

Tuesday 6 June 2023 – Afternoon

A Level Mathematics A

H240/01 Pure Mathematics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator



INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N})$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

- 1 In the triangle ABC , the length $AB = 6$ cm, the length $AC = 15$ cm and the angle $BAC = 30^\circ$.
- (a) Calculate the length BC . [2]
- D is the point on AC such that the length $BD = 4$ cm.
- (b) Calculate the possible values of the angle ADB . [3]
- 2 (a) (i) Show that $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}}$ can be written in the form $\frac{a}{b+cx}$, where a , b and c are constants to be determined. [2]
- (ii) Hence solve the equation $\frac{1}{3-2\sqrt{x}} + \frac{1}{3+2\sqrt{x}} = 2$. [2]
- (b) **In this question you must show detailed reasoning.**
- Solve the equation $2^{2y} - 7 \times 2^y - 8 = 0$. [4]
- 3 (a) Given that $f(x) = x^2 + 2x$, use differentiation from first principles to show that $f'(x) = 2x + 2$. [4]
- (b) The gradient of a curve is given by $\frac{dy}{dx} = 2x + 2$ and the curve passes through the point $(-1, 5)$.
- Find the equation of the curve. [3]
- 4 It is given that $ABCD$ is a quadrilateral. The position vector of A is $\mathbf{i} + \mathbf{j}$, and the position vector of B is $3\mathbf{i} + 5\mathbf{j}$.
- (a) Find the length AB . [1]
- (b) The position vector of C is $p\mathbf{i} + p\mathbf{j}$ where p is a constant greater than 1.
- Given that the length AB is equal to the length BC , determine the position vector of C . [3]
- (c) The point M is the midpoint of AC .
- Given that $\overrightarrow{MD} = 2\overrightarrow{BM}$, determine the position vector of D . [2]
- (d) State the name of the quadrilateral $ABCD$, giving a reason for your answer. [2]

- 5 (a) The function $f(x)$ is defined for all values of x as $f(x) = |ax - b|$, where a and b are positive constants.
- (i) The graph of $y = f(x) + c$, where c is a constant, has a vertex at $(3, 1)$ and crosses the y -axis at $(0, 7)$.
- Find the values of a , b and c . [3]
- (ii) Explain why $f^{-1}(x)$ does not exist. [1]
- (b) The function $g(x)$ is defined for $x \geq \frac{q}{p}$ as $g(x) = |px - q|$, where p and q are positive constants.
- (i) Find, in terms of p and q , an expression for $g^{-1}(x)$, stating the domain of $g^{-1}(x)$. [3]
- (ii) State the set of values of p for which the equation $g(x) = g^{-1}(x)$ has no solutions. [1]

6 A curve has equation $y = e^{x^2+3x}$.

- (a) Determine the x -coordinates of any stationary points on the curve. [4]
- (b) Show that the curve is convex for all values of x . [5]

7 (a) Use the result $\cos(A + B) = \cos A \cos B - \sin A \sin B$ to show that

$$\cos(A - B) = \cos A \cos B + \sin A \sin B. \quad [2]$$

The function $f(\theta)$ is defined as $\cos(\theta + 30^\circ)\cos(\theta - 30^\circ)$, where θ is in degrees.

- (b) Show that $f(\theta) = \cos^2\theta - \frac{1}{4}$. [3]
- (c) (i) Determine the following.
- The **maximum** value of $f(\theta)$
 - The smallest **positive** value of θ for which this maximum value occurs [2]
- (ii) Determine the following.
- The **minimum** value of $f(\theta)$
 - The smallest **positive** value of θ for which this minimum value occurs [2]

- 8 (a) Find the first **three** terms in the expansion of $(4 + 3x)^{\frac{3}{2}}$ in ascending powers of x . [4]
- (b) State the range of values of x for which the expansion in part (a) is valid. [1]
- (c) In the expansion of $(4 + 3x)^{\frac{3}{2}}(1 + ax)^2$ the coefficient of x^2 is $\frac{107}{16}$.
Determine the possible values of the constant a . [4]

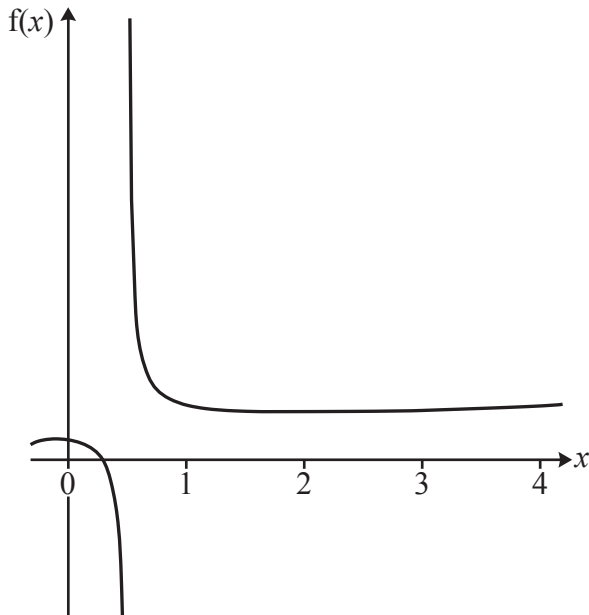
- 9 Conservationists are studying how the number of bees in a wildflower meadow varies according to the number of wildflower plants. The study takes place over a series of weeks in the summer. A model is suggested for the number of bees, B , and the number of wildflower plants, F , at time t weeks after the start of the study.

In the model $B = 20 + 2t + \cos 3t$ and $F = 50e^{0.1t}$.

The model assumes that B and F can be treated as continuous variables.

- (a) State the meaning of $\frac{dB}{dF}$. [1]
- (b) Determine $\frac{dB}{dF}$ when $t = 4$. [4]
- (c) Suggest a reason why this model may not be valid for values of t greater than 12. [1]

10

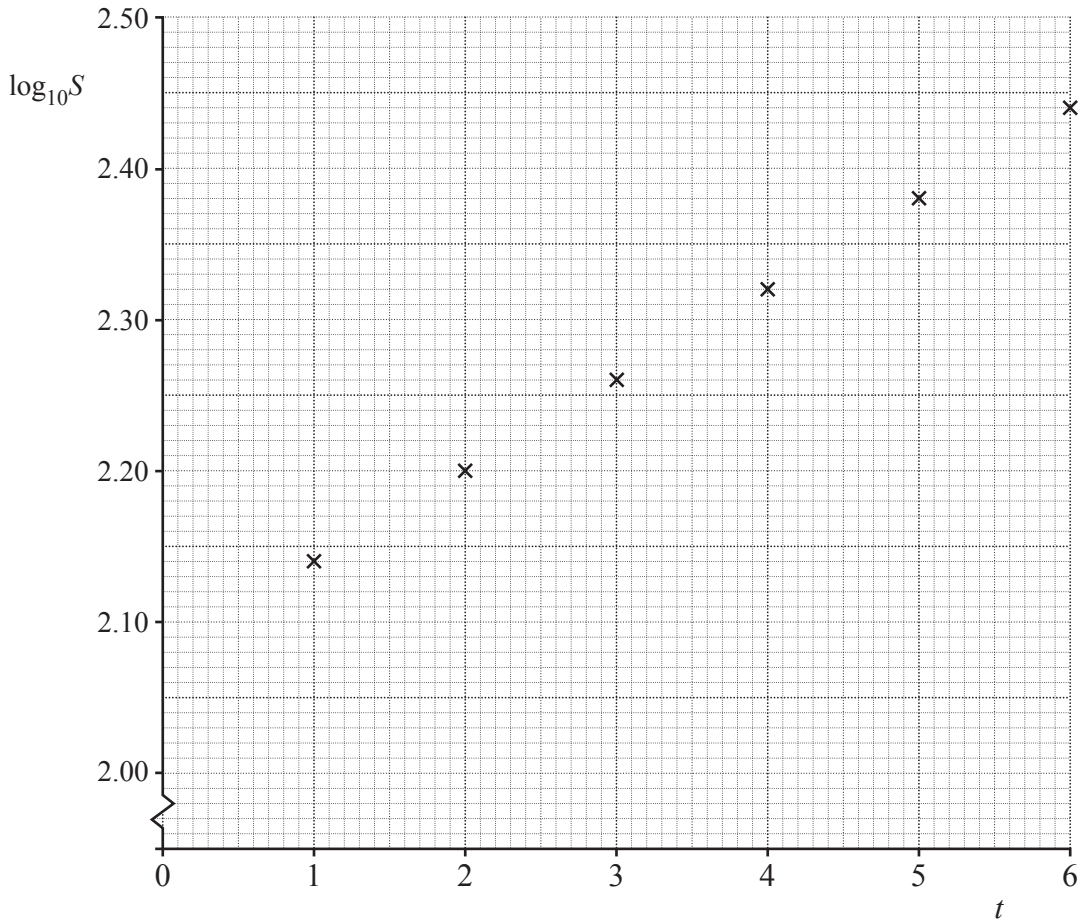


The diagram shows part of the curve $f(x) = \frac{e^x}{4x^2 - 1} + 2$. The equation $f(x) = 0$ has a positive root α close to $x = 0.3$.

- (a) Explain why using the sign change method with $x = 0$ and $x = 1$ will fail to locate α . [1]
- (b) Show that the equation $f(x) = 0$ can be written as $x = \frac{1}{4}\sqrt{4 - 2e^x}$. [2]
- (c) Use the iterative formula $x_{n+1} = \frac{1}{4}\sqrt{4 - 2e^{x_n}}$ with a starting value of $x_1 = 0.3$ to find the value of α correct to 4 significant figures, showing the result of each iteration. [3]
- (d) An alternative iterative formula is $x_{n+1} = F(x_n)$, where $F(x_n) = \ln(2 - 8x_n^2)$.
By considering $F'(0.3)$ explain why this iterative formula will not find α . [3]

- 11 The owners of an online shop believe that their sales can be modelled by $S = ab^t$, where a and b are both positive constants, S is the number of items sold in a month and t is the number of complete months since starting their online shop.

The sales for the first six months are recorded, and the values of $\log_{10} S$ are plotted against t in the graph below. The graph is repeated in the Printed Answer Booklet.



- (a) Explain why the graph suggests that the given model is appropriate. [3]

The owners believe that $a = 120$ and $b = 1.15$ are good estimates for the parameters in the model.

- (b) Show that the graph supports these estimates for the parameters. [2]

- (c) Use the model $S = 120 \times 1.15^t$ to predict the number of items sold in the **seventh** month after opening. [2]

- (d) (i) Use the model $S = 120 \times 1.15^t$ to predict the number of months after opening when the **total** number of items sold after opening will first exceed 70 000. [4]

- (ii) Comment on how reliable this prediction may be. [1]

12 (a) Use the substitution $u = e^x - 2$ to show that

$$\int \frac{7e^x - 8}{(e^x - 2)^2} dx = \int \frac{7u + 6}{u^2(u + 2)} du. \quad [3]$$

(b) Hence show that

$$\int_{1.4}^{1.6} \frac{7e^x - 8}{(e^x - 2)^2} dx = a + 1/b$$

where a and b are rational numbers to be determined. [7]

END OF QUESTION PAPER

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