

**ADVANCED GCE**  
**MATHEMATICS**  
Probability & Statistics 2

**4733**

Candidates answer on the Answer Booklet

**OCR Supplied Materials:**

- 8 page Answer Booklet
- List of Formulae (MF1)

**Other Materials Required:**

- Scientific or graphical calculator

**Tuesday 22 June 2010**  
**Afternoon**

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

**INFORMATION FOR CANDIDATES**

- The number of marks is given in brackets [ ] at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

- 1 (i) The number of inhabitants of a village who are selected for jury service in the course of a 10-year period is a random variable with the distribution  $Po(4.2)$ .
- (a) Find the probability that in the course of a 10-year period, at least 7 inhabitants are selected for jury service. [2]
- (b) Find the probability that in 1 year, exactly 2 inhabitants are selected for jury service. [3]
- (ii) Explain why the number of inhabitants of the village who contract influenza in 1 year can probably not be well modelled by a Poisson distribution. [2]
- 2 A university has a large number of students, of whom 35% are studying science subjects. A sample of 10 students is obtained by listing all the students, giving each a serial number and selecting by using random numbers.
- (i) Find the probability that fewer than 3 of the sample are studying science subjects. [3]
- (ii) It is required that, in selecting the sample, the same student is not selected twice. Explain whether this requirement invalidates your calculation in part (i). [2]
- 3 Tennis balls are dropped from a standard height, and the height of bounce,  $H$  cm, is measured.  $H$  is a random variable with the distribution  $N(40, \sigma^2)$ . It is given that  $P(H < 32) = 0.2$ .
- (i) Find the value of  $\sigma$ . [3]
- (ii) 90 tennis balls are selected at random. Use an appropriate approximation to find the probability that more than 19 have  $H < 32$ . [6]
- 4 The proportion of commuters in a town who travel to work by train is 0.4. Following the opening of a new station car park, a random sample of 16 commuters is obtained, and 11 of these travel to work by train. Test at the 1% significance level whether there is evidence of an increase in the proportion of commuters in this town who travel to work by train. [7]
- 5 The time  $T$  seconds needed for a computer to be ready to use, from the moment it is switched on, is a normally distributed random variable with standard deviation 5 seconds. The specification of the computer says that the population mean time should be not more than 30 seconds.
- (i) A test is carried out, at the 5% significance level, of whether the specification is being met, using the mean  $\bar{t}$  of a random sample of 10 times.
- (a) Find the critical region for the test, in terms of  $\bar{t}$ . [4]
- (b) Given that the population mean time is in fact 35 seconds, find the probability that the test results in a Type II error. [3]
- (ii) Because of system degradation and memory load, the population mean time  $\mu$  seconds increases with the number of months of use,  $m$ . A formula for  $\mu$  in terms of  $m$  is  $\mu = 20 + 0.6m$ . Use this formula to find the value of  $m$  for which the probability that the test results in rejection of the null hypothesis is 0.5. [4]

- 6 (a) The random variable  $D$  has the distribution  $\text{Po}(24)$ . Use a suitable approximation to find  $P(D > 30)$ . [5]
- (b) An experiment consists of 200 trials. For each trial, the probability that the result is a success is 0.98, independent of all other trials. The total number of successes is denoted by  $E$ .
- (i) Explain why the distribution of  $E$  cannot be well approximated by a Poisson distribution. [1]
- (ii) By considering the number of failures, use an appropriate Poisson approximation to find  $P(E \leq 194)$ . [4]

- 7 A machine is designed to make paper with mean thickness 56.80 micrometres. The thicknesses,  $x$  micrometres, of a random sample of 300 sheets are summarised by

$$n = 300, \quad \Sigma x = 17\,085.0, \quad \Sigma x^2 = 973\,847.0.$$

Test, at the 10% significance level, whether the machine is producing paper of the designed thickness. [11]

- 8 The continuous random variable  $X$  has probability density function given by

$$f(x) = \begin{cases} kx^{-a} & x \geq 1, \\ 0 & \text{otherwise,} \end{cases}$$

where  $k$  and  $a$  are constants and  $a$  is greater than 1.

- (i) Show that  $k = a - 1$ . [3]
- (ii) Find the variance of  $X$  in the case  $a = 4$ . [5]
- (iii) It is given that  $P(X < 2) = 0.9$ . Find the value of  $a$ , correct to 3 significant figures. [4]

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