4754 (C4) Applications of Advanced Mathematics

1		$\frac{1+2x}{(1-2x)^2}=(1+2x)(1-2x)^{-2}$ $=(1+2x)[1+(-2)(-2x)+\frac{(-2)(-3)}{1.2}(-2x)^2+...]$ $=(1+2x)[1+4x+12x^2+...]$ $=1+4x+12x^2+2x+8x^2+...$ $=1+6x+20x^2+...$ Valid for $-1<-2x<1$ $\Rightarrow\ -\frac{1}{2}<x<\frac{1}{2}$	M1 A1 A1 M1 A1 M1 A1 **[7]**	binomial expansion power -2 unsimplified,correct sufficient terms
2		$\tan 2\theta=\frac{2\tan\theta}{1-\tan^2\theta}$ $\Rightarrow\ \cot 2\theta=\frac{1}{\tan 2\theta}=\frac{1-\tan^2\theta}{2\tan\theta}$ * $\cot 2\theta = 1+\tan\theta$ $\Rightarrow\ \frac{1-\tan^2\theta}{2\tan\theta}=1+\tan\theta$ $\Rightarrow\ 1-\tan^2\theta=2\tan\theta+2\tan^2\theta$ $\Rightarrow\ 3\tan^2\theta+2\tan\theta-1=0$ $\Rightarrow\ (3\tan\theta-1)(\tan\theta+1)=0$ $\Rightarrow\ \tan\theta=1/3,\ \theta=18.43°,\ 198.43°$ or $\tan\theta=-1,\ \theta=135°,\ 315°$	M1 E1 M1 M1 A3,2,1,0 **[7]**	oe eg converting either side into a one line fraction(s) involving sin θ and cos θ. quadratic = 0 factorising or solving 18.43°, 198.43°, 135°, 315° –1 extra solutions in the range
3	(i)	$\frac{dy}{dt}=\frac{(1+t).2-2t.1}{(1+t)^2}=\frac{2}{(1+t)^2}$ $\frac{dx}{dt}=2e^{2t}$ $\frac{dy}{dx}=\frac{dy/dt}{dx/dt}$ $\Rightarrow\ \frac{dy}{dx}=\frac{\frac{2}{(1+t)^2}}{2e^{2t}}=\frac{1}{e^{2t}(1+t)^2}$ $t=0\Rightarrow dy/dx=1$	M1A1 B1 M1 A1 B1ft **[6]**	

	(ii)	$2t = \ln x \Rightarrow t = ½ \ln x$ $\Rightarrow \quad y = \dfrac{\ln x}{1+\frac{1}{2}\ln x} = \dfrac{2\ln x}{2+\ln x}$	M1 A1 **[2]**	or t in terms of y
4	**(i)**	$\overrightarrow{AB} = \begin{pmatrix}-2\\-1\\-1\end{pmatrix}, \overrightarrow{AC} = \begin{pmatrix}-1\\-11\\3\end{pmatrix}$	B1 B1 **[2]**	
	(ii)	$\mathbf{n}.\overrightarrow{AB} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-2\\-1\\-1\end{pmatrix} = -4+1+3=0$ $\mathbf{n}.\overrightarrow{AC} = \begin{pmatrix}2\\-1\\-3\end{pmatrix}.\begin{pmatrix}-1\\-11\\3\end{pmatrix} = -2+11-9=0$ $\Rightarrow$ plane is $2x - y - 3z = d$ $x = 1, y = 3, z = -2 \Rightarrow d = 2 - 3 + 6 = 5$ $\Rightarrow$ plane is $2x - y - 3z = 5$	M1 E1 E1 M1 A1 **[5]**	scalar product
5	**(i)**	$x = -5 + 3\lambda = 1 \Rightarrow \lambda = 2$ $y = 3 + 2 \times 0 = 3$ $z = 4 - 2 = 2$, so (1, 3, 2) lies on 1st line. $x = -1 + 2\mu = 1 \Rightarrow \mu = 1$ $y = 4 - 1 = 3$ $z = 2 + 0 = 2$, so (1, 3, 2) lies on 2^{nd} line.	M1 E1 E1 **[3]**	finding λ or μ verifying two other coordinates for line 1 verifying two other coordinates for line 2
	(ii)	Angle between $\begin{pmatrix}3\\0\\-1\end{pmatrix}$ and $\begin{pmatrix}2\\-1\\0\end{pmatrix}$ $\cos\theta = \dfrac{3\times2+0\times(-1)+(-1)\times0}{\sqrt{10}\sqrt{5}}$ $= 0.8485\ldots$ $\Rightarrow \quad \theta = 31.9°$	M1 M1 A1 A1 **[4]**	direction vectors only allow M1 for any vectors or 0.558 radians

6	(i)	BAC = 120 – 90 – (90 – θ) = θ – 60 ⇒ BC = b sin(θ – 60) CD = AE = a sin θ ⇒ h = BC + CD = a sin θ + b sin (θ – 60°) *	B1 M1 E1 **[3]**	
	(ii)	$h = a \sin \theta + b \sin(\theta - 60°)$ $= a \sin \theta + b(\sin\theta \cos 60 - \cos\theta \sin 60)$ $= a \sin \theta + ½\, b \sin \theta - \sqrt{3}/2\, b \cos \theta$ $= (a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta$ *	M1 M1 E1 **[3]**	corr compound angle formula sin 60 = √3/2, cos 60 = ½ used
	(iii)	OB horizontal when h = 0 ⇒ $(a + \frac{1}{2}b)\sin\theta - \frac{\sqrt{3}}{2}b\cos\theta = 0$ ⇒ $(a + \frac{1}{2}b)\sin\theta = \frac{\sqrt{3}}{2}b\cos\theta$ ⇒ $\frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}b}{a + \frac{1}{2}b}$ ⇒ $\tan\theta = \frac{\sqrt{3}b}{2a+b}$ *	M1 M1 E1 **[3]**	$\frac{\sin\theta}{\cos\theta} = \tan\theta$
	(iv)	$2\sin\theta - \sqrt{3}\cos\theta = R\sin(\theta - \alpha)$ $= R(\sin\theta\cos\alpha - \cos\theta\sin\alpha)$ ⇒ $R \cos \alpha = 2$, $R \sin \alpha = \sqrt{3}$ ⇒ $R^2 = 2^2 + (\sqrt{3})^2 = 7$, $R = \sqrt{7} = 2.646$ m $\tan \alpha = \sqrt{3}/2$, $\alpha = 40.9°$ So $h = \sqrt{7}\sin(\theta - 40.9°)$ ⇒ $h_{max} = \sqrt{7} = 2.646$ m when $\theta - 40.9° = 90°$ ⇒ $\theta = 130.9°$	M1 B1 M1A1 B1ft M1 A1 **[7]**	

7	**(i)**	$\frac{dx}{dt} = -1(1+e^{-t})^{-2}.-e^{-t}$ $= \frac{e^{-t}}{(1+e^{-t})^2}$ $1-x = 1-\frac{1}{1+e^{-t}}$ $1-x = \frac{1+e^{-t}-1}{1+e^{-t}} = \frac{e^{-t}}{1+e^{-t}}$ $\Rightarrow\ x(1-x) = \frac{1}{1+e^{-t}}\frac{e^{-t}}{1+e^{-t}} = \frac{e^{-t}}{(1+e^{-t})^2}$ $\Rightarrow\ \frac{dx}{dt} = x(1-x)$ When $t = 0$, $x = \frac{1}{1+e^0} = 0.5$	M1 A1 M1 A1 E1 B1 **[6]**	chain rule substituting for $x(1-x)$ $1-x = \frac{1+e^{-t}-1}{1+e^{-t}} = \frac{e^{-t}}{1+e^{-t}}$ [OR,M1 A1 for solving differential equation for *t,* B1 use of initial condition, M1 A1 making x the subject, E1 required form]
	(ii)	$\frac{1}{(1+e^{-t})} = \frac{3}{4}$ $\Rightarrow\ e^{-t} = 1/3$ $\Rightarrow\ t = -\ln 1/3 = 1.10$ years	M1 M1 A1 **[3]**	 correct log rules
	(iii)	$\frac{1}{x^2(1-x)} = \frac{A}{x^2}+\frac{B}{x}+\frac{C}{1-x}$ $\Rightarrow\ 1 = A(1-x) + Bx(1-x) + Cx^2$ $x = 0 \Rightarrow A = 1$ $x = 1 \Rightarrow C = 1$ coefft of x^2: $0 = -B + C \Rightarrow B = 1$	 M1 M1 B(2,1,0) **[4]**	 clearing fractions substituting or equating coeffs for A,B or C $A = 1$, $B = 1$, $C = 1$ www
	(iv)	$\int \frac{dx}{x^2(1-x)}dx = \int dt$ $\Rightarrow\ t = \int(\frac{1}{x^2}+\frac{1}{x}+\frac{1}{1-x})dx$ $= -1/x + \ln x - \ln(1-x) + c$ When $t = 0$, $x = ½ \Rightarrow 0 = -2 + \ln ½ - \ln ½ + c$ $\Rightarrow\ c = 2.$ $\Rightarrow\ t = -1/x + \ln x - \ln(1-x) + 2$ $= 2+\ln\frac{x}{1-x}-\frac{1}{x}$*	M1 B1 B1 M1 E1 **[5]**	separating variables $-1/x + \ldots$ $\ln x - \ln(1-x)$ ft their A,B,C substituting initial conditions
	(v)	$t = 2+\ln\frac{3/4}{1-3/4}-\frac{1}{3/4} = \ln 3+\frac{2}{3} = 1.77$ yrs	M1A1 **[2]**	

1	15	B1	
2	THE MATHEMATICIAN	B1	
3	M H X I Q 3 or 4 correct – award 1 mark	B2	
4	Two from Ciphertext N has high frequency E would then correspond to ciphertext R which also has high frequency T would then correspond to ciphertext G which also has high frequency A is preceded by a string of six letters displaying low frequency	 B1 B1	 oe oe
5	The length of the keyword is a factor of both 84 and 40. The <u>only</u> common factors of 84 and 40 are 1,2 and 4 (and a keyword of length 1 can be dismissed in this context)	M1 E1	
6	Longer strings to analyse so letter frequency more transparent. Or there are fewer 2-letter keywords to check	B2	
7	OQH DRR EBG One or two accurate – award 1 mark	B2	
8 (i) **(ii)** **(iii)**	Evidence of intermediate H FACE Evidence of intermediate HCEG – award 2 marks Evidence of accurate application of one of the two decoding ciphers - award 1 mark $800 = (3\times266)+2$; the second row gives T so plaintext is R	B1 B3 M1 A1	 Use of second row