous
			[6]	

4.	**(i)**	$3.5644 + \frac{0.0203}{4.6692}$	**B1**	Correct method - allow a slip in one value (e.g. 4.692) only – either correct value for k implies this mark
		$= 3.5687$	**B1**	Accept 3.569 (3.5687476…) – no ft from incorrect values – accept if given as the upper bound of an interval
		$3.5687 + 0.0203 \times \left(\frac{1}{4.6692}\right)^2 = 3.5696$ to 5 sf	**B1**	Accept 3.5697 or 3.570 but not 3.57 – no ft from incorrect values – accept if given as an upper bound
			[3]	
4	**(ii)(A)**	$S = \frac{1}{1 - \frac{1}{4.6692}}$	**M1**	Use of the correct formula for the sum to infinity of a GP – allow a slip in the value of r only (e.g. 4.692) – allow r stated to 2dp or better (r = 0.21416945…)
		$S = 1.2725...$	**A1**	Accept 1.273 not 1.27 (1.2725389…) M0 A0 if formula not used
			[2]	
4	**(ii)(B)**	It is an estimate of a value of k for which chaos is occurring	**B1**	Accept any mention of 'chaos'
			[1]	

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