

**Mathematics**

Advanced Subsidiary GCE

Unit **4721**: Core Mathematics 1

**Mark Scheme for January 2011**

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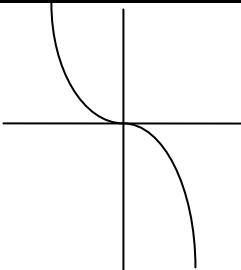
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1 (i)	$\sqrt{(-2-6)^2 + (7-1)^2}$ $= 10$	<b>M1</b> <b>A1</b>	Use of $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ <b>2</b>	3 out of 4 substitutions correct Look out for no square root, $(x_2 + x_1)^2$ etc. <b>M0</b>
(ii)	$\frac{7-1}{-2-6}$ $= -\frac{3}{4}$	<b>M1</b> <b>A1</b>	uses $\frac{y_2 - y_1}{x_2 - x_1}$ <b>2</b> o.e. <b>ISW</b>	3 out of 4 substitutions correct  Allow $-0.75$ $\frac{3}{-4}$ etc.
(iii)	Gradient of given line $= \frac{4}{3}$  $-\frac{3}{4} \times \frac{4}{3} = -1$  So lines are perpendicular	<b>M1</b> <b>B1ft</b> <b>B1</b>	Attempt to rearrange equation to make y the subject <b>OR</b> attempt to find the gradient using points on the line Correct conclusion for their gradients <b>3</b> <b>7</b> States $-\frac{3}{4} \times \frac{4}{3} = -1$ or "negative reciprocal" relating to the correct values <b>www</b>	Must at least isolate y
2	$2x^3 + 9x^2 - 2px^2 - 9px + 10x - 10p$ $= 2x^3 + qx^2 - 8x - 4q$          $p = 2$ and $q = 5$	<b>M1*</b> <b>DM1</b> <b>A1</b>	Attempt to expand both sides <b>OR</b> to substitute 2 values of x into both expressions <b>OR</b> to express at least one side as a product of three factors Valid method to obtain either p or q  <b>3</b> <b>3</b> Both values correct	If expanding, minimum of 5 terms on LHS and 3 terms on RHS  If comparing coefficients, must be of corresponding terms  <b>SR</b> Spotted solutions <b>B1</b> one correct <b>B2</b> other correct
3 (i)	$\frac{1}{8^2}$	<b>B1</b>	<b>1</b>	Allow $8^{0.5}$ Condone $p = \frac{1}{2}$ , just " $\frac{1}{2}$ " seen as answer <b>www</b>
(ii)	$8^{-2}$	<b>B1</b>	<b>1</b>	Condone $p = -2$ , just "-2" seen as answer <b>www</b> $\frac{1}{8^2}$ only not enough
(iii)	$2^8 = \left(8^{\frac{1}{3}}\right)^8$  $= 8^{\frac{8}{3}}$	<b>M1</b> <b>M1</b> <b>A1</b>	$2^8$ or $2^6 = 8^2$ soi  $2 = 8^{\frac{1}{3}}$ soi  <b>3</b> <b>5</b> o.e.	Condone $p = \frac{8}{3}$ , just " $\frac{8}{3}$ " seen as answer <b>www</b>  $2^3 = 8$ not enough for second <b>M</b> mark

4	$u^2 - 5u + 4 = 0$ $(u - 1)(u - 4) = 0$ $u = 1 \text{ or } u = 4$ $3x - 2 = \pm 1 \text{ or } 3x - 2 = \pm 2$ $x = 1 \text{ or } \frac{1}{3} \text{ or } \frac{4}{3} \text{ or } 0$	<b>M1*</b>  <b>DM1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>  <b>A1</b>	Use the given substitution to obtain a quadratic or factorise into 2 brackets each containing $(3x - 2)^2$ Correct method to solve a quadratic Correct values for $u$ Attempt to square root and rearrange to obtain $x$ <b>OR</b> to expand, rearrange and solve quadratic (at least one) 2 correct values All 4 correct values ( $\frac{0}{3} = \mathbf{A0}$ )	<b>No marks</b> if evidence of “square rooting” e.g. “ $(3x - 2)^2 - 5(3x - 2) + 2$ (or $4$ ) = 0” <b>No marks</b> if straight to quadratic formula to get $x = “1”$ $x = “4”$ and no further working <b>SR 1)</b> If <b>M0</b> Spotted solutions <b>www B1</b> each Justifies 4 solutions exactly <b>B2</b> <b>SR 2)</b> If first 3 marks awarded, spotted solutions 2 correct <b>B1</b> Other 2 correct <b>B1</b> Justifies 4 solutions exactly <b>B1</b> <u>Alternative scheme for candidates who multiply out:</u> Attempt to expand $(3x - 2)^4$ and $(3x - 2)^2$ <b>M1</b> $81x^4 - 216x^3 + 171x^2 - 36x = 0$ <b>A1</b> $x = 0$ a solution or $x$ a factor of the quartic <b>A1</b> Attempt to use factor theorem to factorise their cubic <b>M1*</b> Correct method to solve quadratic <b>DM1</b> All 4 solutions correct <b>A1</b>
5 (i)		<b>M1</b>  <b>A1</b>  <b>2</b>	Negative cubic through $(0, 0)$ (may have max and min)  Must have reasonable rotational symmetry. Cannot be a finite “plot”. Allow negative gradient at origin. Correct curvature at both ends.	Must be continuous. Allow slight curve towards or away from y-axis at one end, but not both.
(ii)	$y = -(x - 3)^3$	<b>M1</b>  <b>A1</b>	$\pm (x - 3)^3$ seen or $y = (3 - x)^3$	Must have “ $y =$ ” for A mark <b>SR</b> $y = -(x - 3)^2$ <b>B1</b>
(iii)	Stretch scale factor 5 parallel to y-axis	<b>B1</b> <b>B1</b>  <b>2</b> <b>6</b>	o.e. e.g. scale factor $\frac{1}{\sqrt[3]{5}}$ parallel to the x axis.	Allow “factor” for “scale factor” For “parallel to the y axis” allow “vertically”, “in the y direction”. <b>Do not accept</b> “in/on/across/up/along the y axis”

<p>6 (i)</p> $y = 5x^{-2} - \frac{1}{4}x^{-1} + x$ $\frac{dy}{dx} = -10x^{-3} + \frac{1}{4}x^{-2} + 1$	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>4</p>	<p><math>x^{-2}</math> used for <math>\frac{1}{x^2}</math> <b>OR</b> <math>x^{-1}</math> used for <math>\frac{1}{x}</math> soi,</p> <p><b>OR</b> <math>x</math> correctly differentiated</p> <p><math>kx^{-3}</math> or <math>kx^{-2}</math> from differentiating</p> <p>Two fully correct terms</p> <p>Completely correct</p>	<p>Look out for:</p> <p><math>y = 5x^{-2} - 4x^{-1} + x</math> followed by</p> <p><math>\frac{dy}{dx} = -10x^{-3} + 4x^{-2} + 1</math> and then the correct answer.</p> <p>This is <b>M1 A1 A1 A0</b></p> <p><math>4x^{-1}</math> is <b>NOT</b> a misread</p>
<p>(ii)</p> $\frac{d^2y}{dx^2} = 30x^{-4} - \frac{1}{2}x^{-3}$	<p>M1</p> <p>A1</p> <p>2</p> <p><span style="border: 1px solid black; padding: 2px;">6</span></p>	<p>Attempt to differentiate their <math>\frac{dy}{dx}</math> (one term correctly differentiated)</p> <p>Completely correct</p>	<p>Allow a sign slip in coefficient for M mark</p> <p><b>NB</b> Only penalise “+ c” first time seen in the question</p>

<p>7 (i) <math>4(x^2 + 3x) - 3</math></p> $= 4\left[\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right] - 3$ $= 4\left(x + \frac{3}{2}\right)^2 - 12$	<p><b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b></p>	<p><math>p = 4</math> <math>q = \frac{3}{2}</math> <math>r = -3 - 4q^2</math> or <math>r = -\frac{3}{4} - q^2</math> <math>r = -12</math> (from <math>q = \pm 1.5</math>)</p>	<p>If <math>p, q, r</math> found correctly, then <b>ISW</b> slips in format.  <math>4(x + 1.5)^2 + 12</math> <b>B1 B1 M0 A0</b>  <math>4(x + 1.5) - 12</math> <b>B1 B1 M1 A1 (BOD)</b>  <math>4(x + 1.5x)^2 - 12</math> <b>B1 B0 M1 A0</b>  <math>4(x^2 + 1.5)^2 - 12</math> <b>B1 B0 M1 A0</b>  <math>4(x - 1.5)^2 - 12</math> <b>B1 B0 M1 A1</b>  <math>4x(x + 1.5)^2 - 12</math> <b>B0 B1M1A1</b></p>
<p>(ii) <math>\frac{-12 \pm \sqrt{12^2 - 4 \times 4 \times -3}}{2 \times 4}</math></p> $= \frac{-12 \pm \sqrt{192}}{8}$ $= \frac{-12 \pm 8\sqrt{3}}{8}$ $= -\frac{3}{2} \pm \sqrt{3}$ <p>OR:</p> $4\left(x + \frac{3}{2}\right)^2 - 12 = 0$ $x + \frac{3}{2} = \pm\sqrt{3}$ $x = -\frac{3}{2} \pm \sqrt{3}$	<p><b>M1</b> <b>A1</b> <b>B1</b> <b>A1</b> <b>M1</b> <b>A1ft</b> <b>A1</b> <b>A1</b></p>	<p>Correct method to solve quadratic</p> $\frac{-12 \pm \sqrt{192}}{8}$ or $\frac{-3 \pm \sqrt{12}}{2}$ $\sqrt{192} = 8\sqrt{3}$ or $\sqrt{12} = 2\sqrt{3}$ from correct $b^2 - 4ac$ $\frac{-3 \pm 2\sqrt{3}}{2}$ or $-\frac{12}{8} \pm \sqrt{3}, -\frac{6}{4} \pm \sqrt{3}$ <p>Must have <math>\pm</math> for method mark  <math>x + 1.5</math> <b>ft</b> <math>x + q</math> from part(i) <b>www</b> in LHS in part (ii)  <math>\pm\sqrt{3}</math>  Do not <b>ISW</b></p>	<p>Not for <math>2(x + q) = \dots</math>  <b>SR</b> One correct root <b>www B1</b></p>
<p>(iii) <math>12^2 - 4 \times 4 \times (-k) = 0</math></p> $144 + 16k = 0$ $k = -9$ <p><b>OR (see next page)</b></p>	<p><b>M1</b> <b>A1</b> <b>A1</b></p>	<p>Attempts <math>b^2 - 4ac = 0</math> or <math>\sqrt{b^2 - 4ac} = 0</math> involving <math>k</math>. If <math>b^2 - 4ac</math> not quoted then expression must be correct.  Correct, unsimplified expression</p>	<p><u>Other alternative methods</u>  a) Attempt to factorise into two equal brackets, (may divide by 4 first – must be correct) <b>M1</b>  Equate coefficient of <math>x</math> to 12 (or 3) <b>A1</b> <math>k = -9</math> <b>A1</b>  b) Uses differentiation to find <math>x</math> ordinate of turning point and uses this to form equation in <math>k</math> <b>M1</b>  Correct equation in <math>k</math> <b>A1</b> <math>k = -9</math> <b>A1</b></p>

7(iii) cont.	$4x^2 + 12x = k$	M1	Attempts completing the square in given equation or factorises to $(2x+3)^2 - 9 = k$	Must involve $k$ in their working to gain the method marks in this scheme
	$4(x + \frac{3}{2})^2 - 9 = k$			
	Equal roots when $x = -\frac{3}{2}$	M1	Substitutes $x = -\frac{3}{2}$	
	$k = -9$	A1	<b>3</b> <b>11</b>	
8 (i)	$\frac{dy}{dx} = 6 - 2x$	M1	Attempt to differentiate $\pm y$	One correct non-zero term
		A1	Correct expression <b>cao</b>	
	When $x = 5$ , $6 - 2x = -4$	M1	Substitute $x = 5$ into their $\frac{dy}{dx}$	
	When $x = 5$ , $y = 12$	B1	Correct $y$ coordinate	
	$y - 12 = -4(x - 5)$	M1	Correct equation of straight line through (5, their $y$ ), their non-zero, numerical gradient	
	$4x + y - 32 = 0$	A1	<b>6</b> Shows rearrangement to correct form	Allow $\frac{y-12}{x-5} =$ their gradient If using $y = mx + c$ must attempt at evaluating $c$ Allow any correct form e.g. $0 = 2y + 8x - 64$ etc.
(ii)	$Q$ is point (8, 0)	B1ft	ft from line in (i)	.
	Midpoint of $PQ = \left(\frac{5+8}{2}, \frac{12+0}{2}\right)$ $= \left(\frac{13}{2}, 6\right)$	M1	Uses $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ o.e. for their P,Q	Do not accept $\left(\frac{13}{2}, \frac{12}{2}\right)$
		A1	<b>3</b>	
(iii)	$6 - 2x = 0$	M1	Solution of their $\frac{dy}{dx} = 0$	Alternatives for Method Mark
	(Line of symmetry is ) $x = 3$	A1	<b>2</b> Allow from $\pm[16 - (x - 3)^2]$ , $\pm [6 - 2x = 0]$	a) attempts completion of square with $\pm(x - 3)^2$ b) attempts to solve quadratic (usual scheme) and to find the mid-point of the two roots c) attempts to use $x = -\frac{b}{2a}$ (allow one sign slip on substitution)
(iv)	$x < 3$	M1	$x <$ their3 or $x >$ their3 <b>OR</b> attempt to solve their $\frac{dy}{dx} > 0$	May solve $\frac{dy}{dx} = 0$ then use $\frac{d^2y}{dx^2} < 0$ implies maximum point for the method mark, or sketch of curve
		A1	<b>2</b> <b>13</b> Allow from $\pm[16 - (x - 3)^2]$ , $\pm [6 - 2x = 0]$ in (iii)	Allow $x \leq 3$

9 (i)	Centre (4, 1)	<b>B1</b>	Correct centre	
	$(x-4)^2 + (y-1)^2 - 16 - 1 - 3 = 0$	<b>M1</b>	Correct method to find $r^2$	$r^2 = (\pm \text{their } 4)^2 + (\pm \text{their } 1)^2 + 3 \text{ soi}$
	$(x-4)^2 + (y-1)^2 = 20$	<b>A1</b>	Correct radius	$\pm \sqrt{20}$ is <b>A0</b> Ignore incorrect simplification of $\sqrt{20}$
	Radius = $\sqrt{20}$	<b>A1</b>	Correct radius	<b>A0</b> Ignore incorrect simplification of $\sqrt{20}$
(ii)	$k = 1 \pm \sqrt{20}$	<b>M1</b>	y ordinate of their centre $\pm$ their radius or	<u>Alternatives for method mark :</u> a) Substitutes $k$ for $y$ and uses $b^2 - 4ac = 0$ to obtain quadratic in $k$ b) Recognises $x = 4$ is equation of normal, substitutes into circle equation and solves for $k$ . <b>SR</b> $k = 1 + \sqrt{20}$ or $k = 1 - \sqrt{20}$ or better <b>www B1</b>
	$k = 1 \pm 2\sqrt{5}$	<b>A1ft</b>	Both correct, unsimplified values	
		<b>A1</b>	<b>cao</b>	
		<b>3</b>		
(iii)	$MT^2 = r^2 - 2^2$	<b>M1</b>	Correct use of Pythagoras' theorem involving MT (or SM)	<b>SR</b> $ST=8$ from particular S and T co-ordinates [e.g. horizontal chord calculated as (0,3) and (8,3)] <b>B1</b> Justifies solution the same for all possible chords <b>B2</b>
	$MT = 4$	<b>A1ft</b>	Correct value of $MT$ for their $r$	
	$ST = 8$	<b>A1</b>	<b>cao</b>	
		<b>3</b>		
(iv)	$x = 2y + 12$	<b>M1*</b>	Attempt to solve equations simultaneously	Must be a clear attempt to reduce to one variable using equation of line and either form of equation of circle. Condone poor algebra for first mark. <u>If y eliminated:</u> $(x-4)^2 + \left(\frac{1}{2}x-7\right)^2 = 20$  Or $x^2 + \left(\frac{1}{2}x-6\right)^2 - 8x - 2\left(\frac{1}{2}x-6\right) - 3 = 0$  Leading to $x^2 - 12x + 36 = 0$
	$(2y+8)^2 + (y-1)^2 = 20$	<b>A1</b>	Correct unsimplified expression, may be	
	$4y^2 + 32y + 64 + y^2 - 2y + 1 = 20$		$(12+2y)^2 + y^2 - 8(12+2y) - 2y - 3 = 0$	
	$5y^2 + 30y + 45 = 0$	<b>A1</b>	Obtain correct 3 term quadratic	
	$y^2 + 6y + 9 = 0$			
	$(y+3)^2 = 0$	<b>DM1</b>	Correct method to solve quadratic of form $ax^2 + bx + c = 0$ ( $b \neq 0$ )	
	$y = -3$	<b>A1</b>	y value correct, no extra solutions	
	$x = 6$	<b>A1</b>	x value correct <b>ISW</b>	
	<b>OR</b>			
	$y-1 = -2(x-4)$	<b>M1</b>	Attempt to find equation of radius/normal	
	<b>A1</b>	Correct equation		
	Solve simultaneously with $y = \frac{1}{2}x - 6$	<b>M1</b>		
	$x = 6$	<b>A1</b>		
	$y = -3$	<b>A1</b>		
	States line is tangent as meets at one point or verifies (6, -3) lies on circle	<b>B1</b>	Allow showing distance between (6,-3) and (4,1) = $\sqrt{20}$	<b>SR</b> Correct coordinates spotted or from trial and improvement <b>www B2</b>



Allocation of method mark for solving a quadratic

e.g.  $4x^2 + 12x - 3 = 0$

By factorisation

– when expanded, quadratic term and one other term must be correct (with correct sign):

$(2x+1)(2x-3) = 0$

M1  $4x^2$  and  $-3$  obtained from expansion

$(4x+4)(x+2) = 0$

M1  $4x^2$  and  $+12x$  obtained from expansion

$(4x-1)(x-3) = 0$

M0 only  $x^2$  term correctBy formula

- if the formula is quoted correctly first, allow one sign slip in substituting values into it:

$a = 4, b = 12, c = -3$

$$\frac{12 \pm \sqrt{(12)^2 - 4 \times 4 \times -3}}{8}$$

gains M1 (minus sign incorrect at start of formula)

$$\frac{-12 \pm \sqrt{(12)^2 - 4 \times 4 \times 3}}{8}$$

gains M1 (3 for  $c$  instead of  $-3$ )

$$\frac{12 \pm \sqrt{(12)^2 - 4 \times 4 \times 3}}{8}$$

M0 (2 sign errors: initial sign and  $c$  incorrect)

- if the formula is not quoted, then no errors at all are allowed in substitution.

By completing the square

$$4x^2 + 12x - 3 = 0$$

$$4 \left[ \left( x + \frac{3}{2} \right)^2 - \frac{9}{4} \right] - 3 = 0$$

$$\left( x + \frac{3}{2} \right)^2 = 3$$

$$x + \frac{3}{2} = \pm \sqrt{3}$$

The method mark is awarded only at the last line of working  
i.e. when  $\pm\sqrt{\text{combined constants}}$  is seen.

N.B. The value of the combined constants does not have to be correct for the M1 mark

Condone “invisible brackets” if justified by correct later working

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