General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC1**
Unit Pure Core 1

Friday 9 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The points A and B have coordinates $(1, 6)$ and $(5, -2)$ respectively. The mid-point of AB is M.

(a) Find the coordinates of M. *(2 marks)*

(b) Find the gradient of AB, giving your answer in its simplest form. *(2 marks)*

(c) A straight line passes through M and is perpendicular to AB.

(i) Show that this line has equation $x - 2y + 1 = 0$. *(3 marks)*

(ii) Given that this line passes through the point $(k, k + 5)$, find the value of the constant k. *(2 marks)*

2 (a) Factorise $2x^2 - 5x + 3$. *(1 mark)*

(b) Hence, or otherwise, solve the inequality $2x^2 - 5x + 3 < 0$. *(3 marks)*

3 (a) Express $\dfrac{7 + \sqrt{5}}{3 + \sqrt{5}}$ in the form $m + n\sqrt{5}$, where m and n are integers. *(4 marks)*

(b) Express $\sqrt{45} + \dfrac{20}{\sqrt{5}}$ in the form $k\sqrt{5}$, where k is an integer. *(3 marks)*

4 (a) (i) Express $x^2 + 2x + 5$ in the form $(x + p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Hence show that $x^2 + 2x + 5$ is always positive. *(1 mark)*

(b) A curve has equation $y = x^2 + 2x + 5$.

(i) Write down the coordinates of the minimum point of the curve. *(2 marks)*

(ii) Sketch the curve, showing the value of the intercept on the y-axis. *(2 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = x^2$ onto the graph of $y = x^2 + 2x + 5$. *(3 marks)*

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t, \qquad 0 \leqslant t \leqslant 4$$

(a) Find:

(i) $\dfrac{dx}{dt}$; *(3 marks)*

(ii) $\dfrac{d^2x}{dt^2}$. *(2 marks)*

(b) Verify that x has a stationary value when $t = 3$, and determine whether this stationary value is a maximum value or a minimum value. *(4 marks)*

(c) Find the rate of change of x with respect to t when $t = 1$. *(2 marks)*

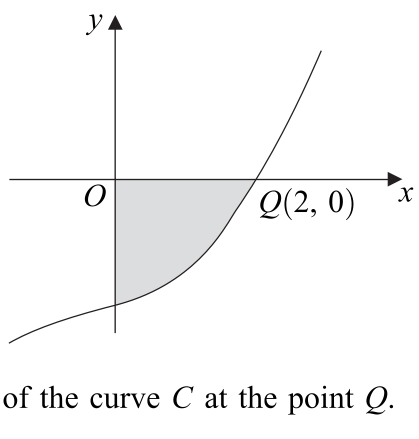
(d) Determine whether the distance of the car from O is increasing or decreasing at the instant when $t = 2$. *(2 marks)*

6 (a) The polynomial $p(x)$ is given by $p(x) = x^3 + x - 10$.

(i) Use the Factor Theorem to show that $x - 2$ is a factor of $p(x)$. *(2 marks)*

(ii) Express $p(x)$ in the form $(x - 2)(x^2 + ax + b)$, where a and b are constants. *(2 marks)*

(b) The curve C with equation $y = x^3 + x - 10$, sketched below, crosses the x-axis at the point $Q(2, 0)$.



(i) Find the gradient of the curve C at the point Q. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve C at the point Q. *(2 marks)*

(iii) Find $\int (x^3 + x - 10)\, dx$. *(3 marks)*

(iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. *(2 marks)*

Turn over for the next question

7 A circle with centre C has equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) Write down:

(i) the coordinates of C;

(ii) the radius of the circle. *(2 marks)*

(c) The point D has coordinates $(7, \ -2)$.

(i) Verify that the point D lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point D, giving your answer in the form $mx + ny = p$, where m, n and p are integers. *(3 marks)*

(d) (i) A line has equation $y = kx$. Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$ *(2 marks)*

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots. *(5 marks)*

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). *(1 mark)*

END OF QUESTIONS